PDHonline Course C676 (1 PDH)

The Law of Buoyancy

Instructor: Ruben A. Gomez, PE

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The Law of Buoyancy

Ruben A. Gomez, P.E.

1.0 PREFACE

When it came to shipbuilding, there was a generalized belief until a few centuries before Christ, that if a certain material had the quality of floating on the water, that would qualify it as a desirable material for boat building. Conversely, a material that would not float was worthless in the minds of the boat builders. Consequently, wood was decidedly considered the ideal material for such purpose. On that same line of thought, nobody from that era would consider any of the known metals and minerals of the time to be any suitable, for they did not float.

It was not until and after Archimedes' discovery in about 250 BC, that the ancient man figured that the material did not matter as originally perceived, since flotation was rather controlled and determined by the factors of volume and density. Yet, for hundreds of years after that they continued to use wood for several other reasons, such as availability, technology, workability, proper tools and their own self-assurance.

On the other hand, needs such as size, hermeticity, strength and requirements from the art of war brought out the reality of having to consider other more efficient materials such as steel, aluminum, fiberglass and even wire-reinforced concrete (this latter material has been known for nearly one-hundred years by its coined Italian name of *ferro-cemento*) as part of the boat building industry.

2.0 THE PRINCIPLE OF ARCHIMEDES

The Law of Buoyancy, or by another name, The Law of Flotation, is what the Principle of Archimedes is also called for and known as. Archimedes' discovery came to change the generalized common perception of flotation that had existed from the beginning of man's development.

As far as history goes, the Greek philosopher and mathematician Archimedes sometime around the third century BC is credited with the observation that led to the principle named after him. King Hieros of Syracuse had decided he needed a new gold crown with a revised design deserving of his achievements. For that purpose he commissioned a goldsmith named Niarchos for the task of melting his crown for the gold and recast it under his new approved design.

When the new crown was finished and delivered to the king, he felt that the crown was somewhat lighter as if Niarchos had cheated by alloying the gold with other cheaper metals and had kept the difference in value, in addition to his fee for the work. King Hieros asked Archimedes to make the determination without damaging the crown.

Archimedes was perplexed, although he knew that by the goldsmith having mixed the pure gold of the crown with another precious or semi-precious metal, it should have resulted in a reduced density which was the point he needed to prove. Further, he also knew well the consequence of his findings and how it would affect the fate of the goldsmith. Yet, he had no idea of how to prove it in a decidedly and conclusive manner. He needed both weight and volume for such a purpose and while he could easily obtain the exact weight, he could not determine the volume of such intricately conceived object.

Suddenly one day, as Archimedes submerged himself into a water filled bathtub to take a bath, he was stunned by observing how as he lowered himself into the tub the displaced water overflowed over the sides. All of the sudden the answer dawned on him. He ran naked (as the storytelling goes) through the streets of Syracuse shouting: *Eureka, I have got it!* Right then and there the Archimedes' Principle was born!

The rest was easy, first he weighed the crown as accurately as the methodology of the time permitted, and then, as he immersed the crown in a pail of water, he measured the rise in water level. As a third step, he took a bar of solid gold of equal weight and repeated the experiment and found that there was a reduced density in the gold crown. That was a conclusive proof and evidence that Niarchos had cheated and therefore, had to endure the consequences of his dishonest act.

The Principle of Archimedes in all its wisdom predicates that: *when a body is submerged in a liquid, it receives an upward force equal to the weight of liquid displaced.* Such an upward force exerted by liquids against their submerged objects, is in fact the phenomenon so called *buoyancy.*

It took nearly two thousand years for the next needed principle to couple with the Law of Buoyancy, so as to bring practical use and to inject life of its own to the science of Hydraulics. French mathematician Blaise Pascal (1623-1662 AD), after a lot of trial and error experimenting is when he conceived what is now known as the *Pascal's Principle.* This principle can be expressed as follows: *Pressure exerted anywhere in a confined liquid is transmitted unchanged to <u>every</u> portion of the interior and to all the walls of the containing vessel; and such pressure is <u>always</u> exerted at right angles to such walls.*

We can reason here that since all liquids will flow in response to an applied force; therefore, any given force will be transmitted through the liquid in a form of constant pressure acting against all surfaces of the container or vessel. By using such principle we can take advantage of liquid pressure in the same manner as we have used the lever.

3.0 ARCHIMEDES' PRINCIPLE EXPRESSED IN MATHEMATICAL FORM

The weight of a solid or liquid body (W) is equal to its volume (V) multiplied by the density (D) of the material it is made of, in other words:

$$W = V \cdot D$$

Referring now to the contents of Figure 3.1 where we show a certain container filled up with water (could be any other liquid for that matter) and a submerged body subject to different material densities and/or interior characteristics. These are the governing parameters:

W_1 = weight of the submerged body
W_2 = weight of displaced liquid
V_1 = volume of the submerged body
D_1 = density of the submerged body (a variable)
V_2 = volume of the displaced liquid (as a function of W_1)
D_2 = density of the displaced liquid (a constant)

