



PDHonline Course E336 (6 PDH)

Calculating Currents in Balanced and Unbalanced Three Phase Circuits

Instructor: Joseph E. Fleckenstein, PE

2020

PDH Online | PDH Center

5272 Meadow Estates Drive
Fairfax, VA 22030-6658
Phone: 703-988-0088
www.PDHonline.com

Calculating Currents in Balanced and Unbalanced Three Phase Circuits

Joseph E. Fleckenstein, P.E.

Table of Contents

Section Description	Page
1. Introduction.....	1
2. General Information.....	2
2A. Common Electrical Services.....	2
2B. Instantaneous Voltage and Instantaneous Current	3
2C. RMS Voltage and RMS Current	5
3. Single Phase Circuits	8
3A. Single Phase Resistive Loads.....	8
3B. Leading and Lagging Power Factor	10
3C. Phasor Diagrams of Single Phase Circuits.....	12
3D. Parallel Single Phase Loads	15
3E. Polar Notation.....	17
4. Balanced Three Phase Circuits	18
4A. Voltages in Three Phase Circuits - General.....	18
4B. Calculation of Power in a Balanced Three Phase Circuit	20
4C. Phasor Diagrams of Three Phase Circuits.....	22
4D. Calculating Currents in a Balanced Three Phase Delta Circuit –General	23
4D.1 Resistive Loads	23
4D.2 Capacitive Loads.....	27
4D.3 Inductive Loads.....	29
4D.4 Two or More Loads	31
4E. Calculating Currents in a Balanced Three Phase Wye Circuit - General.....	38
4E.1 Resistive Loads.....	39
4E.2 Inductive Loads	40
4E.3 Capacitive Loads	41
5. Unbalanced Three Phase Circuits	41
5A. Unbalanced Three Phase Circuits - General	41

5B Unbalanced Three Phase Delta Circuits with Resistive, Inductive or Capacitive Loads - General.....	41
5B.1 Unbalanced Three Phase Delta Circuits with Resistive, Inductive or Capacitive Loads.....	42
5B.2 Unbalanced Three Phase Delta Circuit with Only Resistive Loads.....	51
5C. Unbalanced Three Phase Wye Circuit	54
5D. Combined Unbalanced Three Phase Circuits	56
5E. Power Computation and Power Factor.....	67
6. Summary of Course Content.....	69
7. Summary of Symbols and Equations.....	70
7A. Symbols.....	70
7B. Equations.....	71
8. References.....	81

COURSE CONTENT

1. Introduction

The importance of three phase circuits is well recognized by those who deal with electricity and its use. Three phase electrical sources are the most effective means of transmitting electrical current over long distances and three phase motors offer many advantages over single phase motors. While the electrical service delivered to residences in the United States is commonly single phase, larger users typically are served with a three phase electrical service.

In general three phase loads are considered either “balanced” or “unbalanced”. A three phase circuit is considered balanced if the voltages, currents and power factors in all three phases are identical. Conversely, when any of these parameters are not identical the circuit is classified as unbalanced. The computations of electrical properties of balanced loads are relatively straightforward and may be performed by simple computations. On the other hand, the calculations of the electrical properties of unbalanced three phase circuits become somewhat more complicated. To determine currents in unbalanced circuits a greater understanding of the subject is required.

For a variety of reasons it often becomes necessary to calculate the currents in both balanced and unbalanced three phase circuits. For example, the magnitude of the currents may be needed to properly size conductors, conduits, relays, fuses, circuit breakers, transformers and the like. Furthermore, the calculations of currents are often needed to demonstrate that an installation will be in accordance with applicable codes, as the National Electrical Code (NEC).

