PDHonline Course C662 (4 PDH)

Stairs & Stairways

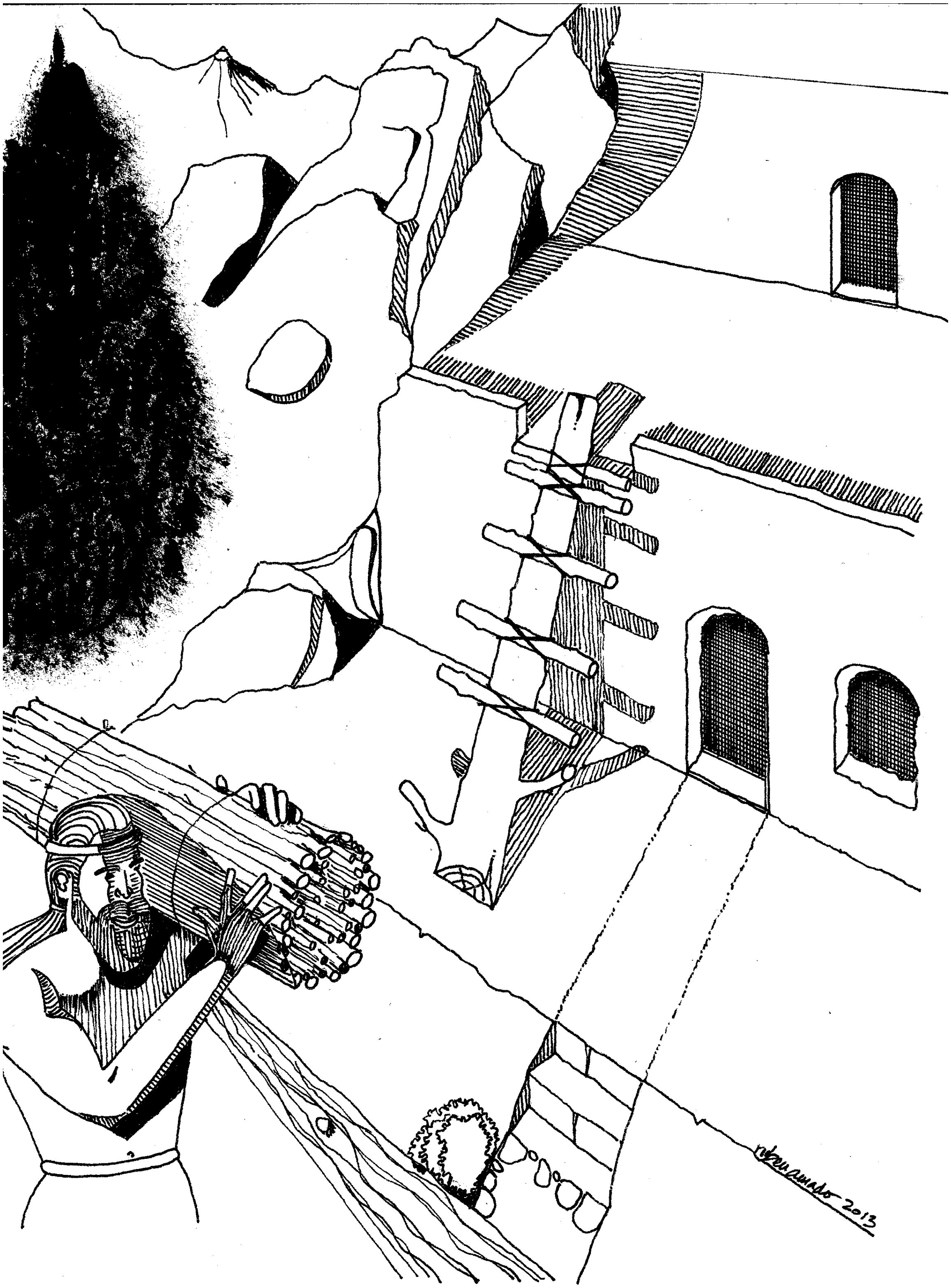
Instructor: Ruben A. Gomez, PE

2020

PDH Online | PDH Center

5272 Meadow Estates Drive
Fairfax, VA 22030-6658
Phone: 703-988-0088
www.PDHonline.com

An Approved Continuing Education Provider



1.0 PROLOGUE

The old civilizations with enclaves in the mountainous areas of Europe, Asia, South and North America, such as the tribes of Catal Huyuk in today's Turkey or the Incas of Machu Pichu in Los Andes, found the need to carve steps in the mountain rocks to facilitate escalation through the rough terrain.

In building construction, stairs became a necessity the moment civilized man decided to elevate the house floor above the surrounding ground as in the Sumerian Ziggurats or the Harapan cities of the Indus Valley. He first started with a single stepping stone and kept on adding up as the need arose, and little by little we ended up with the stairs and stairways we have all around us today.

Stairs come in all kinds, shapes, configurations and materials. In this course we will cover all the basic information that is needed to know about stairs and be successful in planning an efficient, comfortable and reasonably safe set of stairs.

When you plan your building layout and start calculating your floor to floor heights by adding the required finished floor to ceiling space, as well as the interstitial space plus the structural thickness allowances so to arrive to the "magical figure". While in that process you need to keep in mind the stairway(s) configuration and details, for you do not want to end up with risers with fractional dimensions ending in 1/16ths or 1/32nds of an inch. Your draftsmen, builders, stair fabricators and carpenters coming after you will be appreciative of your anticipation.

In the "old times" a 7" x 10" combination of riser and tread was considered ideal, but as time passed and experience accumulated, the practitioners figured that in order to accommodate the shoe size found in 95% of the human adult population, an 11" tread was a more convenient dimension, therefore, in these "new modern times" you will hear that the most comfortable riser/tread combination should be as close as possible to 7" x 11", which the reader will find often as being used in the pages of this course.

Another important factor which should be kept in mind is the workings of the human mind. As a person ascends or descends (particularly the latter) a stairway, he (or she) develops an involuntary cadence where there is a mindset based on an unconscious expectation of uniformity when it comes to the size of the steps. Such a cadence is particularly important to help maintain the person's balance, efficiency and a feeling of safety. Therefore, any break in the uniformity that sponsors disruption of said cadence may provoke missteps, unwanted accidents and therefore, should be avoided at all cost.

That is why when a forensic engineer is first confronted with a stair accident, he will pull his tape measure and look for inconsistencies in riser dimensions and unevenness, before he continues on the pursuit of other factors such as: stair width, configuration, tread depth, friction factors, condition of the surface, code compliance and so on.

2.0 OLD CODES' FORMULAE

Old building codes were very eager to furnish "fit all sizes" formulas that worked well for as long as the riser/tread configuration was kept within the proximity of the 7 x 10 proportion. However, when either the riser or the tread were taken to the outer edges of those limits, the resulting stairs were borderline cases of absurdity and were close to impossible to maintain in compliance with the principle of continuity and "mental cadence" mentioned in the prologue above.

Below are the most known and used of those rules and formulas. Please bear in mind that any of the tread dimensions resulting from their application were exclusive of nosing size:

#1- "The sum of tread and riser shall not exceed a total of seventeen-and-a-half inches": T + R = 17½".

While by using a 6½" riser and 11" tread would sponsor developing proper "cadence", a combination of 3½" riser and a 14" tread would not.

We have plotted some of the parameters to better illustrate the case on Figure 2.1 and as you may observe, by holding the formula to its extreme application of T + R = 17½, the selection of a 7 inch riser results in a tread of 10½ inches, a little short of what is now considered the ideal tread.

#2- "The product of tread and riser shall not exceed 75: T x R = 75". Here again, by holding the formula to its extreme case of T x R = 75 and assuming a riser of 7 in. we ended up with a tread of 10.71 inches, and by lowering the standard to T x R = 70 we obtained a tread of 10 inches, as by the old standards.

#3- Later on, the above formula was modified so as to maintain the results within more reasonable limits:

T x R = 75, but never less than 70

Which is in effect what we did as part of the chart shown on Figure 2.2.

#4- The fourth rule known to some as the *hyperbolic rule* used to claim that "When R is fixed, T can be obtained from the hyperbola formula":

$$\{T = 5 + \sqrt{[1/7\,(9-R)^2 + 9]}\}^*$$

The use of such formula did not bring any better results, if any, than those of the above rules.

---

*This formula was reproduced from a 1940 New York City Code. We suspected the distinct possibility of a typographical error and tried adjusting the first constant (5) by replacing it for several numerals and found that by using 8 instead, the results were more reasonable.

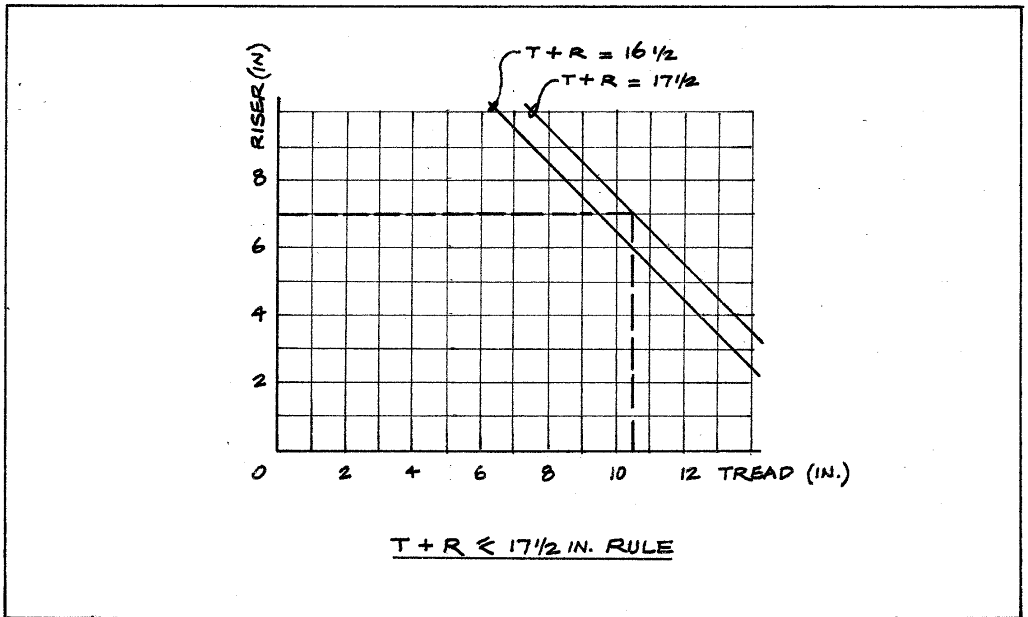
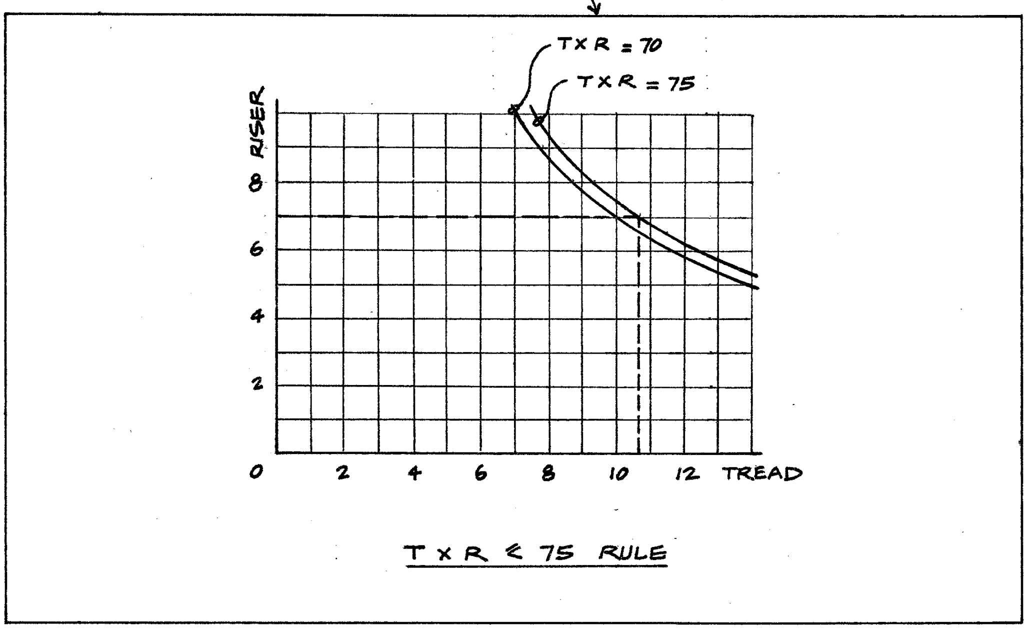


FIGURE 2.1

FIGURE 2.2



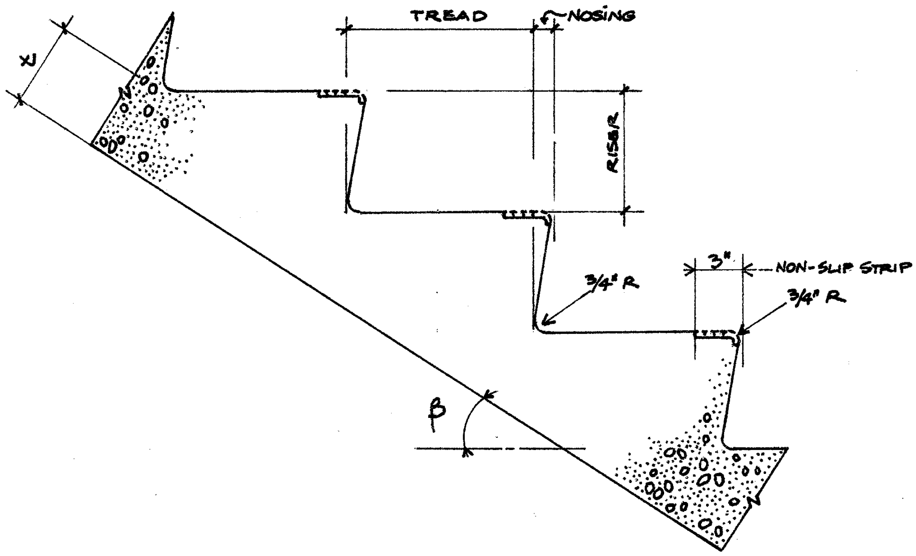
3.0 NEW CODES' APPROACH

In recent times, both the 2006 International Building Code and the 2007 Florida Building Code have abandoned the old formulas and have come to agree in their Section 1009 where they both read in exactly the same manner:

"Stair riser heights shall be 7 inches maximum and 4 inches minimum. Stair tread depths shall be 11 inches minimum. The riser height shall be measured vertically between the leading edges of adjacent treads. The tread depth shall be measured horizontally between the vertical planes of the foremost projection of adjacent treads and at a right angle to the tread's leading edge."