Therefore:

$$W_1 = V_1 . D_1 \text{ (out of the water) or,}$$

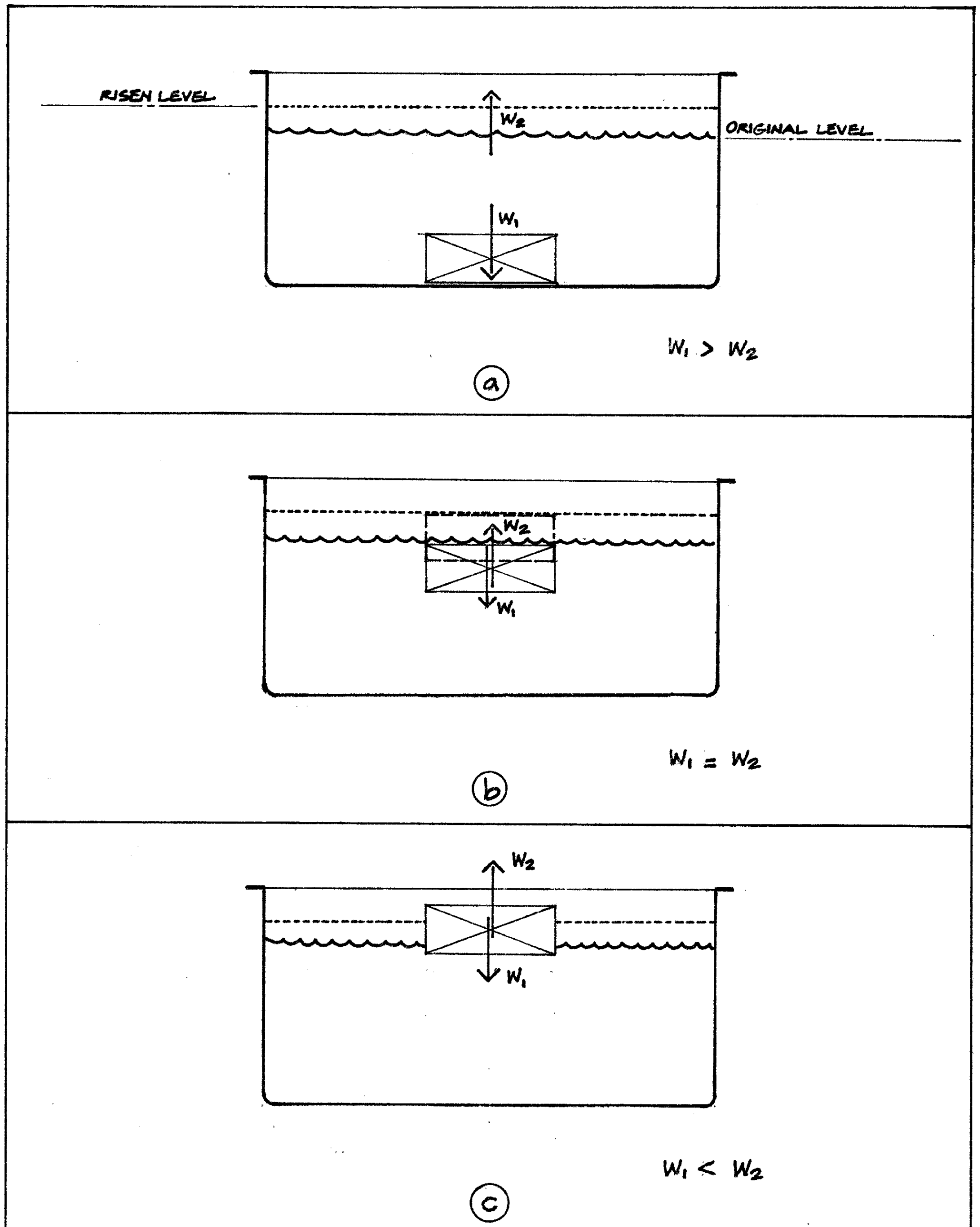
$$W_1 = V_1 . D_1 - V_2 . D_2 \text{ (when submerged)}$$

We can now place Archimedes' original idea in a mathematical form and find D_1 this way:

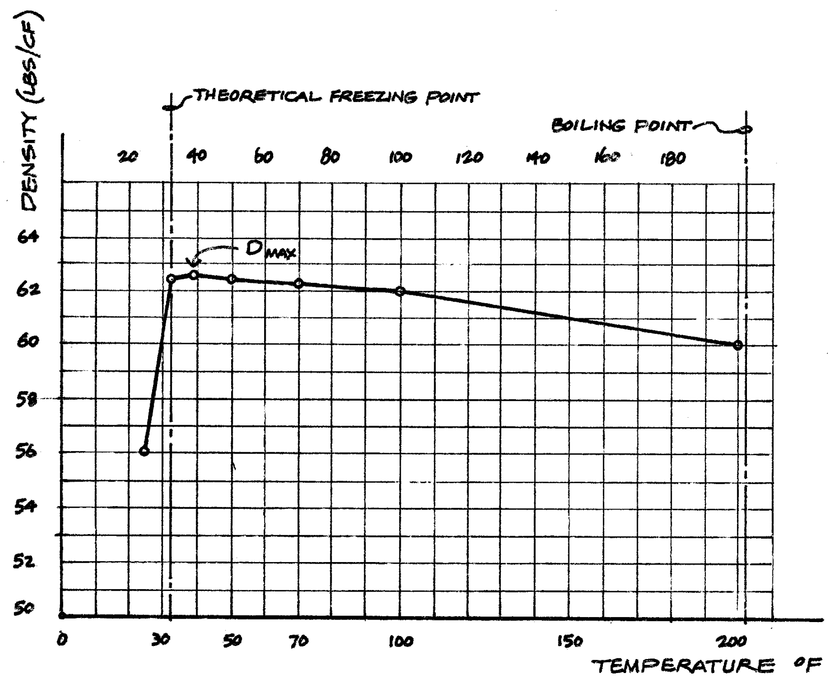
$$D_1 = [W_1 + (V_2 . D_2)]/V_1$$

Figure 3.1 also reflects the fact that for flotation to take place, the weight of liquid displaced must be equal of larger than the weight of the submerged body.