This course presents the means for calculating currents in the conductors of both balanced and unbalanced three phase circuits. Numerous diagrams and examples are used to illustrate the principles that are involved in relatively simple concepts. Balanced circuits are treated first. The principles pertinent to balanced circuits provide a convenient basis for the principles used to analyze the more complicated unbalanced circuits. The concept of phasors is introduced first with balanced circuits. Subsequently, the step to using phasors diagrams to analyze unbalanced circuits is easily taken.

As demonstrated in the course, phasors diagrams assist a person to visualize what is happening in an electrical circuit. By a technique commonly known as “vector-algebra,” phasor diagrams are combined with algebraic expressions to explain, in simple terms, how currents are calculated in the respective three phase circuits. The resulting equations that are applicable to the various types of circuits are introduced in “cookbook” fashion. The result is that currents may be calculated by easily applied methods.

The course considers the two common types of three phase circuits, namely the common “delta” circuit (which is so named because of the resemblance of the configuration to the Greek symbol “ Δ ”) and the “wye” circuit which is also called a “star” or “Y” circuit.

Unbalanced three phase circuits often present the need to calculate line currents based on knowledge of phase currents and power factors. Another frequently encountered need is the requirement to determine net line currents in a feeder that delivers power to a mix of two or more three phase loads each of which may be in a delta or a wye circuit and balanced or unbalanced. The course offers methods to meet all of these needs by means of easily followed procedures.

Complex variables as well as polar notations are often found in texts on the subject of three phase electricity. At times both can be helpful to understand and resolve three phase computations. On the other hand, their use can introduce complications and confusion. For this reason neither complex variables nor polar notation are used in the computations of the course. Nevertheless, the relationship between the often-used polar notation and the symbology of the course is briefly explained.

2. General Information

2A. Common Electrical Services

In the United States, electrical utilities usually supply small users with a single phase electrical source. A residence would typically be serviced with a three-wire 120/240 VAC source and the electrical service would commonly be divided within the residence into both 120 VAC and 240 VAC circuits.

Within a residence the 240 VAC branch circuits would be used to power

larger electrical appliances as ranges, air conditioning units, and heaters. The 120 VAC circuits would be used for convenience outlets and smaller loads.

Users of large amounts of electrical power, as commercial buildings or industrial installations, are generally supplied with a three phase electrical supply. The three phase services could be either the three-wire or the four wire type. Within commercial and industrial installation, circuits would typically be divided into both single phase and three phase circuits. The three phase circuits would be used to power motors whereas the single phase branches of the three phase service would typically be used for lighting, heating and fractional horsepower motors. A common electrical service to commercial and industrial users would be 480-3-60. (The “480” designates 480 volts, the “3” designates three phase, and the “60” designates 60 hz.) If the user has individual motors greater than, say, 500 HP, the voltage of the electrical service would very likely be much greater than 480 volts and could be as high as 13,800 volts.

Before moving onto three phase circuits, it is helpful to first review and understand the principles and terminology applicable to single phase circuits.

2B. Instantaneous Voltage and Instantaneous Current

Consider in the way of illustration, a typical electrical service to a residence. In the United States a 120/240 VAC service to a residence would normally be similar to the schematic representation of Fig. 1. The service would consist of three conductors. The neutral conductor would be very near or equal to ground potential and would be connected to ground either at the utility transformer or at some point near the residence. If the guidelines of the NEC are being observed, the neutral conductor within the building must be colored either gray or white in color. There is