On Figure 3.1 it is shown the ideal configuration and proportions of a desirable stair step as defined by the codes. The nosing is not mandatory, but it is strongly recommended and while any less than ¾ in. would be ineffective, more than 1¼ in. is not only prohibited but right down cumbersome for the user.

Still on the same Figure 3.1, when the riser R = 7 in. and the tread T = 11 in., then the tangent of angle β would be 0.63636 and the actual angle close to 32° 30′.



TYP. STEP DETAIL

FIGURE 3.1

4.0 FINDING THE BEST RISER

Although the 7 in. riser will not always be attainable, you should try to be as close to it as possible. In order to do that you should closely prepare or examine (if you already have one) the building typical floor section showing all the components that have any incidence in the vertical dimensions. For that purpose and as a matter of illustration, we have prepared a generic section which appears below as Figure 4.1.

You will notice that such section has three parts that play important and significant roles in the determination of the finished-floor-to-finished-floor height:

#1- The Headroom (or overhead clearance), which is the distance from the finished floor to the ceiling. Such dimension is determined by use, for instance, in a residential building it has a minimum of 8′-1″, as conceived by a standard sheet of drywall plus the necessary tolerance to install it without cutting.

#2- The Interstitial Space, the distance from the ceiling to the structure. In this space there must be adequate room to place the air conditioning ductwork, crossing plumbing pipes, fire sprinklers and electrical lighting fixtures.

#3- The Structural Allowance, the space dedicated to the supporting structure. Such space shall also include the floor finish above, as a matter of completeness. On the figure we have arbitrarily assumed a structural framework consisting of a flat plate, however, it could be any other type of solution, such as concrete joists, steel framing or what have you.

Once the total height is established, the design professional may proceed to the determination of the adequate riser by a few rounds of trial and error. The enclosed Table IV.1 may help to make the calculations easier and the selection faster.

Table IV.1

Floor to Floor Height (ft.)	**Number of Risers**	**Riser "R" (in.)**	**Tread "T" (in.)**	**Total Straight Run (ft.)**
8′-7½″ (103½″)	14	7-3/8	10	10-10
Do.	15	6-7/8	11	12-10
9′-0″ (108″)	16	6-3/4	11	13-9
9′-6″ (114″)	16	7-1/8	10½	13-1½
10′-0″ (120″)	16	7½	10	12-6
10′-6″ (126″)	18	7	11	15-7
11′-1″ (132″)	19	7	11	17-5

The finished floor to finished floor height of 8′-7½″ shown on the first row of the table is very tight and would only work for residential buildings where the typical apartment layout either does not call for air conditioning, or it is so that the kitchen and bathrooms are both contiguous and permit a dropped ceiling down to a headroom of 7′-0″. That condition would barely make room for the air conditioning main supply duct and the secondary duct distribution serving the different rooms in the dwelling.

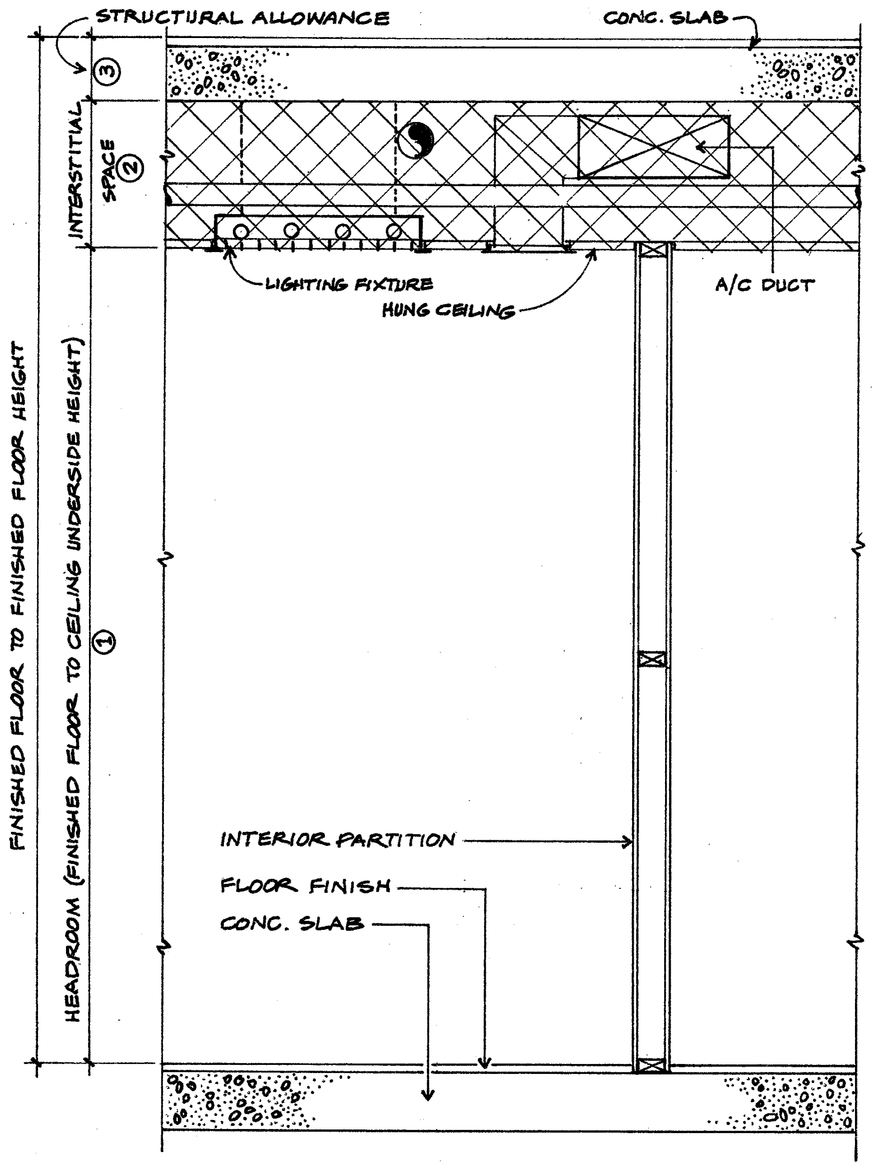

STRUCTURAL ALLOWANCE
CONC. SLAB
3
INTERSTITIAL SPACE 2
LIGHTING FIXTURE
HUNG CEILING
A/C DUCT
FINISHED FLOOR TO FINISHED FLOOR HEIGHT
HEADROOM (FINISHED FLOOR TO CEILING UNDERSIDE HEIGHT)
1
INTERIOR PARTITION
FLOOR FINISH
CONC. SLAB

FIGURE 4.1

5.0 THE IDEAL STAIRWAY

Since stairways are the most likely places to stage "trip & fall" and "slip & fall" accidents, every possible provision and precaution shall be taken to minimize such an occurrence. Therefore, it should be an unequivocal duty of the designer to conceive stairs as close as possible to the ideal image of safety.

With that thought in mind, we have put together a collection of those basic conditions and features which are conducive to creating the ideal stair:

#1- Use the correct rise-to-tread proportion. Try to get as close as practical to the 7/11 ratio.

#2- Make sure that the *cadence flow* is kept uninterrupted through the entire stairway.

#3- Avoid winders, for they are not only cadence interrupters but also accident prone.

#4- Maintain adequate and constant stair width, those widths are code regulated and based on occupant load. Although code's widths are generally adequate in high-rise buildings where stairs are mostly used just for emergency egress since elevators take care of the daily volume of vertical traffic, however in rental walk-ups where all the foot traffic is done through the stairways with the added constant furniture moving in and out, and especially when railings and newels are encroaching into the actual stair width, we recommend the use of a minimum width of four feet (4'- 0").

#5- The stair structure must be sturdy, secure, and absent of vibrations, shakings, swaying or undue deflections.

#7- Landings shall be spaced in such a way that they do not create breaks in cadence.

#8- Tread and landing surfaces shall be finished with a non-slip material and having a minimum *static coefficient of friction* of 0.60. The coefficient of friction between two materials μ, represents the ratio of the force F required to move one over the other, to the total force W pressing the two together, thus:

$$\mu = F/W$$

This knowledge is particularly important and relevant in the practice and application of forensic engineering.

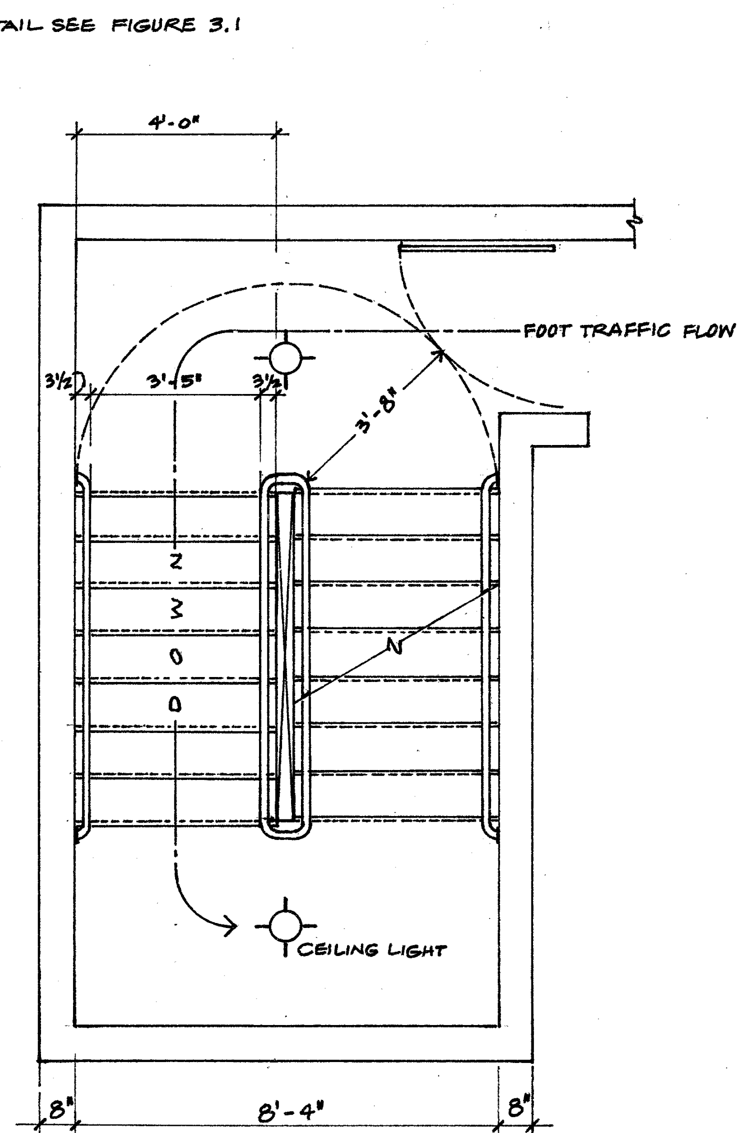
#9- Provide adequate railings made of the proper material, height and design. They are code regulated, therefore, follow those requirements.

#10- Riser and tread dimensions must be of strict uniformity and with a dimensional tolerance not to exceed one-eighth of an inch (⅛") between two consecutive steps.

#11- Staircases should be provided with adequate illumination in compliance with the requirements of the National Electric Code (NEC).

#12- The stair framing shall be built of a non-combustible material and be enclosed within a non-combustible confinement.