Although Archimedes never had the opportunity to experiment with the effect of temperature fluctuations on the density of fluids, we now know that not only density varies from liquid to liquid, but also that temperature changes tend to affect density and therefore, volumetric changes within the same liquid. Enclosed Figure 3.2 (as well as Table 3.1) show those variations particularly applied to fresh water. Temperatures have been shown within a spectrum from 25 to 200 degrees Fahrenheit, with the maximum attainable density occurring at 39 deg F. Further, it must be stated here that diluted sodium chloride (salt) as it is the case in sea water, acts as temperature retardant and volume stabilizer.



SUBMERGED BODY IN LIQUID MEDIUM
FIGURE 3.1



$D_{MAX.}$ = FRESH WATER MAX. ATTAINABLE DENSITY.

FRESH WATER VOLUMETRIC CHANGES INDUCED BY TEMPERATURE FLUCTUATIONS.

FIGURE 3.2

TABLE 3.1*
Density of different liquids

Liquid	Temperature (Deg. F)	Density (lbs/CF)	Remarks
Pure Alcohol		49.0	
Mineral Oils		58.0	
Vegetable Oils		59.0	
Freezing Fresh Water	About 25	56.0	
Fresh Water	32	62.40	
Fresh Water	39	62.50	Max. attainable density.
Fresh Water	50	62.40	
Fresh Water	70	62.30	
Fresh Water	100	62	
Fresh Water	200	60	
Brackish Water		63	As in the Biscayne Canal.
Sea Water		64	

*Above values have been directly reproduced, or either derived or interpolated from sources such as: "Standard Density Tables" of the U.S. Bureau of Standards and the ASCE Manual for Hydraulic Studies.

In the case of a steel barge, as in the example we will have the opportunity to examine at the end of this course, although the density of the steel was a lot greater than that of water, it still floated because said barge was hollow and therefore capable of displacing a large volume of water, which weight was considerably larger than the weight of the steel hull plus the payload.

4.0 CASE HISTORY

One of the largest steady clients of our office in South Florida was providing services in three areas of business: road construction, erection of bridge structures and marine construction. In order to perform at a high level of efficiency and maintain a production volume of over twenty million dollars a year, it was necessary for them to have in addition to an adequate working capital, a well trained personnel team and a wise and considerable investment in construction equipment.

When Hurricane Andrew swept by the area in August 24, 1992, they sustained painful damage to their equipment, especially those pieces that were on water in several locations of the South Florida coast. It took some time for the company to recover, salvage and repair all of their equipment. Early in the year 1994 they requested from us to prepare an evaluation of a diesel pile driver unit mounted on a steel barge and determine its worthiness.

During the hurricane, the ½ in. thick steel hull was impacted several times by flying objects, some were insignificant dents but in two cases the hits affected some of the welded joints which in turn allowed water seepage and slow yet steady incoming leaks. Since the barge was left inactive in the aggressive saline environment of the Biscayne Canal for a year and a half with either very little or no maintenance at all, rust had started to get the best of it and some of the steel beams and purlins were showing significant loss of section in critical areas of the vessel, while two of the sealed chambers were partially flooded at the time of our examination.

The total dead weight was approximately 136 tons and the payload estimated as 282 for a combined total load of 418 tons.

Following the Principle or Archimedes and based on a displaced water volume of 19,310 cubic feet, the *ultimate buoyancy capacity* was calculated as 604 tons and the *critical sinking load* after subtracting the dead weight, as 468 tons, with a resulting *safety factor* of 1.65 for the operational loads. This safety factor although too modest for a sea-worthy barge was however adequate for a stationary vessel, as it was the case at hand.

Please notice that on calculation sheet #1 we used a water density of 62.50 lbs/cf to determine buoyancy but could have well used 63.0 representing the density of brackish water according to the herein Table 3.1, with a resulting deviation of +0.8% which could be added to the safety factor shown in the above paragraph, as well as in calculation sheet #2 and in the cover letter dated 03/21/1994.

Since the largest portion of the live load was carried by the framing over chambers 4 and 5 towards the center of the vessel, we examined that part of the structure in detail and considering the section losses due to steel corrosion, conservatively estimated a 50% equivalent reduction in computed bending stresses.

Our assessment of signs of overstress in some structural components brought attention on the need to adopt a program for basic repairs mainly consisting of sandblasting, grinding and brushing off all rust, replacement of corroded members, reinforcement of those with lesser damage, repair of all steel hull leaks, application of two rust inhibiting coatings and finishing paint to complete the refurbishing process.

The enclosed cover letter dated March 21, 1994 addressed to the client's representative, as well as calculation sheets #1 through #10 covered in part, the extent of our work on the above described case.

RUBEN A. GOMEZ, P.E.