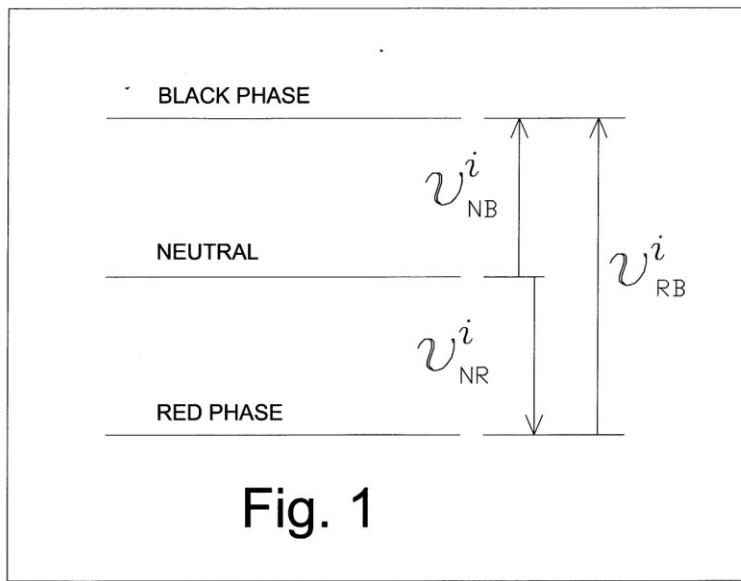


Fig. 1

no requirement in the NEC for color coding of the two “hot” (120 VAC to ground) conductors, but these conductors are often colored red and black, one phase being colored black and the other colored red.

If, say, an oscilloscope would be used to view the instantaneous voltages of a single phase residential service, a trace of the voltages would resemble the depiction of Fig. 2. A pair of leads from the oscilloscope would be connected to the neutral wire and a black phase conductor with the (–) lead common to a neutral conductor and the (+) lead common to a black phase conductor. The oscilloscope would show a trace similar to the V_{NB}^i trace of Fig. 2. If another set of the oscilloscope leads is connected to the neutral and the red phase, with the (–) lead on the neutral and the (+) lead on the red conductor, the trace would be similar to that shown for V_{NR}^i in Fig. 2. With the (–) lead on the red conductor and the (+) lead on the black conductor the trace would be that shown as V_{RB}^i in Fig. 2.

The traces of Fig. 2 represent a single cycle. (The “*i*” notation is used to distinguish instantaneous values of voltage or current from rms values, as explained below.)

The time period from $t = 0$ to $t = t_4$ in Fig. 2 would be the time for a single cycle. In the United States, the common frequency of alternating current is 60 hertz (60 cycles/second). Thus, the time for a single cycle would be 1/60 second, or 0.0166 second and the time from $t = 0$ to $t = t_1$ would be (1/4) (1/60) second, or 0.004166 second.

As mentioned above, the trace of V_{NB}^i in Fig. 2 is a representation of the instantaneous voltage of a typical 120 VAC service. The trace can be described by the algebraic relationship

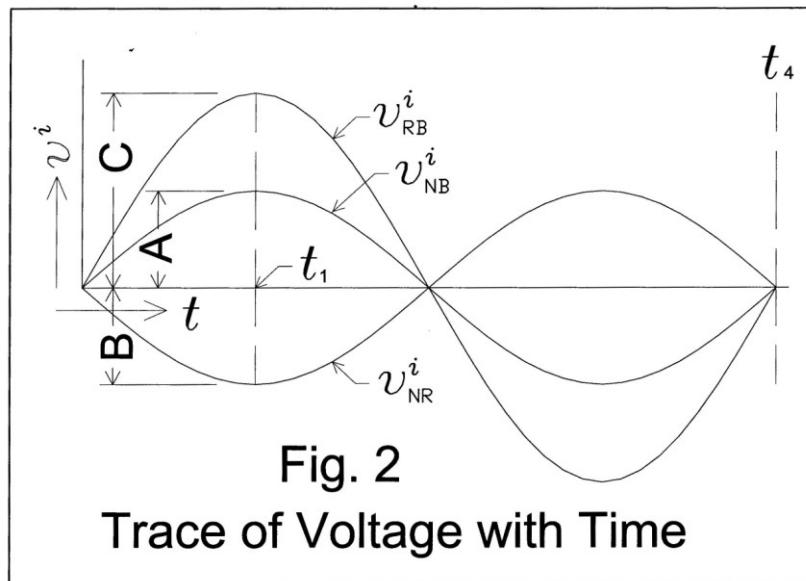


Fig. 2
Trace of Voltage with Time

$v^i_{NB} = A \sin \omega t$, where

A = value of voltage v^i_{NB} at time t_1

Since the value of v^i_{NB} at time t_1 is equal to the peak value of v^i_{NB} ,

$v^i_{PK} = A$, and

$v^i_{BN} = (v^i_{PK}) \sin \omega t$

Where,

$\omega = 2\pi f$ (radians)

f = frequency (hz)

t = time (sec)