Figure 5.1 depicts a plan view of a traditional stairway of comparable standard safety features as what the reader is likely familiar with. Later on the reader will have the opportunity to compare this design against those less conventional stairs shown on the chapters and sections coming ahead in the development of this course.



CONVENTIONAL STAIRWAY PLAN

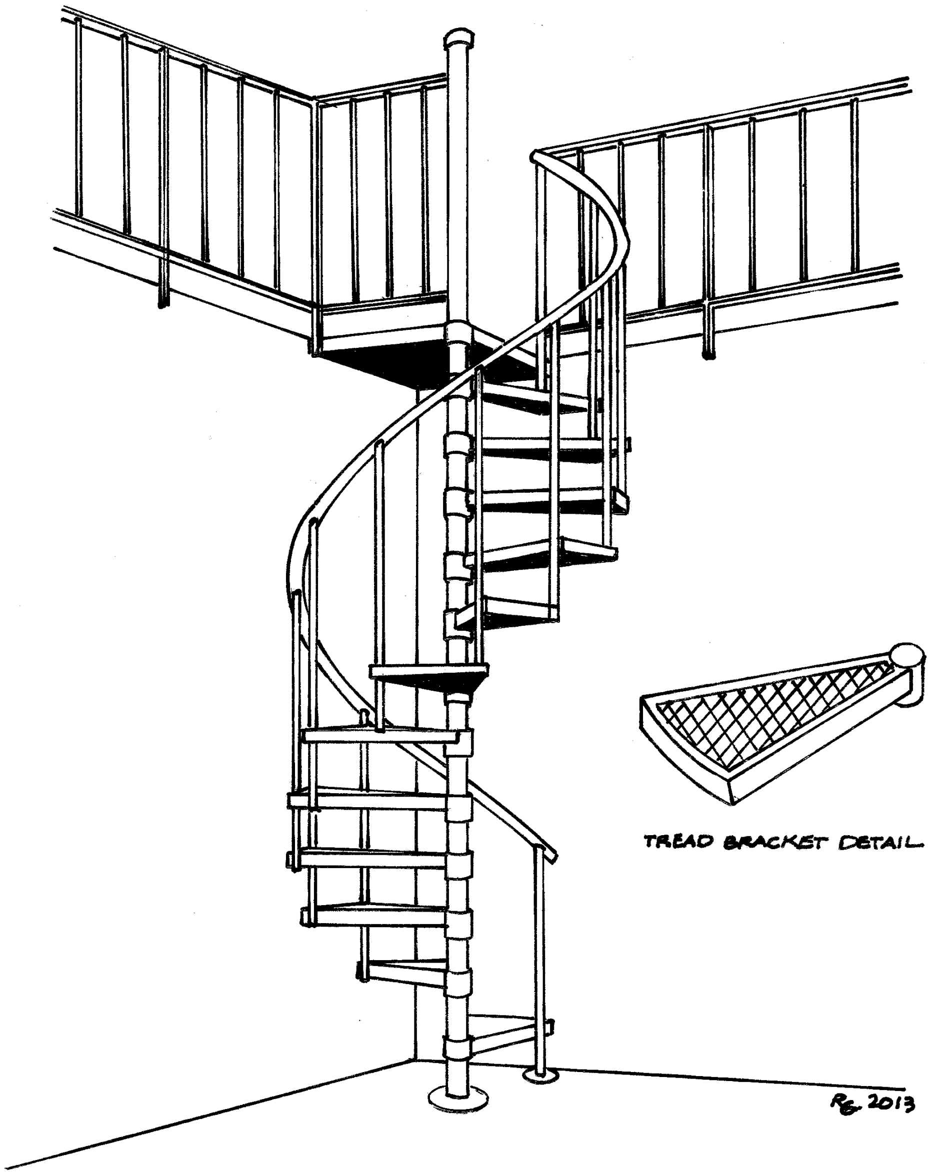
FIGURE 5.1

6.0 SPIRAL STAIRS

Although they are not part of a group that we could designate as safe stairs nor are they even comfortable. Moreover, spiral stairs have a much higher risk for falls and injuries than those stairways constructed with straight runs, particularly in emergency situations when the rate of foot travel needs to be increased. However, they are quite different, exotic, somewhat elegant and sophisticated enough to attract the attention of many homeowners and decorators. They are also space savers and therefore become handy for two level apartments, townhomes, or to access lofts, attics and bonus rooms. Nevertheless, they should never be intended to take the place of a straight main stairway.

Spiral stairs are available in many styles and materials, such as hardwoods, steel, cast iron, brass, cast aluminum and even bronze. The user may order them all assembled and welded together, which is the better option, or in a kit form by components to be assembled at the site.

Spiral stairs have no solid risers and are basically bracketed treads projecting out of a central supporting pole. These stairs are designed in such a way that the dead load is balanced around the pole and any unbalance brought up by the moving live loads is counteracted by the bolted connections at the lower floor and the upper landing. Figure 6.1 depicts a regular spiral stair servicing a loft and located somewhere in a convenient corner of a living room, den or family room.



SPIRAL STAIR
FIGURE 6.1

7.0 ORTHOPOLYGONAL STAIRS

Most of us have in our garages those pull-down ladders for the purpose of climbing in our attics and retrieve some of those dusty items we have not seen for years and had almost been forgotten. As we climb over the ladder it shakes, crackles, sags and although they usually do not fall, one gets the feeling of uncertainty and insecurity.

This author climbed for the first time an orthopolygonal stair in 1958 while visiting the University of Havana. It felt the same way as the pull-down ladder of the analogy above, there was that same psychological feeling of insecurity as if he was "walking on air" and the added perception that something was missing under his feet. In fact, something was missing indeed: a concrete slab underneath. The following Figure 7.1 depicts a stair with a configuration very similar to the one described in the story.

At the time, Cuban engineers called this type of stair: "escalera sin losa" as some English speakers would call it in their own same way and for the same reasons: "slab-less stair".

Admittedly, they are rare, intriguing and even mind boggling to many. Unfortunately, just from the structural design point of view, the calculations could turn to be a bit pesky and convoluted, for internally the structure has a high degree of redundancy.

Normally, an experienced design engineer would approach the problem by first examining the structure and look for conditions that may simplify his calculations by taking advantage of them to the maximum. To illustrate that point, let us assume a design assignment of an orthopolygonal stair flight with the following characteristics:

Number of treads: 13
Tread depth: 11 in.
Number of risers: 12
Riser height: 7 in.
Overall stair width: 3'- 8" (44 inches)
Material: Reinforced Concrete (f_c' = 4,000 PSI; f_s' = 60,000 PSI)

Figure 7.2(a) gives us a schematic view of the stair flight in question. The stair is loaded with a set of loads P applied at the centroid of the risers, and it has been assumed fixed at the supports where end moments are generated accordingly. The end reactions at the supports (R_A and R_Z) equal to 6P. By mere inspection we can start to simplify the process to our advantage by following these steps:

#1- Since the stair is going to be built homogeneously and out of the same material: reinforced concrete. Therefore, for being a constant factor, the modulus of elasticity (E) can be either cancelled out or reduced down to a value of 1.0.

#2- For the same reason, the moment of inertia (I) of the different components can be minimized to a proportionality factor. Therefore, the feared (EI) term is then reduced to a small and inoffensive fractional value of less than 2.0 as we will see ahead.

#3- Although in principle gravity loads (DL + LL) are uniformly distributed loads, there is a valid simplification that would help and in the end will not make that much of a difference in the results. Such shortcut consists of converting all distributed loads into concentrated loads located at the centroid of the risers as indicated above. Since the treads are all equal, the span from riser to riser (center to center) is a constant 11 in., thus, making all concentrated loads equally spaced along the length of the stair's flight.

#4- Another valid shortcut: since every flight of stairs is symmetrical in two ways; first, about a diagonal axis passing by its geometrical center, thus, reducing the number of redundants to half and therefore, allowing the design engineer to cut his computation work in half; second, the same symmetry is also found along the stair flight's own orthopolygonal axis.

#5- Once the concentrated loads are determined, the next step would be to figure out the fixed-end-moments (FEM's). By taking the first member of stair flight: A-B, the end moment would be:

$$-M_a = M_{afem} + 2EI/L\ (2\theta_a + \theta_b - 3\Delta/L)$$

Where:
M_a = end moment at joint A
M_{afem} = fixed end moment at joint A
E = modulus of elasticity
I = moment of inertia
L = span (11 in.)
θ_a = rotational angle at joint A
θ_b = rotational angle at joint B
Δ = relative deflection at the risers

As it has been found in common construction practice that it makes a world of difference if the stair flights are prefabricated (poured on top of each other to save forming and therefore intense labor), finished in a precast yard and then shipped to the site for erection. In such case, the moment M_{afem} would be reduced if not eliminated altogether, depending entirely on the connection method utilized between the precast stair flight and the supporting spandrel beam.

#6- In order to simplify the process even further, in the analysis we recommend the use of the *analogous column method* for the determination of the design stresses. Because of symmetry, the centroid of the analogous column can be found with ease. Furthermore, since the stair is comprised of a repetitive number of identical treads and risers of constant equal dimensions, they become a constant factor in the operative.

#7- In order to facilitate the conception of the elastically deflected structure, we will assume the acceptable premise *that the corner joints are originally square (90 deg) and that they shall remain square after deformation.* Please examine Figure 7.2(b) and bear in mind that the deflections have been largely exaggerated to give a better idea of the *elastic line.*

Now we will proceed with the example:

Since the failure mechanism of this stair suggests that collapse would take place following the tendency of the steps to straighten themselves out flat, therefore the corner joint stiffness is paramount in the safe behavior of the structure. For this reason and also for the purpose of allowing more space for the placing of the reinforcing steel, the riser's thickness has been increased from 4 to 5 inches as shown back in Figure 7.1. The added concrete volume is very small as the reader can see from the computations below.

Unit concrete weight taken as: 150 lbs/cubic foot
Live Load: 100 lbs/square foot

<u>Dead Loads</u>

Treads: 0.33′ x 1.33′ x 3.67′ x 150 =	242 lbs.
Risers: 0.42′ x 0.25′ x 3.67′ x 150 =	58 "
Total DL:	300 lbs

<u>Live Loads</u>

Treads: 0.92′ x 3.67′ x 100 =	338 lbs
(DL+ LL) = P =	638 lbs

Modulus of Elasticity: E = 1.0

Relative Moment of Inertia:

Treads: I_t = 1/12 (44 x 4^3) = 235 (1.0 in relative terms)

Risers: I_r = 1/12 (44 x 5^3) = 458 (1.95 in relative terms)

Therefore,

I_r/I_t = 458/235 = 1.95, also: (EI) = 1.0 x 1.95 = 1.95

As the resultant load is applied to the centroid of the analogous column, we obtain the expression:

$$M_x = M_i - W_a/A$$

Where:
M_x = bending moment at any given point "x" of the structure
M_i = isostatic moment at point "x"
W_a = total load on the analogous column
A = area of the analogous column.

Now referring to Figure 7.3 where we have made some progress in our abbreviated calculations by using the analogy of an idealized simple supported stair string and the determination of the main moments needed for member sizing.