Structural Engineer
1375 W. Hillsboro Blvd., Suite 131
Deerfield Beach, FL 33442
(407) 362-8210

March 21, 1994

Mr. Nelson Fulcher
[redacted] Corporation
1200 N.E. 105th Street
Miami Shores, Florida 33138

Re: Bridge over Biscayne Canal
-Barge Analysis-

Dear Mr. Fulcher,

At your request, the barge at above site was analyzed to determine performance based on its present condition.

A total distributed load of 564,020 lbs. was used to determine buoyancy, which resulted in a safety factor of 1.65. The largest part of such load was considered applied to the central chambers marked 4 & 5, with an allowance of 334, 824 lbs., including the crane weight, a concrete pile, spreaders, as well as an impact factor.

On the other hand, a 50% reduction in allowable stresses was introduced to account for section loss and welding deterioration. Based on such scenario, the steel purlins were found to be 5% overstressed, while beams were 29% overstressed.

Some diagonals at the end chambers need to be either replaced or reattached. Due to flooding conditions, some low areas of the hull could not be properly inspected.

Should you have any comments or questions regarding this matter, please do not hesitate to contact me at your earliest convenience.

Very truly yours,

Ruben A. Gomez, P.E.

RG:kc.
Encl.

RUBEN A. GOMEZ, P.E.
STRUCTURAL ENGINEER

JOB: BARGE ANALYSIS
SHEET NO. ___ OF 10
CALCULATED BY: R.G. DATE: 03/21/94
CHECKED BY: ___ DATE: ___
SCALE: ___

DEAD WEIGHT:

DECK:	110x30x16.37 =	54,021	LBS.	(3/8" PL.)
	110x30x14.90 =	49,170	"	
HULL:	2x643.65x21.47 =	27,638	"	(1/2" PL.)
	2x2x30x21.47 =	2,576	"	
	2x643.65x8.02 =	10,324	"	
	2x2x30x8.02 =	962	"	
	111.20x30x21.47 =	71,624	"	(1/2" PL.)
	111.20x30x14.95 =	49,873	"	
STABIL.:	(4x10x65)+(4x400) =	4,200	"	
MISCEL.:		800	"	
	W =	271,188	LBS.	

TOTAL VOLUME: 75x30x6.58' = 14,805 CF
[1/2(2+6.58)17.50x30]2 = 4,505 "
V = 19,310 CF

TOTAL SINKING WEIGHT: 19,310x62.50 = 1,206,875 LBS
LESS (DEAD WEIGHT): (271,188)

NET SINKING LOAD: 935,687 LBS

NET OPERATIONAL LOAD:

CRANE:		200,000	LBS.
PRECAST PILES:	6x45,000 =	270,000	"
PILE DRIVER:		40,000	"
AUGER:		20,000	"
TIMBER:	27x1,260 =	34,020	"
	P =	564,020	LBS.

PRODUCT 204-1 NEBS Inc., Groton, Mass. 01471

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JOB: BARGE ANALYSIS
SHEET NO.: 2 OF: 10
CALCULATED BY: R.G. DATE: 03/21/94
CHECKED BY: DATE:
SCALE:

SAFETY FACTOR (BUOYANCY): $\frac{935,687}{564,020} = 1.65$

LOAD ON CHAMBERS: 4 & 5

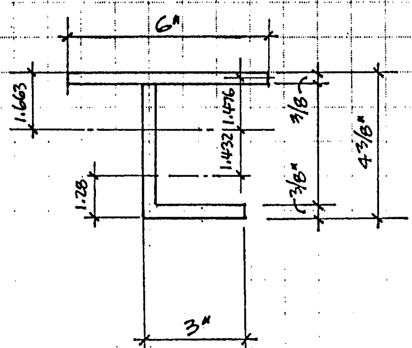
CRANE:	200,000 LBS.
CONCRETE PILE:	45,000 "
TIMBER SPREADERS:	34,020 "
	279,020 LBS.
IMPACT FACTOR:	× 1.20
	334,824 LBS.

LIVE LOAD: $\frac{334,824}{750} = 446.43$ PSF

DEAD LOAD: 31.27 ~ 478 PSF

PURLINS: L4×3×3/8"

$M_{MAX} = 0.10 \times 956 \times 8.33^2 = 6,633$ LB-FT.



$\Sigma A = (6 \times 0.375) + (7 \times 0.375) = 4.87$ in²

$A_{PLATE} = 2.25$ in² $I_{PLATE} = 0.026$ in⁴

$A_{ANGLE} = 2.48$ in² $I_{ANGLE} = 3.96$ in⁴

$(2.25 \times 0.1875) + (2.48 \times 3.095) = 4.87\, Y_g$

$(0.4218 + 7.6756) = 4.87\, Y_g$ $Y_g = \frac{8.0974}{4.87} = 1.663$ in

$\Sigma I_x = [0.026 + (2.25 \times 1.476^2)] + [3.96 + (2.48 \times 1.432^2)]$

$= (0.026 + 4.9016) + (3.96 + 5.0854) = 13.97$ in⁴

$S = \frac{13.97}{2.71} = 5.15$ in³

$M = 6,633 \times 12 = 79,596$ IN-LB $= f \cdot S$

$f = \frac{79596}{5.15} = 15,455.5$ PSI. ~ 15,456 PSI. } OVERSTRESSED!

$R = 1.10 \times 956 \times 8.33 = 8,763$ LBS.