Similarly,

$v^i_{NR} = B \sin \omega t$, and

$v^i_{RB} = C \sin \omega t$

In general,

$v^i = (v_{PK}) \sin \omega t \dots \text{Equation 1}$

where,

v^i = instantaneous value of voltage (volts)

v_{PK} = peak value of voltage (volts)

Much as with instantaneous voltage, instantaneous current can also be described as a function of time by the general relationship,

$i^i = i_{PK} \sin (\omega t + \theta_{SP}) \dots \text{Equation 2}$

Where,

i^i = instantaneous value of current (amps)

i_{PK} = peak value of current “ i^i ” (amps)

θ_{SP} = angle of lead or angle of lag (radians) (current with respect to voltage in a single phase circuit) (subscript “SP” designates single phase)

for a lagging power factor, $\theta_{SP} < 0$

for a leading power factor, $\theta_{SP} > 0$

2C. RMS Voltage and RMS Current

A trace of instantaneous voltage as obtained with an oscilloscope is of interest and educational. An oscilloscope trace provides a true visual picture of voltage and current as a function of time. Nevertheless, it is the values of root

mean square (rms) voltage and root mean square current that are of the most practical use. This is due to the common and useful analogy to DC circuits. In DC circuits, power dissipation is calculated by the relationship,

$$P = I^2R, \text{ or}$$

$$\text{since } V = IR, \text{ and } I = V/R,$$

$$P = [V/R] (IR) = VI$$

By common usage, these same formulas are also used for determining power as well as other parameters in single phase AC circuits. This is possible only by use of the rms value of voltage and, likewise, the rms value of current. The relationship between rms and peak values in AC circuits are given by the relationship,

$$V = (1/\sqrt{2}) V_{PK}^i = (0.707) V_{PK}^i, \text{ and}$$

$$I = (1/\sqrt{2}) i_{PK}^i = (0.707) i_{PK}^i$$

where

V = rms voltage, and

I = rms current

In general, rms voltage can be described as a function of time by the equation,

$$V(t) = V \sin(\omega t) \dots \text{Equation 3}$$

where,

$V(t)$ = voltage expressed as a function of time (rms volts)

V = numerical value of voltage (rms)

[Equation 3 assumes that at $t = 0$, $V(t) = 0$.]

Current in a circuit may lag by the amount θ_{SP} . So, the rms current may be described as a function of time by the relationship,

$$I(t) = I \sin(\omega t + \theta_{SP}) \dots \text{Equation 4}$$

where,

$I(t)$ = current expressed as a function of time (rms amps)

I = numerical value of current (rms)

θ_{SP} = angle of lead or angle of lag (radians) (current with respect to voltage in a single phase circuit)

for a lagging power factor, $\theta_{SP} < 0$

for a leading power factor, $\theta_{SP} > 0$

(Note: The expressions for rms voltage and rms current stated in Equation 3 and Equation 4, respectively, are consistent with common industry practice and are generally helpful in understanding electrical circuits. However, these expressions are not true mathematical descriptions of the rms values of voltage and current as a function of time. Expressed as a function of time, the curves of the rms values of voltage and current would have an entirely different appearance.)

Example 1

Problem: Mathematically express as a function of time the typical residential voltages of Fig. 1 and Fig. 2, given that the source voltage is a nominal “120/240 VAC” service.