Analogous Column Loads:

$W_t = \delta_{mt}/EI = 14{,}052{,}358/1.0 = 14{,}052{,}358$

$W_r = \delta_{mr}/EI = 8{,}940{,}932/1.95 = 4{,}585{,}093$

$W_a = W_t + W_r = 18{,}637{,}451$

Analogous Column Areas:

Since: n = 13 and n -1 = 12, then

$A_t = (n)\ (I_t)\ (L_t) = 13 \times 1.0 \times 11 = 143$
$A_r = (n - 1)\ L_r/I_r = 12(7/1.95) = 43$

Thus,

$$A = 186$$

Since the moment at any point of the string is derived from the equation:

$$M_x = M_i - W_a/A$$

Considering that at the supports the isostatic moment $M_i = 0$, the resultant negative moment at both ends would then be:

$$-M = 0 - W_a/A = 18{,}637{,}451/186 = 100{,}201 \text{ lb-in} = 100.2 \text{ kip-in}$$

The positive moment at the center of the span can be deducted the same way:

$$+M = 100{,}201 - 13{,}398 = 86.8 \text{ kip-in}$$

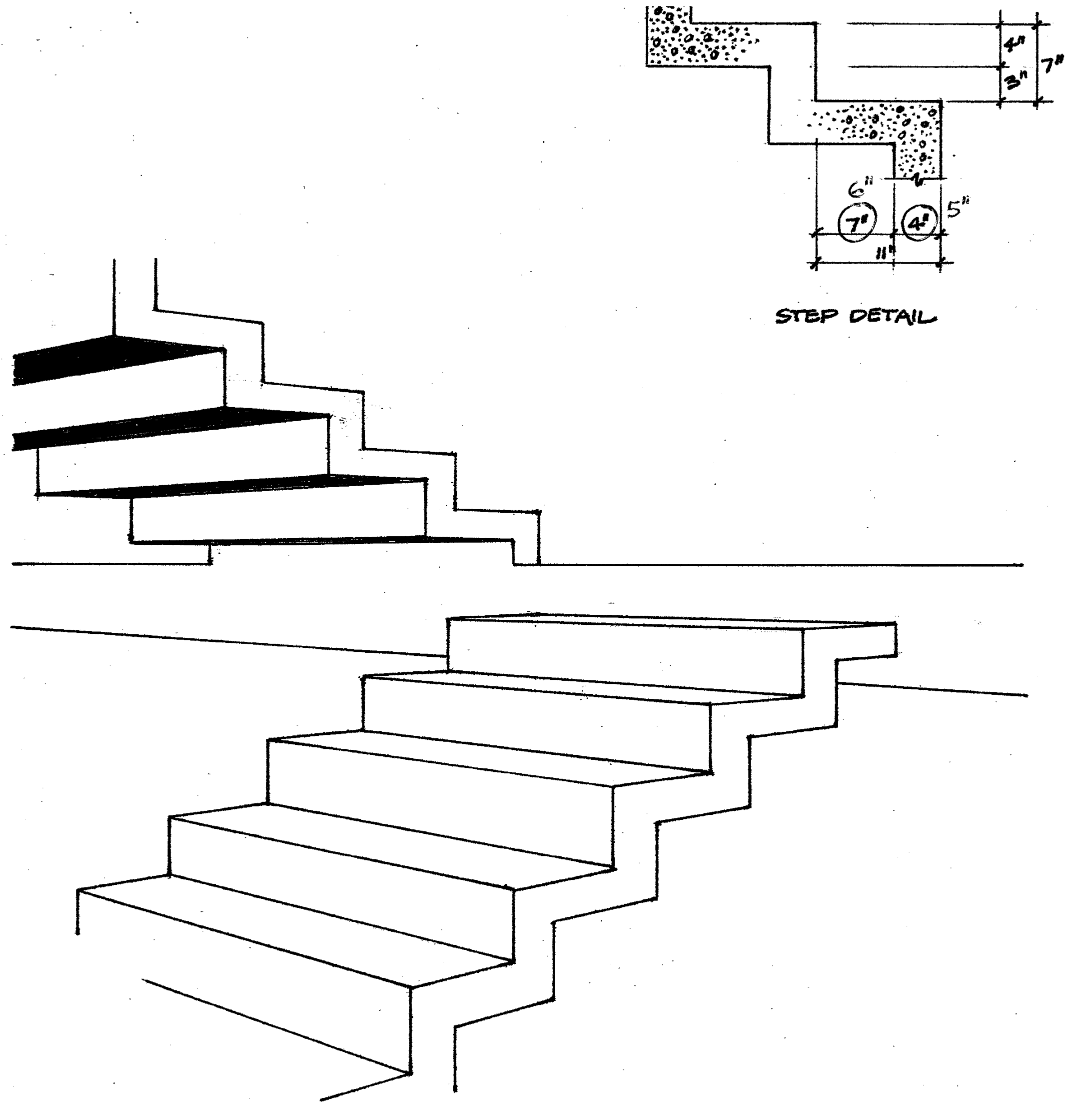
Consequently, we can now proceed to conclude the member sizing:

Slab effective depth: d = v 100,201/11,484 = 2.95 in.

And the slab reinforcing: $A_s = 100{,}201/8{,}831 =$ 12 #3

Some design engineers have solved the orthopolygonal reinforcement in the past by providing a single line of reinforcing bars. Although such solution could be borderline adequate for short spans, say 4 to 5 feet; however, due to the frequent moment reversals taking place along the axis, we recommend the use of a double layer as shown ahead.

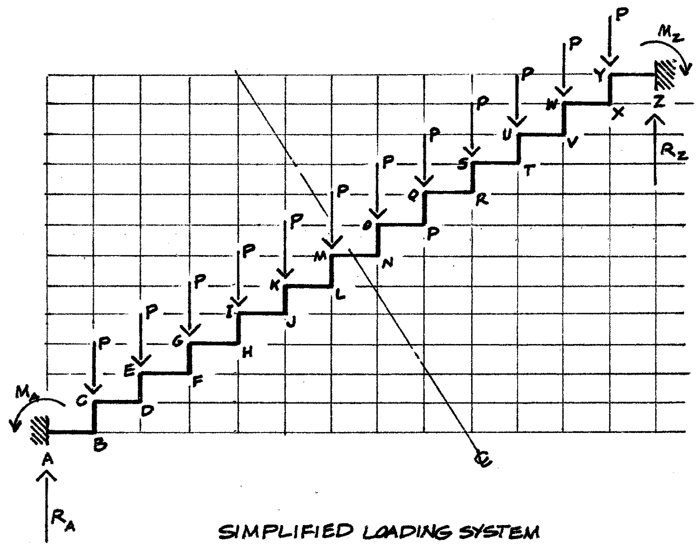
Although moment magnitudes vary at every step, all the way from the end supports to the center of span as in the example at hand, however, we recommend to maintain a constant reinforcement pattern for the sake of construction simplicity and consisting of three typical sets of jig-preformed and shop welded #3 stirrups and additionally, a precut 42 in. long straight temperature bars as called for on Figure 7.4.



RAILINGS OMITTED FOR CLARITY.

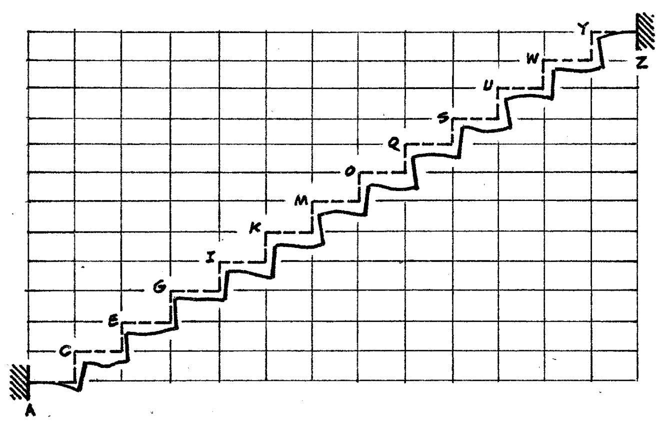
ORTHOPOLYGONAL STAIR

FIGURE 7.1



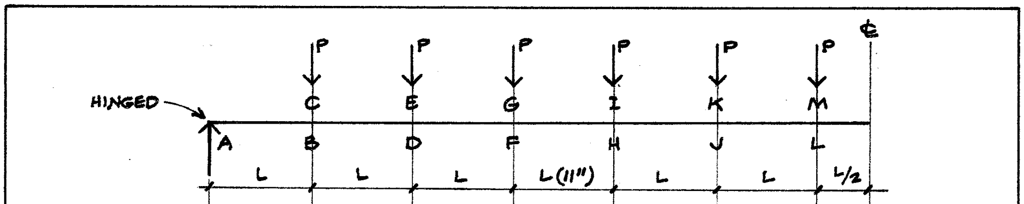
SIMPLIFIED LOADING SYSTEM

FIGURE 7.2 (a)

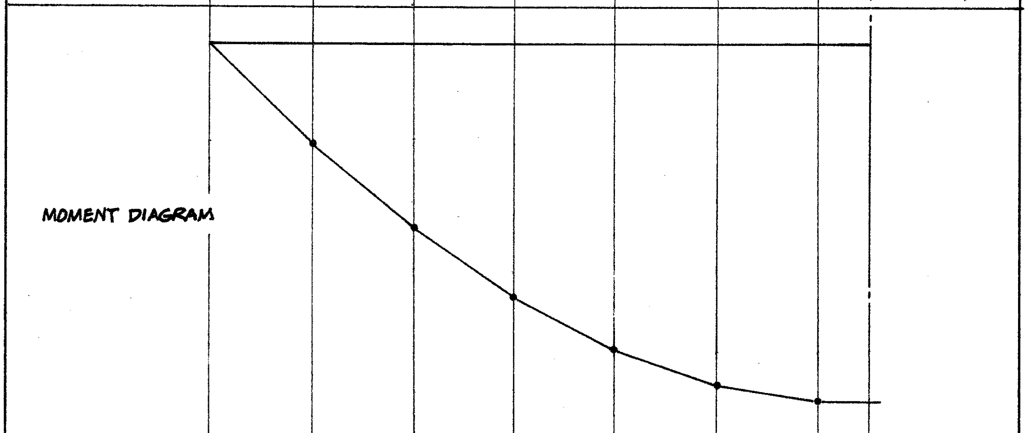


DEFLECTED STAIR AFTER LOADING

FIGURE 7.2 (b)



LOAD (P)	0	-638	-638	-638	-638	-638	-638	POUNDS
SHEAR (V)	3828	3190	2552	1914	1276	638	0	POUNDS
MOMENT (M)	0	3828	7018	9570	11484	12760	13398	PNDS (-L)



C = 6M - P	3828	22 330	41 470	56 782	68 266	75 922	79 750	PNDS $(-\frac{L^2}{6})$

$L_t = 11"$ $L_r = 7"$

3,828
7,018
9,570
11,484
12,760
13,398 $\Sigma M = 58,058$

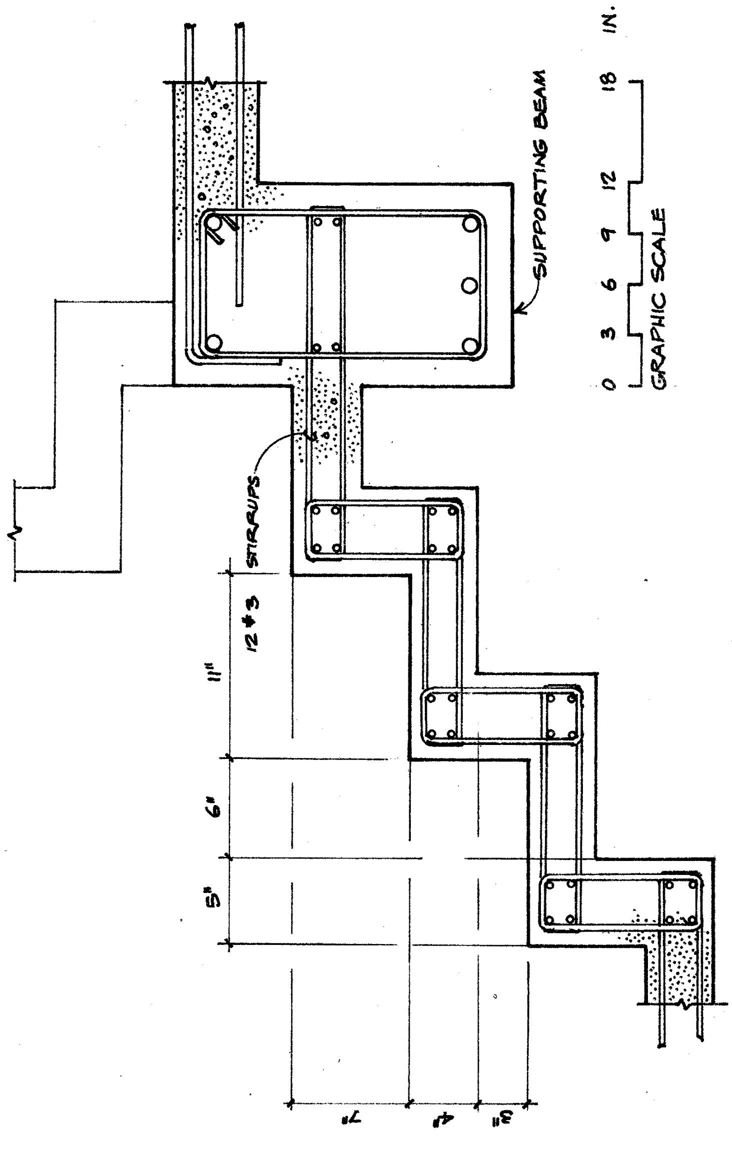
3,828
22,330
41,470
56,782
68,266
75,922
79,750 $\Sigma C = 348,348$

$$\delta_{mt} = \frac{L_t^2}{6}(\Sigma C) = \frac{11^2}{6}(2 \times 348,348) = 20.17 \times 696,696 = 14,052,358$$

$$\delta_{mr} = (L_t)(L_r)(\Sigma M) = 7 \times 11 (2 \times 58,058) = 8,940,932$$

ABBREVIATED CALC'S ON THE COLUMN ANALOGY.

FIGURE 7.3



SECTIONAL VIEW OF ORTHOPOLYGONAL STAIR

FIGURE 7.4

8.0 CIRCULAR STAIRS

Although some authors would call these “helical stairs”, however, not without first doing some “soul searching” we have decided to call them circular stairs for three main reasons. First, the case we are about to see is in reality a quarter of a turn, or better said a central 90 degree angle, and therefore not much of a helix. Second, for reasons which will become obvious during your reading, we also decided to use the analogy of the circular beam as a simpler approach to analysis and design. Third, we are reserving the name for the following case where we will discuss the true helical stairs, which are shapes of or in excess of a full turn of 360 degrees.