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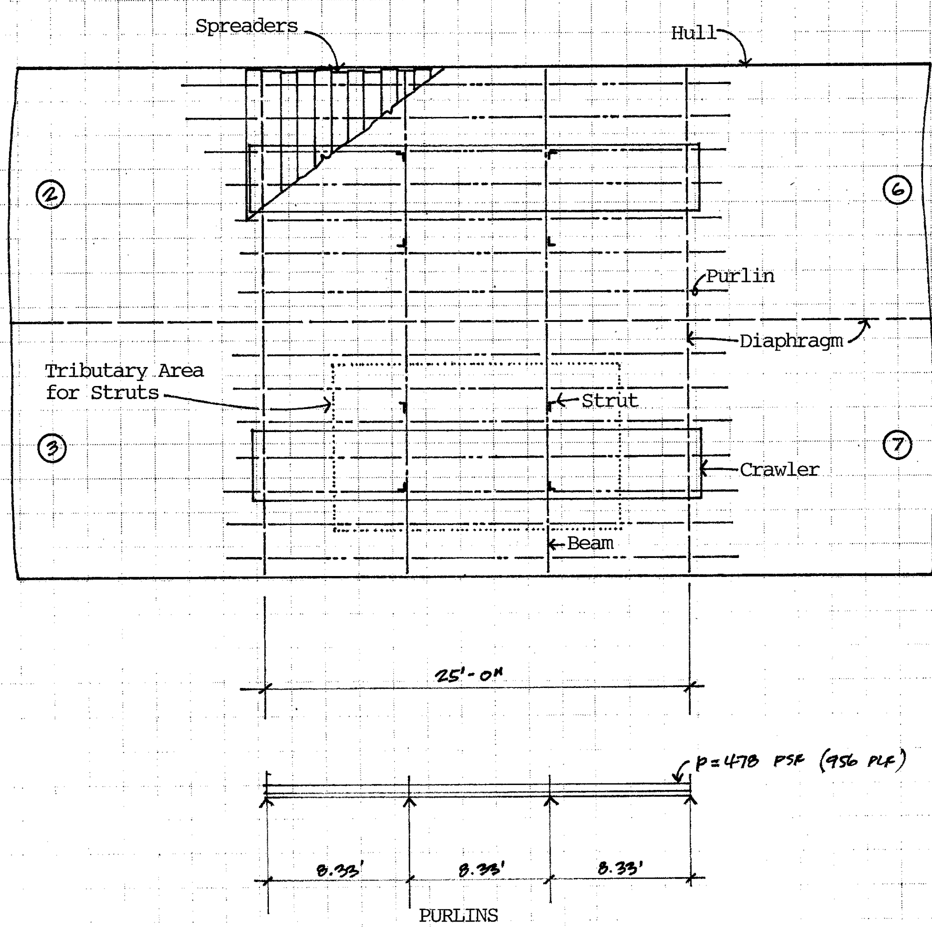
JOB BARGE ANALYSIS
SHEET NO. 3 OF 10
CALCULATED BY R.G. DATE 03/21/94
CHECKED BY DATE
SCALE



LOADING DIAGRAMS, CHAMBERS 4 & 5

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SHEET NO. 4 OF 10
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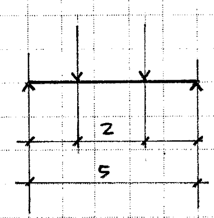
SCALE ____

BEAMS: (C 8 × 11.50)

$S = 8.14 \text{ in}^3$



WEB = 1/4 × 8 = 2 in^2

R = 8763#

SHEAR: $V = \frac{8763}{2} = 4,382$ PSI.

$M = 8763 \times 1.50 = 13,145$ LB-FT.
$= 157,740$ LB-IN

$f = \frac{157,740}{8.14} = 19,378$ PSI. OVERSTRESSED!

STRUTS: L3 × 3 × 5/16"

h = 5.01

CAP. = 23,000 LBS (f_c = 18,000 PSI).

LOAD: 10 × 17 × 478 × 1/4 = 20,315 LBS.

STRESS CRITERIA

A-36 STEEL

$f_u' = 36,000$ PSI

LESS: 50% LOSS DUE TO RUSTING/LOSS OF SECTION WELDING DETERTN.

$f_u'' = 18,000$ PSI.

SAFETY FACTOR: 1.20

$f_{ALLOW.} = 15,000$ PSI.

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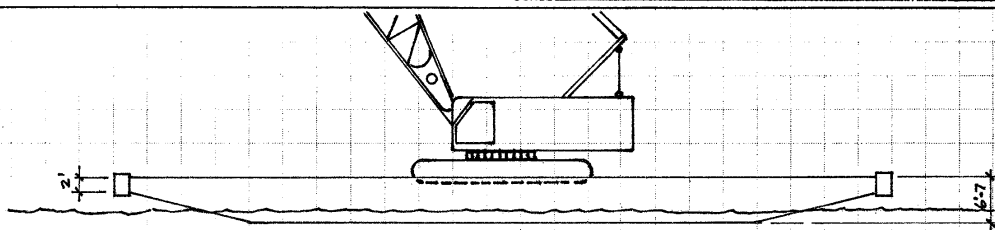
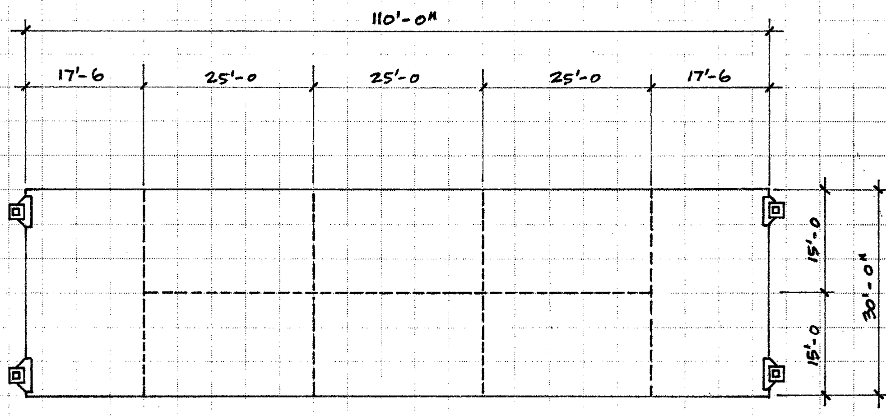
JOB: BARGE ANALYSIS
SHEET NO. 5 OF 10
CALCULATED BY: R.G. DATE: 03/21/94
CHECKED BY: DATE:
SCALE:



LATERAL VIEW
Scale: 1" = 20'



PLAN VIEW
Scale: 1" = 20'

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JOB: Barge Analysis
SHEET NO. 6 OF 10
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SCALE ______

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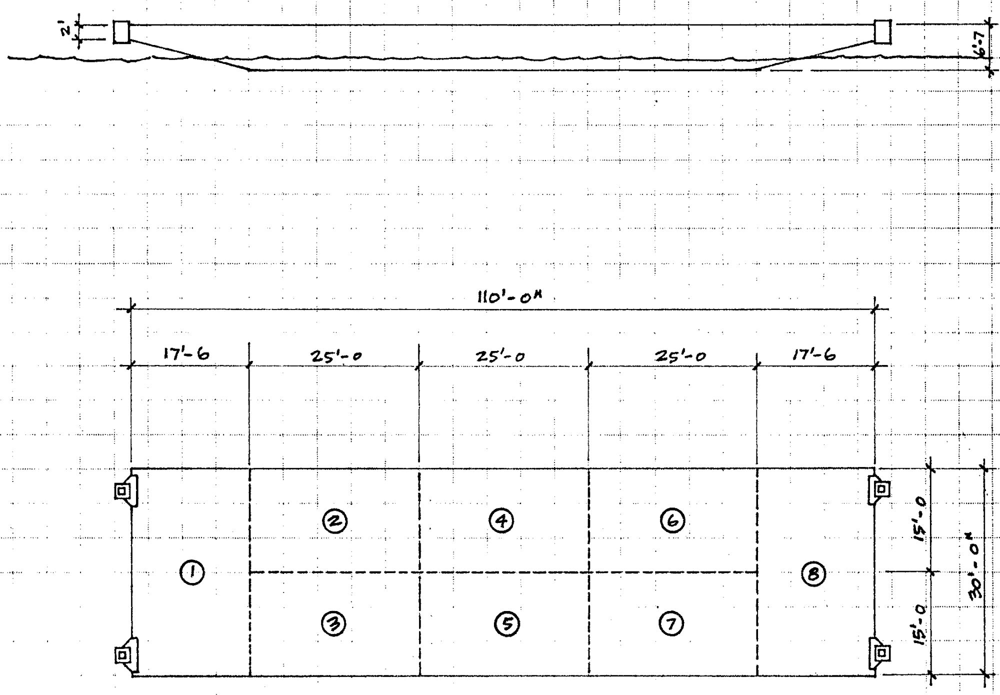