In preparation for this work, we examined a publication way back from the year 1960 and titled *Comparison of Analyses of Helical Stairs* authored by an English engineer named V. A. Morgan. We also had the opportunity to review the results of certain experimental work conducted by research engineers such as Holmes, Bergman, Cusens, Young and Scordelis.

Although there was consensus amongst those four authors (Young & Scordelis worked together using a fiberglass model for their laboratory tests, so we counted them as one) when it came to the basic idealization of the helicoidal beam (or slab) as a three dimensional structure, redundant and indeterminate to the sixth degree. They all also agreed, as expected, on the fact that by using the benefits of symmetry, redundancy could be cut in half.

There was disagreement however, in how to deal with the problem posed by the fact that the center of gravity of the loads did not fall over the centerline of the stairway, thus introducing an eccentricity that needed to be dealt with. That eccentricity added an additional torsional moment to the analysis.

Everyone of those researches had their own ideas and used their own methods of analysis to predict the vertical, lateral and torsional moments. However, the most complete investigation was conducted by A. R. Cusens, who used a real reinforced concrete model to half scale, which was incrementally and continually loaded to failure. In the end, the final results indicated that the ultimate failure moments were between 65 and 70 percent higher than those calculated in advance by empirical formulas.

For the development of this chapter we have selected a 90° circular stair with 15 steps from top to bottom, which is fully end-fixed to the supporting spandrel beams at both lower and upper levels, as it has been shown on the enclosed Figure 8.1. In this figure we have picked a quarter of a circle with 15 treads as already indicated, which means that every tread occupies a segment of 6 degrees. The inner radius is 7’- 0” and the stair width is 4’- 0”, which gives us an inner circular perimeter of:

$$\tfrac{1}{2}\Pi R = \tfrac{1}{2} \times 3.1416 \times 7 = 10.9956 \text{ ft.}$$

and an outer circular perimeter of 17.2787 ft.

Since there are 15 treads, they would be 0.733' wide at the inner edge and 1.1519' wide at the outer edge.

There also are 16 risers that would give us a finished floor to finished floor height of (16 x 7 = 112") 9'- 4".

Most authors recognize the fact that the presence of monolithic steps do contribute to increase the torsional strength of the stair, however, they do not quantify such contribution and would rather use it as an added safety factor. On second thought, ignoring the contribution of the steps and their existence altogether is not a bad idea. Since the forming of the steps is such of an involved and labor intensive carpentry job, in the Greater Antilles of the Caribbean Basin, namely Cuba, Hispaniola (especially in the Dominican Republic) and Puerto Rico, in most cases they finish such stairways with either marble, travertine, granite or terrazzo; consequently, they do not bother to form or pour the steps at all, for they would rather cut the finishing slabs of the appropriate size and lay them on the stair concrete slab, so to form the steps the same way they would mud-lay the tiles on a regular floor.

In a perfect theoretical world, for a given height and a constant radius, all risers and treads should be identical, however, in the real world of construction small differences and inconsistencies in width, thickness and lengths are common and to be expected. Fortunately, there are some areas where those blemishes can be easily hidden by adjusting the riser panel without altering the riser height as shown in Figure 8.2 where the reader can also appreciate how the components are fit together as they are erected in place.

When it comes to the use of materials such as marble, it needs to be highly polished so to bring up its highest attainable beauty. At the same time, while doing so, it becomes slippery and dangerous for the user, therefore such material must either be chemically treated to reduce the possibility of a slip & fall or provided with non-slip strips adhered near the thread nosing.

Regarding the analysis and design part of the solution, because of its relative inherent simplicity, we decided to take sides with Bergman and use the traditional circular beam (or bow-girder as called by some) formulae to solve the analysis quagmire. In Figure 8.3 the reader will find the pertinent loading and moment diagrams.

When it comes to torsional stresses, section configuration is of utmost importance. As the old analogy goes: "a plank has very little torsional strength", in the same manner and for the same reasons, thin slabs have no place as a solution to circular or helical stairs. Therefore, the design engineer needs to pay particular attention to those slabs where the thickness to width ratio (α) falls below 0.15.

Based on the formula: $M = (K)(W)(R^2)$

Where:

K is a multiplier given by the table below
W is the uniformly distributed load (DL+LL) on the stair, and
R is the radius to the centerline of the slab.

Moment Factor (K)

Moment	α	**-M**A	**+M**1	**-T**A	**+T**1
Flexural	0.50	0.225	0.0962		
	0.20	0.222	0.0994		
	0.15	0.221	0.0998		
Torsional	0.50			0.010	0.027
	0.20			0.007	0.029
	0.15			0.006	0.030

As a matter of example, assuming a full stair width of 4 ft., R = 9 ft. and a total load of (DL+LL) = 800 pounds per linear foot and a shape factor α = 0.15, the negative moment at support A would be:

$$-M_A = 0.221 \times 800 \times 9^2 = 14{,}321 \text{ lb-ft}$$

Above moment applies to the full slab width of 4 ft.

While it is quite true that Bergman's proposal reduces a tridimensional problem to a planar approach and not only ignores the eccentricity between the center of loads and the centerline of the stair (on the negative side) as described above; but it also ignores, this time with positive results, the compressive capacity of the quarter helix and fortunately, both effects tend to cancel each other out.

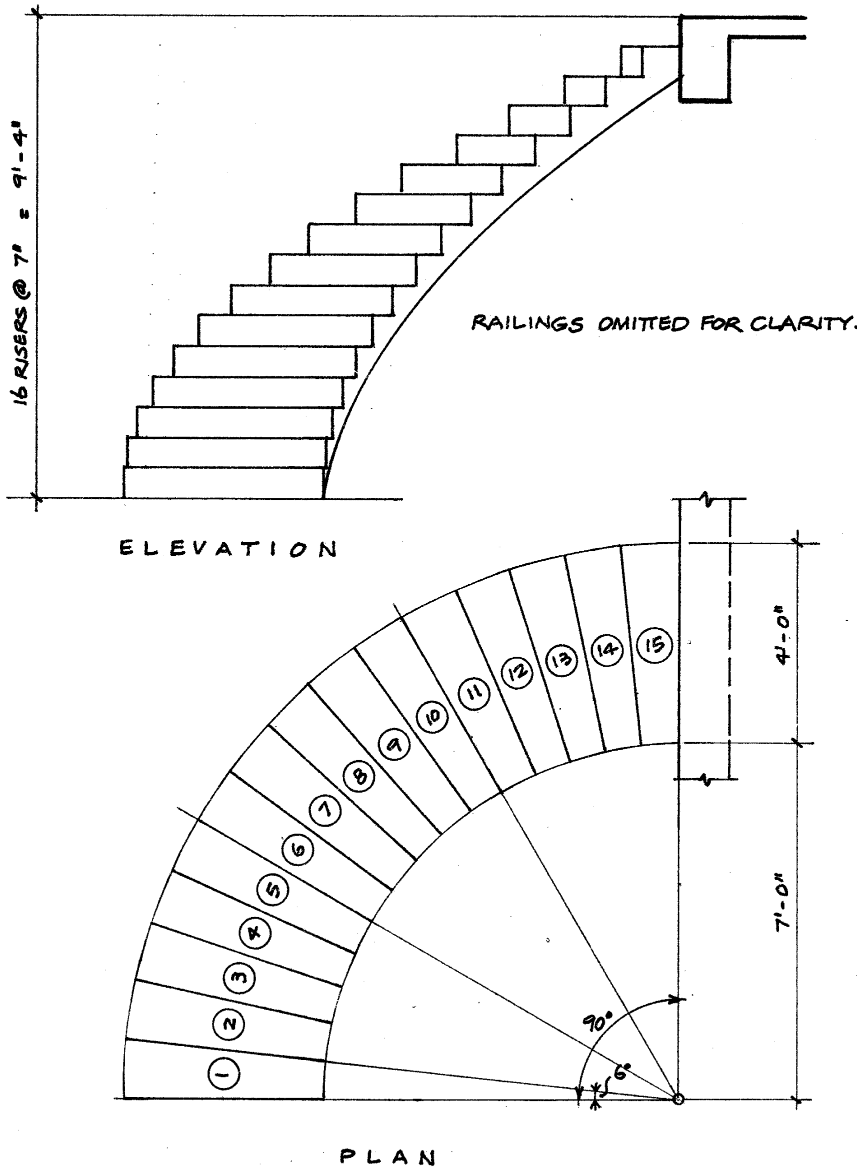
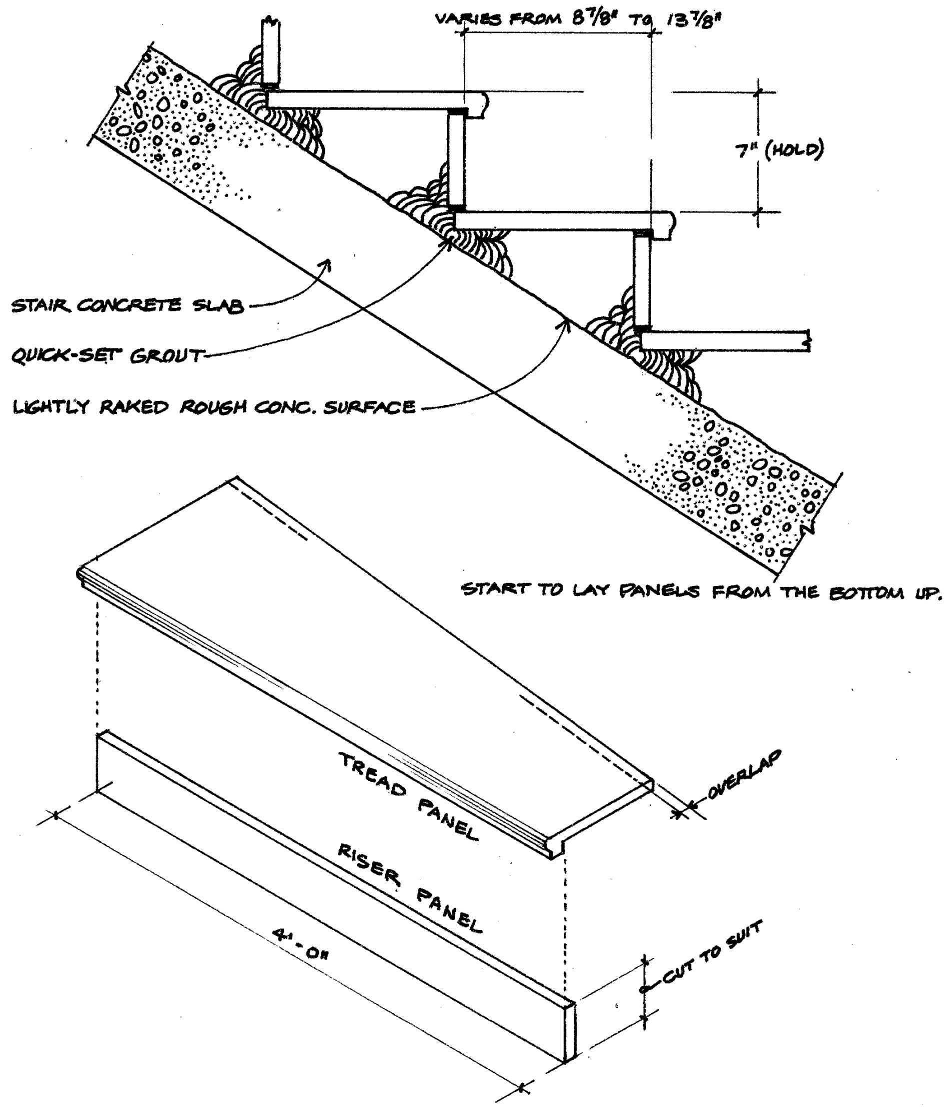

16 RISERS @ 7" = 9'-4"
RAILINGS OMITTED FOR CLARITY.
ELEVATION
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
4'-0"
7'-0"
90°
6°
PLAN

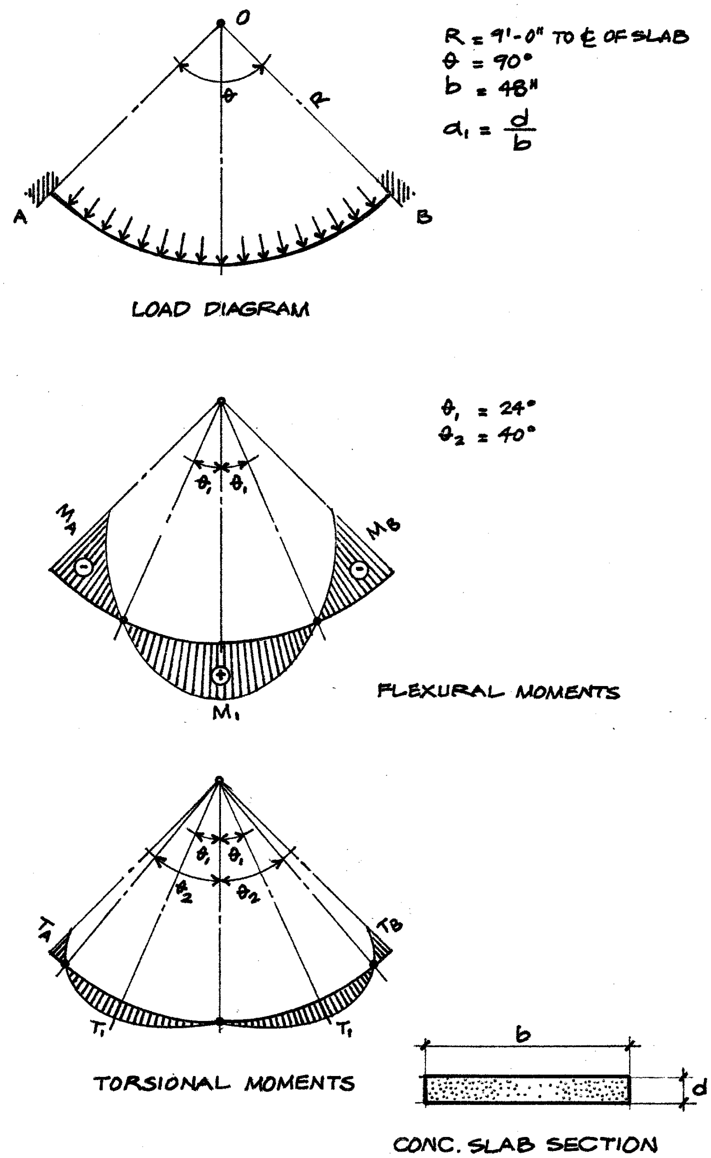
FIGURE 8.1



ALTERNATIVE TO POURING STEPS

FIGURE 8.2

FOR BETTER REPRESENTATION FORCES AND MOMENTS HAVE BEEN ROTATED 90°.



FLEXURAL AND TORSIONAL MOMENTS IN CIRCULAR BEAMS

FIGURE 8.3

9.0 HELICAL STAIRS

Once the central angle θ exceeds the limit of 90 degrees, Bergman's method is no longer workable and we must turn to the Morgan's method as a reasonable solution for and up to a central angle of 360 degrees as shown in Figure 9.1. Morgan's formulae give full consideration to the tridimensional aspect of the problem, the eccentricity between the load center and the centerline of the slab, as well as to the added compression capacity contributed by the concrete helix. However, his formulas will not work passed the limit of 360°, after such boundary helical stairs are of little practical use anyways.

Figure 9.2 depicts a full revolution concrete helical slab as a precursor to the stair with the same name. Hand railings and steps have been omitted for a better comprehension of the structural concept.

The formulas given below use the following notations and nomenclature:

b = total width of the stair's concrete section
C_1, C_2, C_3 = redundant moment coefficients
d = total depth of stair's concrete section
E = modulus of elasticity of concrete (in tension or compression)
G = modulus of elasticity of concrete (in shear)
H = horizontal redundant force at midspan
k = torsional constant = $0.33 - 0.21(d/b)[1- 0.083(d/b)^3]$
M_o = redundant moment acting in tangential plane at midspan
M_l = lateral moment
M_v = vertical moment
P = thrust normal to tangent
S = shearing force across section of stair
T = torsional moment
R_1 = radius to centerline of load
R_2 = radius to centerline of stair
w = total loading per unit of projected width and length along the centerline
W = total loading for the full width of the stair
θ = stair central angle
φ = slope of the stair with respect to the horizontal plane

Vertical moment:
$$M_v = M_o \cos\theta + HR_2\,\theta \tan\varphi \sin\theta - wR_1^2(1- \cos\theta)$$

Lateral moment:
$$M_l = M_o \sin\theta \sin\varphi - HR_2\,\theta \tan\varphi \cos\theta \sin\varphi - HR_2 \sin\theta \cos\varphi + (wR_1 R_1 \sin\theta - w R_1 R_2\theta) \sin\varphi$$

Torsional moment:
$$T = (M_o \sin\theta - HR_2\,\theta \cos\theta \tan\varphi + wR_1.R_1 \sin\theta - wR_1 R_2\,\theta) \cos\varphi + HR_2 \sin\theta \sin\varphi$$

On the other hand,

$$M_o = C_1\, w\, (R_2)^2$$

and

$$H = C_2\, w\, R_2$$

Now, we will put those formulas to a test all the way to their limit of 360° by using the example as described in figures 9.1 and 9.2. Such stair has a slope of 26°- 20′ about the horizontal plane, carries a uniformly distributed live load of 100 pounds per square foot (PSF) and a dead load of 120 PSF for a total combined DL+LL = 220 PSF. These are the rest of the parameters:

Stair width: 3 ft.
Slab thickness: 6 in. (net)
Equivalent stair thickness: 9½ in. (if steps are cast integrally with the slab)
Radius to centerline of slab: R_2 = 4.50 ft.
2ΠR = 3.1416 x 9 = 28.2744′
Floor to floor height: 14 ft
Tan φ = 14.0/28.2744 = 0.4952, thus: φ = 26°- 20′
Radius to inside edge of stair: R_i = 3 ft.
Radius to outside edge of stair: R_o = 6 ft.
Central angle: 360°
Radius to center of load: $R_1 = 0.67\, [(6^3 - 3^3)/(6^2 - 3^2)] = 4.69$ ft.
Thus: $R_1 / R_2 = 4.69/4.50 = 1.042$

We will assume a ratio of 1.05 to conform to the empirical tabulations provided by Morgan and Cusens. The difference between R_1 and R_2 gives us the load eccentricity (e):

$$e = 4.69 - 4.50 = 0.19' = 2\tfrac{1}{4}''$$

Since b = 36″ and d = 6″, then: b/d = 6.0

We used the charts shown in Morgan-Cusens publications to obtain the corresponding values of coefficients C_1, C_2 and C_3. Some interpolating and extrapolating were actually necessary to accommodate our values to those on the charts.

$$C_1 = -\,0.49;\;\; C_2 = +\,2.0;\;\; \text{and}\;\; C_3 = -\,1.68$$

Hence,

$$M_o = -0.49 \times 220 \times 4.50^2 = -2{,}183 \text{ lb-ft}$$
$$H = +2.0 \times 220 \times 4.50 = +1{,}980 \text{ lbs}$$

The predominant moment for design purposes is the negative moment at the lower and upper ends of the stairway:

$$-M = -1.68 \times 220 \times 4.50^2 = -7{,}485 \text{ lb-ft}$$

Those moments and forces apply to a one-foot strip of slab and have been found to be of reasonable value and within 5% of the computer generated results.

When it comes to the construction process, if you have never done it before, you must listen carefully before entering the “mine field“ it represents. First and foremost, an anticipating and thoughtful layout is of paramount importance in any given case, but it is particularly decisive when you are building a helical stair.

You should start by taking a 5 to 6 in. finely lathed wooden (because wood is easily markable and nailable) pole of the proper length and height, fasten it to a 1″x 24″x 24″ plywood base, make sure it is plumbed along in all directions, then firmly attach it to the floor slab right at the center of the stair. Verify plumbness again before fastening the top. You may start your layout and every time you take a horizontal radial dimension, do not forget to subtract one-half of the pole’s diameter.

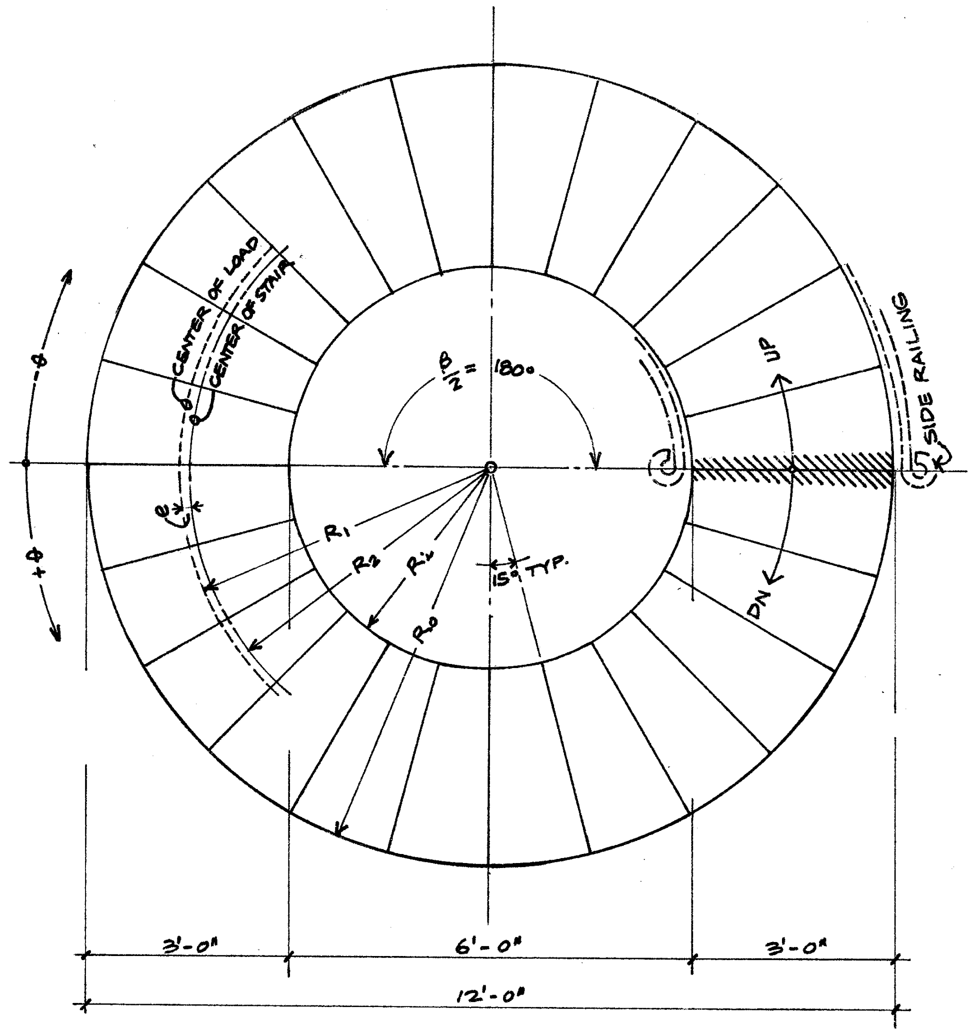
Prepare your framing by laying the 2″x 6″ stringers from the central pole to your outer line of shores. Lay the stringers flat leveled while keeping in mind that you have three thicknesses to contend with to determine their top elevation: concrete slab, plywood sheathing and joist depth. Once your stringers are complete and checked for position, level and length, proceed with the 2″x 4″ joists @ 16″ on centers, then the plywood sheathing and finally the edge forms.

Whether the steps are going to be formed simultaneously and integrally with the main formwork is a decision that depends on many factors. Naturally, if the stair is going to have just bare concrete finish, then the steps must be formed in, unless a second concrete pour is contemplated. The design engineer may have a role to play in such decision, since integral steps will increase the torsional strength of the slab. However, if there are some other finishes involved, such as marble, travertine, granite, ceramic or man-made materials, then their choice of thicknesses will come into playing. Again, as shown on Section 8.0, there are ways to cut and fabricate the finishing materials to form the steps accordingly if that would be the preferred method.

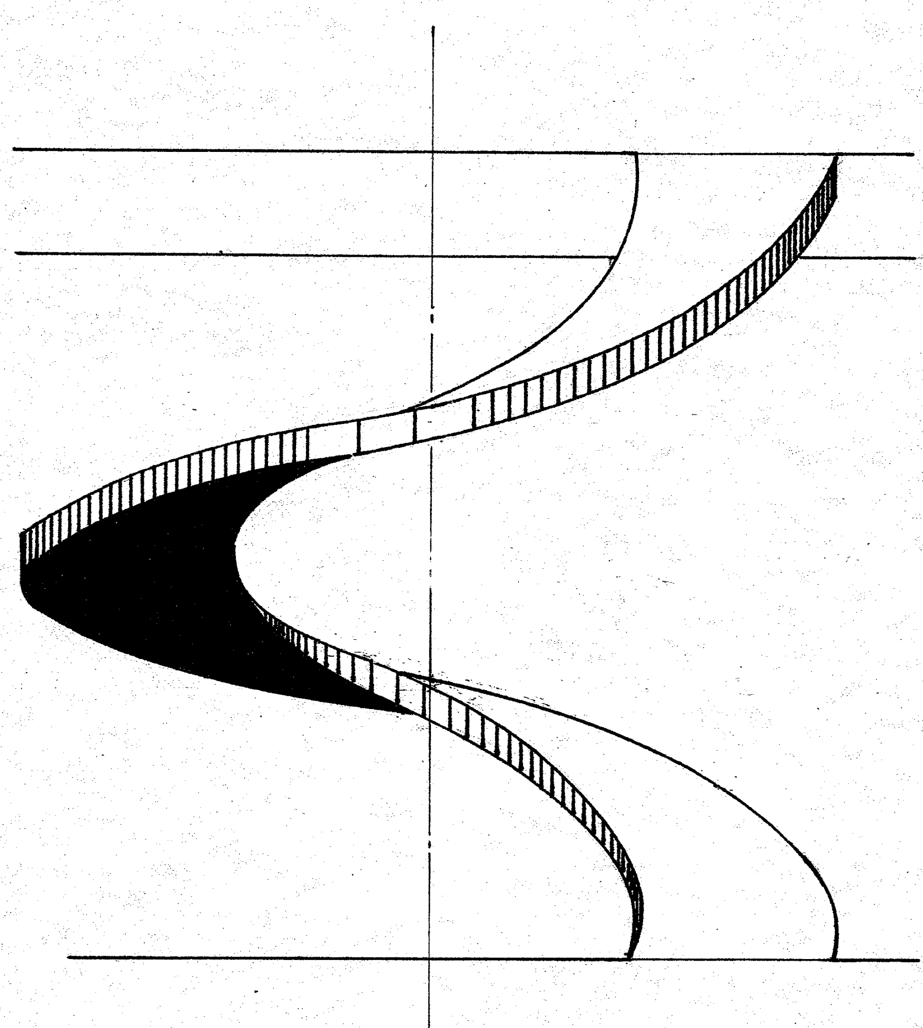
Once the formwork is complete, well secured and inspected, the next step would be to install the reinforcement. Bars must be cut to the right length and placed neatly in size and spacing according to the construction drawings. Use chairs to maintain the proper clearance to the forms and observe the proper concrete coverage at the bottom of the reinforcing bars.