DESIGNATED CHAMBERS

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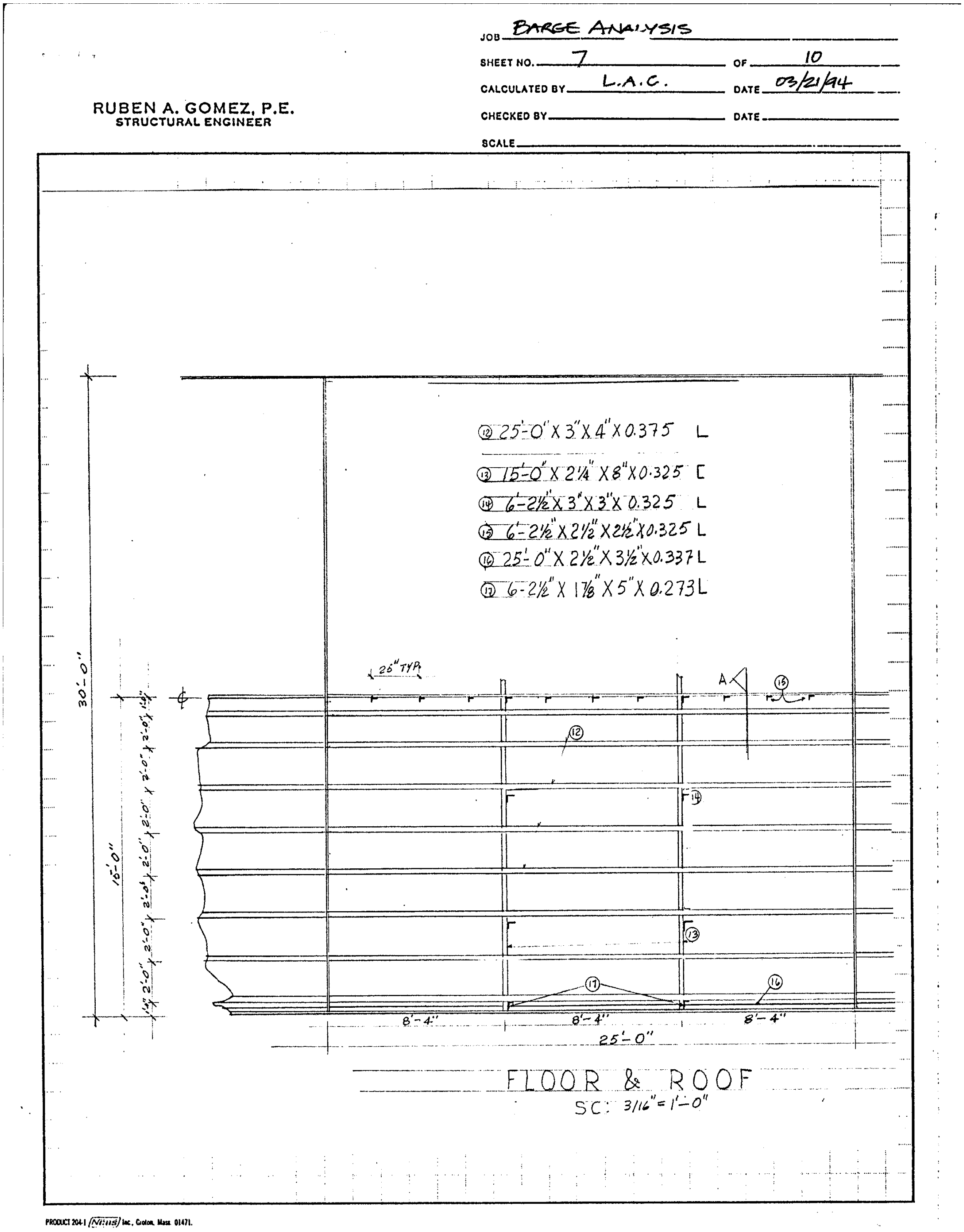
JOB BARGE ANALYSIS
SHEET NO. 7 OF 10
CALCULATED BY L.A.C. DATE 03/21/94
CHECKED BY DATE
SCALE
RUBEN A. GOMEZ, P.E.
STRUCTURAL ENGINEER
⑫ 25'-0" X 3" X 4" X 0.375 L
⑬ 15'-0" X 2¼" X 8" X 0.325 [
⑭ 6'-2½" X 3" X 3" X 0.325 L
⑮ 6'-2½" X 2½" X 2½" X 0.325 L
⑯ 25'-0" X 2½" X 3½" X 0.337 L
⑰ 6'-2½" X 1⅞" X 5" X 0.273 L
25" TYP.
30'-0"
15'-0"
8'-4"
25'-0"
FLOOR & ROOF
SC: 3/16" = 1'-0"

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SHEET NO. 8 OF 10
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SCALE

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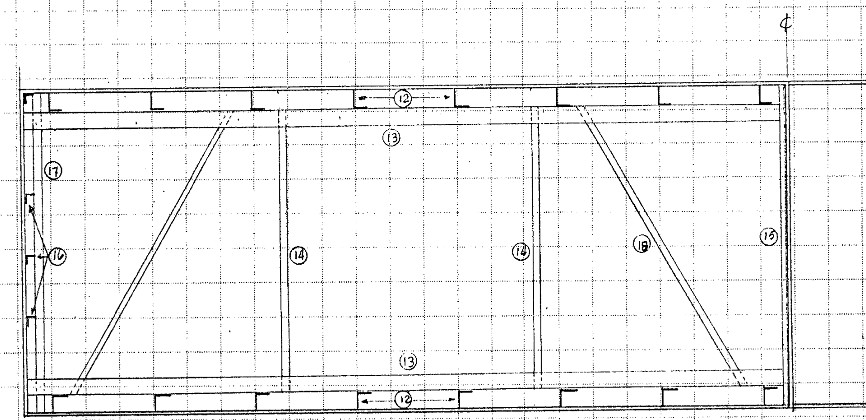

SECTION A (CHAMBERS 2 THRU 7)
SC. 3/8" = 1'-0"