Following the inspection and approval of the reinforcing bar placing comes the careful preparation for pouring. We recommend a minimum concrete strength of 3,500 PSI at 28 days (however desirable 4,000 PSI). The concrete mix should be properly designed, water content carefully controlled, vibrated and compacted to its maximum density. Another plus of forming the steps is that the riser forms will help maintaining the concrete in place and without sliding downhill. Form stripping should not take place any sooner than the mandatory 4 weeks after pouring.

In closing, do things the right way and in the end you will have a stairway that will give you a full service and will last for the next five-hundred years.



FULL TURN HELICAL STAIR
FIGURE 9.1



FULL TURN HELICAL SLAB
FIGURE 9.2

10. FREE STANDING STAIRS

This is the last type we have selected for this course; they are provocative to the viewer and treacherous for the builder. In the early 1970's we had the opportunity to examine a case where things went wrong and the consequences unpleasant for all involved, the owner, the engineer and the builder. We will bring up some brief details about the case at the end of this section.

Free standing reinforced concrete stairs without an enclosure and landing supports are structurally challenging and architecturally attractive. The first rational attempt of a logical analysis is credited to a German engineer named Fuchsteiner who in 1954 presented his procedure by assuming the stair "supported" along the intersection line between landing and flights. Moments were calculated as the flights were having one support at each end, as well as another one at the landing and the rest treated as a regular cantilevered beam. The reaction at the inexistent support at the landing had to be counteracted by a fictitious force equal in magnitude and of opposite direction, then solving the resulting equation system for a rigid frame undetermined to the sixth degree.

In 1960, Liebenberg improved on the concept by introducing his idea of the "space interacting plates" where the torsional moments were small and treated as secondary stresses. We will comment on this understatement later along the development of our material. Shortly in 1962, Siev extended Liebenberg's theory by further refining the mathematical procedure for the determination of the neglected secondary stresses. Then in 1964, Franz Sauter came up with his own analysis applied to Fuchsteiner's frame and using the principle of *least work.*

Both, horizontal displacements and vertical deflections are very significant in the final behavior of any free standing stair. Further to structural safety, the psychological effect on the user seems to be the responsible for that feeling of instability expressed by most of the surveyed interviewees. There are documented stair cases where although found structurally adequate, due to undue vibrations they have had to be strengthened to overcome some of the prevailing perceptions of insecurity.

Using as a prototype the SouthEnd Country Club's Stair #4 as shown in Figure 10.1, it depicts a two-flight and a landing negotiating a single floor height. Moreover, the singularity of this type of stair gets even further enhanced when used in a multistoried application. As it becomes readily apparent by directly observing said figure, free standing stairs are not self standing or self balancing, for there is a large overturning moment being transmitted from the stair to the main building structure that the design engineer must also take into consideration.

In our example ahead you will see that the combined (DL+LL) of every flight is:

DL: 9.17 x 4.58 x 104 = 4,368 lbs
LL: 7.68 x 4.58 x 100 = 3,600 "
For a total of: 7,968 lbs

While the landing by itself is:
DL: 10.50 x 4.58 x 104 = 4,208 lbs
LL: 10.50 x 4.58 x 100 = 4,809 "
For a total of: 9,017 lbs

The total load from the stair is: 2(7,968) + 9,017 = 24,953 lbs which multiplied by the appropriate lever arm, delivers an overturning moment of hundreds of thousands of foot-lbs.

Figure 10.2(a) shows a general plan view of the stair and the two unknowns, namely the axial force H and the moment M_o. In Figure 10.2(b) we show a side view of the idealized frame where (½ Wc) is the cantilever moment. W and w are the total loads acting on the landing and flights, respectively. Figures 10.2(c) and (d) are self-explanatory.

The following are the bending moment formulas applicable:

$$M_B = -\tfrac{1}{2} W(\tfrac{1}{2} b + b_1 - y)^2$$

and,

$$M_{AB} = Hs \sin \alpha - W(\tfrac{1}{2} b + b_1) s \cos \alpha - \tfrac{1}{2} Wc (\tfrac{1}{2} b + b_1) - \tfrac{1}{2} (ws^2 \cos^2 \alpha)$$

To solve the unknowns, two simultaneous equations were formulated:

$$0.0614H - 0.00452M_o = +514.99$$
$$0.0182H + 0.01356M_o = -11.80$$

Which we solved by using matrices:

$$\Delta = \begin{vmatrix} 0.0614 & -0.00452 \\ 0.0182 & +0.01356 \end{vmatrix} = 0.0009174$$

$$\Delta_1 = \begin{vmatrix} 514.99 & -0.00452 \\ -11.80 & +0.01356 \end{vmatrix} = 6.9299 \quad \text{thus,} \quad H = 6.9299/0.0009174 = 7{,}554 \text{ lbs}$$

$$\Delta_2 = \begin{vmatrix} 0.0614 & 514.99 \\ 0.0182 & -11.80 \end{vmatrix} = 10.09734 \quad \text{thus,} \quad M_o = 10.09734/0.0009174 = 11{,}006 \text{ ft-lb}$$

Therefore:

$$M_A = -\tfrac{1}{2} Hb. \cos \alpha - M_o \sin \alpha + W (\tfrac{1}{2} b + b_1) . \tfrac{1}{2} \{(b_1 + \tfrac{1}{2} b) - b\} \sin \alpha$$

Moment M_A not only would be used to determine the negative reinforcing bars at point of support A, but also will be transferred to the carrying spandrel beam as a torsional moment.

DEFLECTIONS

Deflections of the different parts of the free standing stair should be evaluated with a great degree of caution. Figure 10.3 shows a deflected diagram of the prototype stair used as an example and after being fully loaded. Those deflections shown on the figure have been taken from the experimental model results obtained by Kuang-Cusens, but they are not the absolute numbers obtained, they are rather relative to the deflection of point F taken as the largest deflection and assumed as unity. The deflections of all other points are relative to that number. For instance, deflection of point G happens to be 87% of such deflection at Point F and so on.

Kuang-Cusens team found that in addition to the expected vertical deflections, there were also horizontal displacements at points F and G which suggested a counter-clockwise rotation of the landing platform about point O which has been blamed on the different angular relationships between the landing and the two connecting flights on line COD.

The upper flight AB is subjected to a large negative moment as well as tension forces, while the lower flight gets under torsion and compression forces as suggested by the same Figure 10.3.

TYPICAL REINFORCEMENT PATTERN

As of this point, the reader has likely figured out that we used the stair dimensions and geometry of Stair #4 at the SouthEnd Country Club, and that is a correct deduction. We have tried to blend the results of that experience with the available experimental results and use it for the benefit of this course. Some of those learned facts have also being incorporated in the proposed steel reinforcing arrangement shown in Figure 10.4.

Please notice that every other closed #3 stirrup on the upper and lower flights have been extended into the concrete steps as a crack arrester. The reinforcing bar marked Q is a #3 x 2′- 6″ added to provide for small moment reversals caused by the possible flapping of the landing. In the same manner, bar marked S is a #4 x 10′- 4″, which has been added to prevent hairline crack propagation from the landing to the steps of the upper and lower flights.

FORMWORK

Still referring to Figure 10.4. Although formwork for this case is not much different than that required for a regular and customary stairway, two comments need to be made that has to do with aesthetics and the final quality of the product:

#1. A half-round drip is recommended to be added at the far underside end of the landing. Such a small feature will prevent rainwater runoff from rolling down the slope and eventually producing an unsightly dark mold stain.

#2. In order to assure a straight line at point P, some provisions have to be made by either adjusting the landing's bottom form or stepping it down as suggested.

DOCUMENTED CASE

Reinforced concrete cantilevering structures have to be carefully handled by the design engineer. They are unforgiving at best for they have little ductility reserves and no defense against progressive collapse; once the reinforcing steel reaches its yield point there is no turning back.

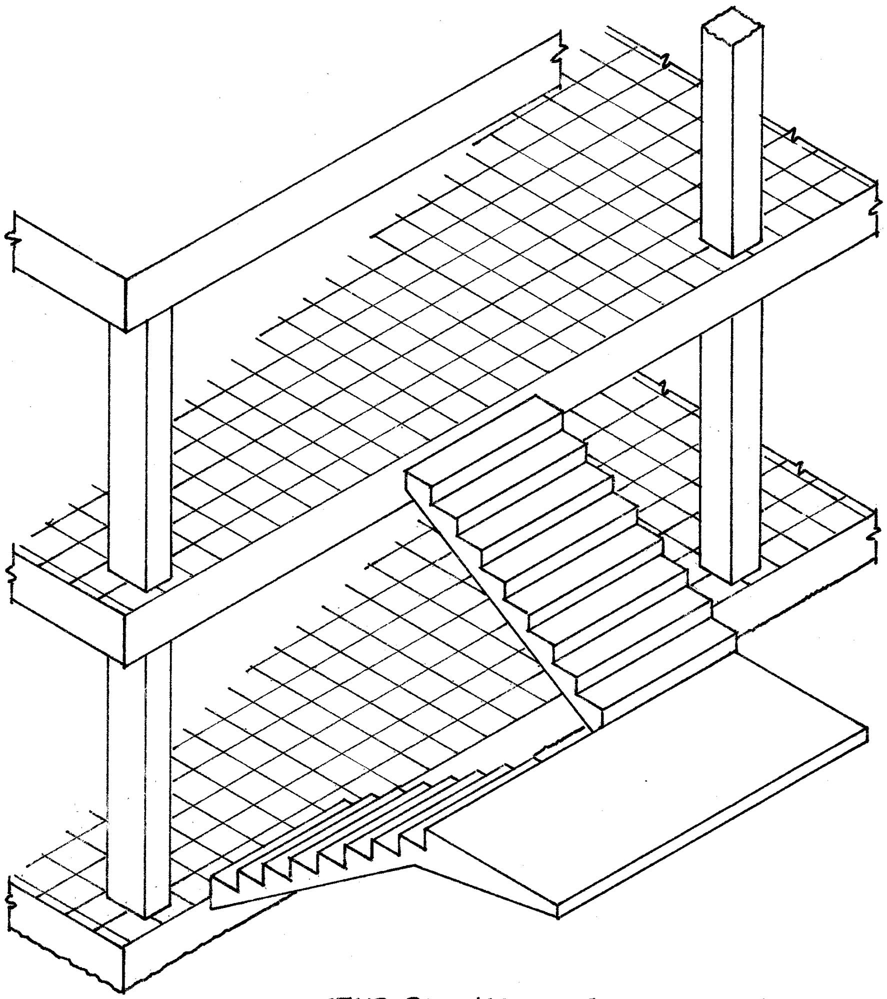
In October 1991 we were called to a country club facility to evaluate a one year old free standing stairway showing signs of distress. As reported, since the very beginning and because of the same psychological sense of insecurity mentioned before and produced by the bouncing effect perceived by the users, the stair became a source of concern. However, soon enough and in some momentarily unsupervised adolescent children activity, the perceived vibrations had an unanticipated opposite effect, they thought it was very exciting to their purposes and started to jump as in a trampoline, so to induce vibrations and whiplash. It was not immediately known how long they kept up the game, but obviously long enough to create many cycles of forced vibrations to likely reaching the fundamental period until material fatigue started to show up in the form of hairline cracks.

At our arrival and upon the proper examination, we found some alarming cracks (3-4 mm. wide) along line CD much as shown on Figure 10.5. Those cracks originating in the vicinity of point marked O and extending across to the edge of the landing; and in the opposite direction propagating through the steps of the lower and upper flights, were caused by the torsion stresses generated in the same area. We asked the management to completely and effectively block public access to the stairway and temporarily install shoring jacks at points C, D, F and G. From the original design engineer, we were able to secure a set of working drawings and with such information at hand went back to the office to do our evaluation. We will be brief in our description since this case will be abundantly covered as part of a new course we are preparing on structural concrete repairs.

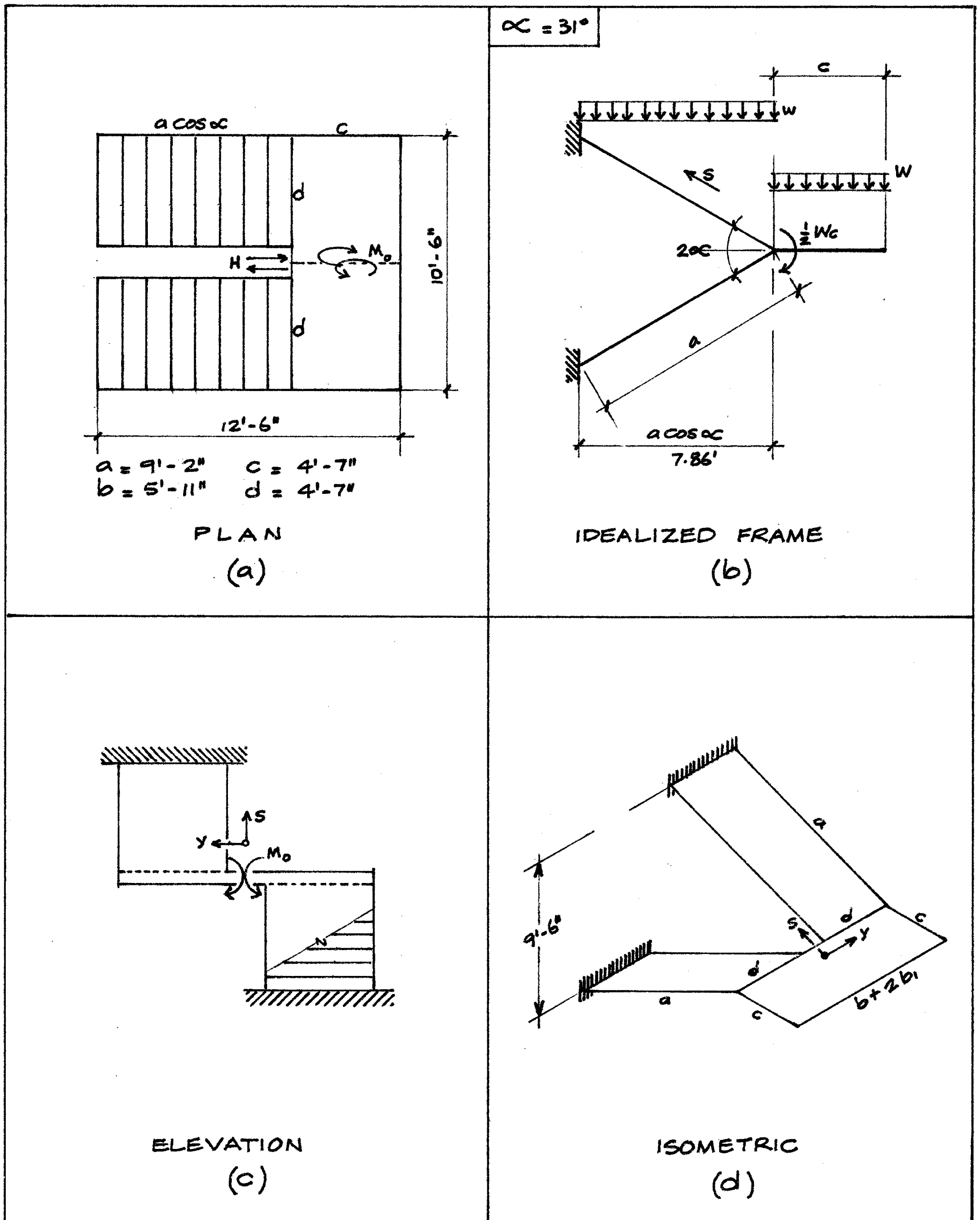
We went through the entire analysis and design exercise and although found that there was some 16% under designing, however, in all fairness to the engineer-of-record we must say that in the experiments conducted by Kuang & Cusens, it took about 6 times the DL+LL to cause the failure of their test model, therefore that level of under-design could not have been the culprit. In summary, we found that the deflections were still within the elastic zone and therefore the reinforcing steel yield point had not be been reached, so we proceeded to prepare and issue some repair details basically consisting of adding a "T" shaped supporting structural steel column at point O as it has been indicated on the same Figure 10.5. Such support was set and rigidly bolted to a poured in place concrete foundation and erected in such a way that allowed a 2 inch gap below the existing landing, which gap was then filled up and solidly packed with expanding grout. The cracks were injected and sealed with an epoxy adhesive formulated for that type of use and then lightly cosmetically chiseled, ground and re-finished.

As result of the above described repairs, slab vibrations were substantially reduced, the propagation of cracks was stopped and the stair had a useful life that lasted for many years after completion of the repairs.

Despite the mishap above described and considering the fact that there are many factors that may contribute to negative results, such as, gross negligence in design protocol, human error, poor concrete quality, inadequate reinforcement, formwork failure, improper curing procedures, etc., the percentage of this type of occurrence is very small. Nevertheless, the design engineer must be aware and prepared to know what to do and how to solve those problems, should they occur in the course of his practice.

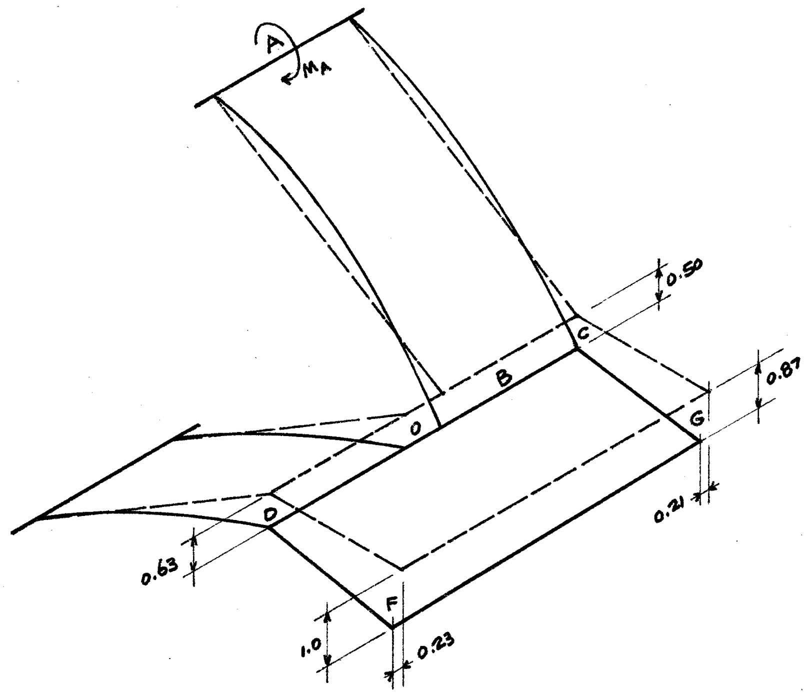


FREE STANDING STAIRWAY
FIGURE 10.1



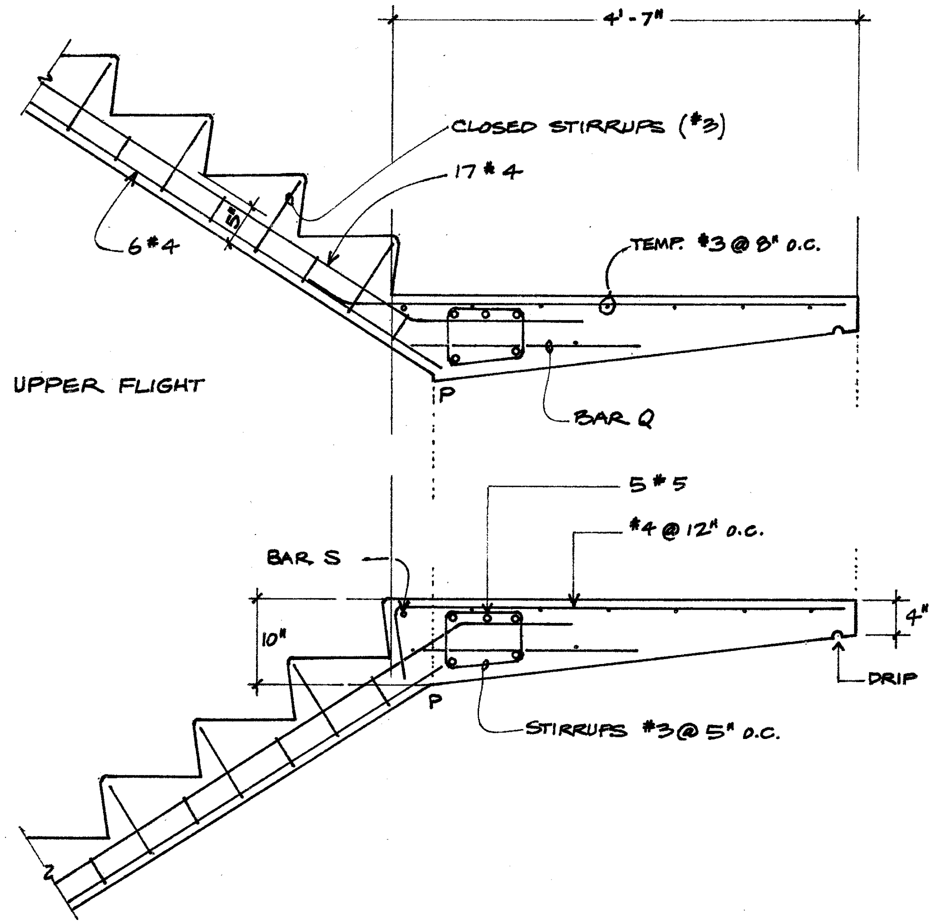
STAIR SCHEMATICS

FIGURE 10.2



DEFLECTED STAIR AFTER FULL LOADING

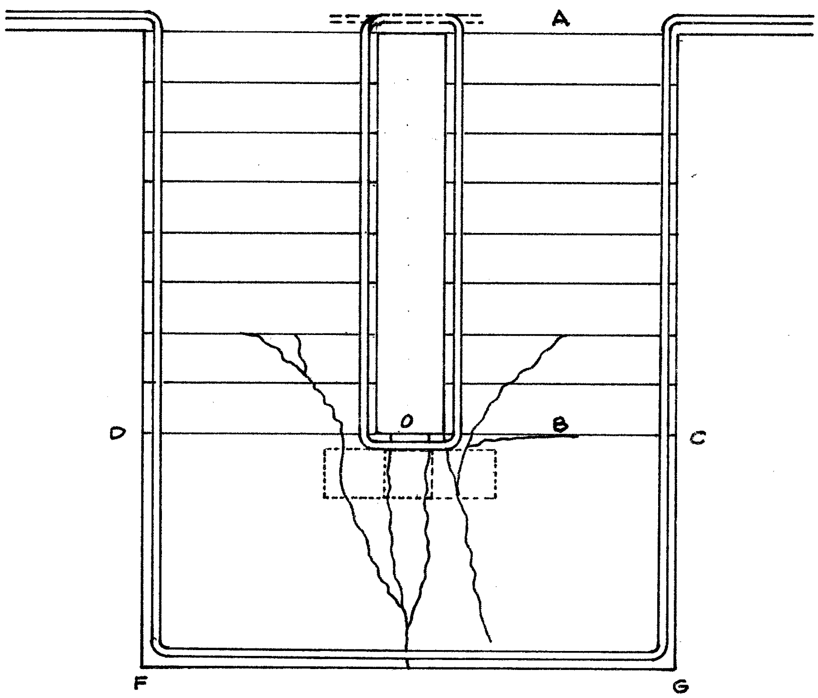
FIGURE 10.3



DESIGN MOMENTS (FT-LB)	
M_A	- 2,980
M_B	- 6,450
M_o	- 11,690
CANTILEVER	- 1,164 / FT
M_{AB}	- 29,810

TYPICAL REINFORCEMENT

FIGURE 10.4



SOUTHEND COUNTRY CLUB
1991

STAIR #4

CRACK PATTERN
3MM. GAP ON LINE CD
HAIRLINE ON LINE FG

PLAN VIEW

DOCUMENTED CASE

FIGURE 10.5

11.0 CONCLUSION

Unquestionably, a set of well designed stairs could add a singular interest and even beauty to a building's main lobby or entranceway. In fact, some buildings from the distant past or this recent era, are primarily known for their sumptuous stairways. Nevertheless, such beauty and distinction should never be achieved by sacrificing the safety of the users or motivate the greed of those gold-diggers who seek the advantage of the adequate and captive stage for the implementation of their rehearsed "accidents".

To those architects who have enough talent to blend "form & function" to the point of a balanced safety, congratulations are in order. At the same time, to those consulting engineers who possess the skills to bring together their design to meet both, the requirements of structural integrity as well as the attribute of build-ability, our due respect and admiration!

END