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SCALE

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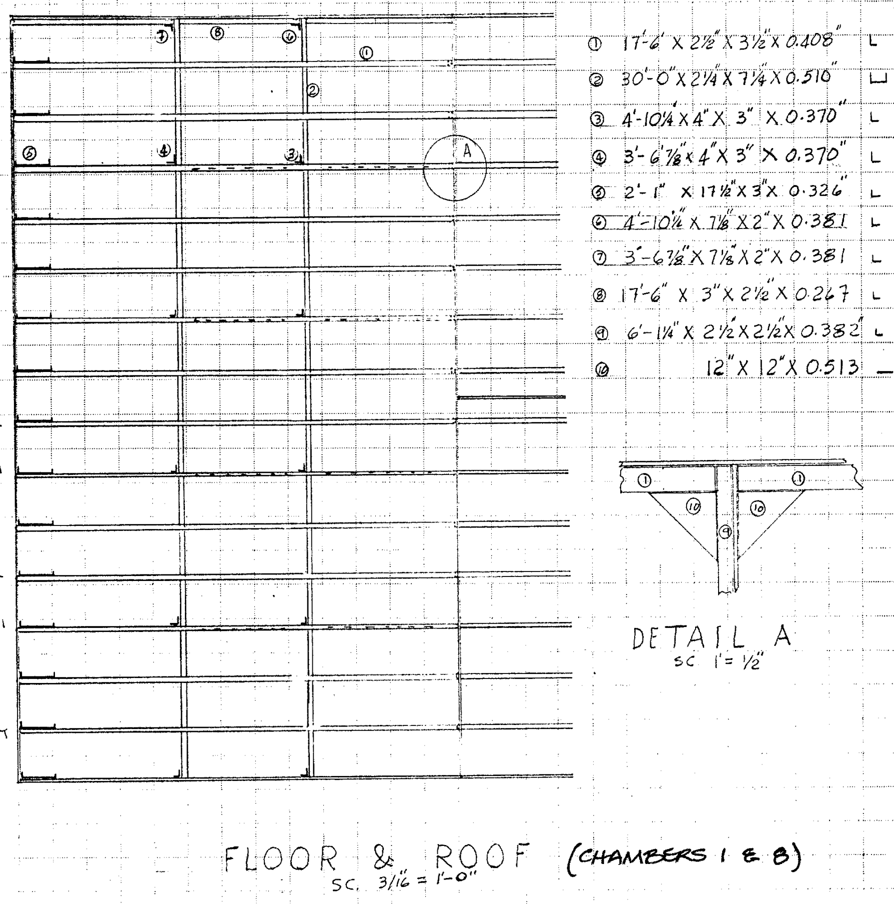

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SHEET NO. 10 OF 10
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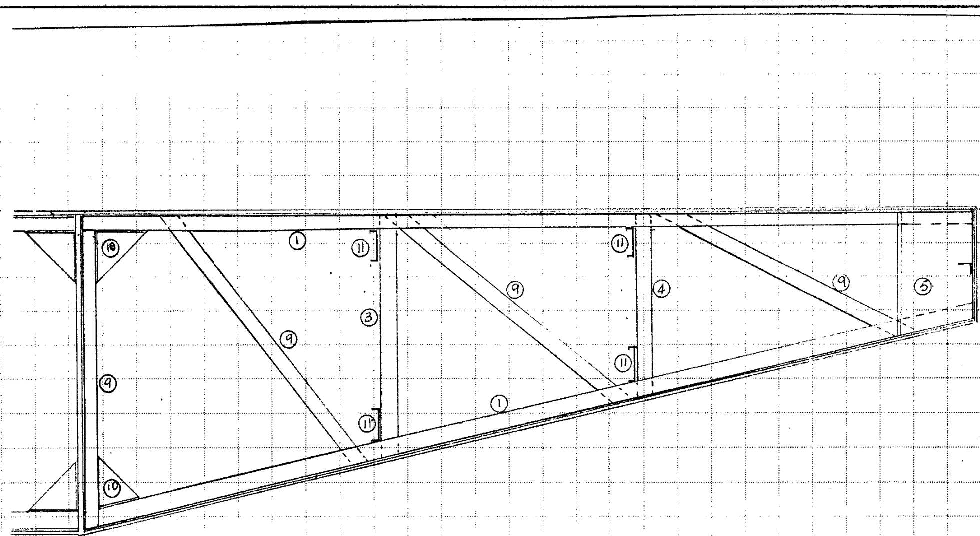
SCALE:



END TRUSS DETAIL
SCALE: 3/8" = 1'-0"

(11) CHANNEL 2 1/4" x 7 1/4 x 0.50"

5.0 CONCLUSION

Archimedes (287-212 B.C.) is credited with being the greatest mind that Greece offered to the world, his genius surpassed Aristotle and even Democritus. He was not only the greatest physicist of the ancient world, but one of the greatest mathematicians of all times. Further, his well known treatise titled "On Equilibrium" firmly establishes him as the undisputed founder of *statics.* He also introduced the concept of the *center of gravity,* and his work in geometry provided the foundation for Newton and Leibnitz to develop *infinitesimal calculus.*

Archimedes' Principle which we have used in this course, has provided us engineers in general with a foundation for *hydraulics* and particularly to *naval engineers* with the concept of buoyancy, which is of vital importance in the practical application of *vessel design* and *shipbuilding.*

In closing, it must be said that sometimes we forget that all this knowledge we have at hand and use on daily basis to assist us in our routine work, is possible and partially paraphrasing the words of Sir Isaac Newton, "because we are [indeed midgets] standing on the shoulders of giants". Consequently, we would like these final words to stand as a humble tribute to all those ingenious minds of the past for their awesome and immeasurable legacy of scientific knowledge.

APPENDIX A

In the third paragraph of the foreword we have made reference to vessels built out of materials other than wood, such as, steel, aluminum, fiberglass and wire reinforced concrete or ferro-cemento as it is commonly called in the countries of southern Europe.

Further, the matter appears again in the course quiz. Therefore, we felt necessary and convenient that the density of those materials should be made part of the herein given information, so here they are:

Material	**Density**	
	lbs/ci	**lbs/cf**
Ponderosa wood (wet)	0.024	41
Carbon steel	0.284	490
Aluminum (alloy 5456)	0.096	166
Fiberglass	0.055	95
Wire reinforced concrete	0.084	145