Solution: If the nominal (i.e. rms voltage) is “120 VAC” then the “peak” value of voltage would be

$$V_{PK}^i = (1.414) V = (1.414) (120) = 169.7 \text{ volts}$$

Therefore the instantaneous value of voltage described as a function of time is given by the equation,

$$V^i = V_{PK}^i \sin \omega t = (169.7) \sin \omega t \text{ (volts)}$$

The 120 VAC (rms) voltage as a function of time would be,

$$V(t) = (120) \sin \omega t \text{ (volts), and}$$

The 240 VAC voltage as a function of time would be,

$$V(t) = (240) \sin \omega t \text{ (volts)}$$

Check!

At t_1 , $t = 0.004166$ (as stated above), and

$$\omega = (2\pi f)t = (2\pi)(60)(0.004166) = 1.570 \text{ radians}$$

So, $\sin \omega t = 1.0$, and

$$V_{PK}^i = V_{PK} \sin \omega t = 169.7 \text{ volts (peak value),}$$

$$V(t) = (120)(1) \text{ (volts)} = 120 \text{ volts, and}$$

$$V(t) = (240)(1) \text{ (volts)} = 240 \text{ volts}$$

Thus, the computation checks!

It may be also seen that

$$\mathcal{V}_{NR}^i = -\mathcal{V}_{NB}^i, \text{ and}$$

$$\mathcal{V}_{RB}^i = 2\mathcal{V}_{NB}^i$$

Since, \mathcal{V}_{NR} is the mirror image of \mathcal{V}_{NB} , the instantaneous voltage would be described by the relationship,

$$\mathcal{V}_{NR}^i = (-169.7) \sin \omega t \text{ (volts), and}$$

the voltage \mathcal{V}_{RB} would be described by the relationship,

$$\mathcal{V}_{RB}^i = (1.414)(240) \sin \omega t = (339.4) \sin \omega t \text{ (volts)}$$

Unless specifically stated otherwise in documents, voltages and currents are always assumed to be rms values. This would also be true, for example, of a multimeter that reads voltage or current unless the meter is set to read “peak” values. (Some multimeters have the capability to read peak voltages and peak currents as well as rms values.)

3. Single Phase Circuits

3A. Single Phase Resistive Loads

Application of a single phase voltage to a load would typically be represented as depicted in Fig. 3. The AC voltage source is represented by a symbol that approximates a single cycle of a sine wave. The load in Fig. 3 is represented by the letter “L”. If the load is purely resistive, the current is in phase with the voltage. Consider a single phase application in which the load is purely resistive. A typical

resistive load would consist of incandescent lighting or electrical resistance heaters. A trace of an applied (rms) voltage and the typical resultant (rms) current, expressed as a function of time, is represented in Fig. 4.

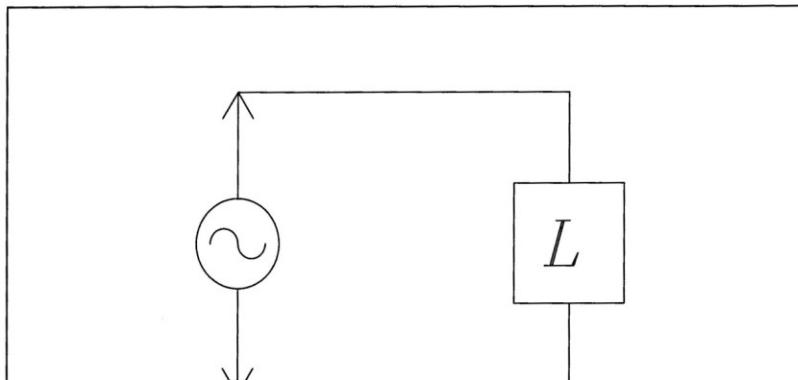


Fig. 3
AC Voltage Source and Load

Example 2

Problem:

Assume a nominal single phase source voltage of 240 VAC and a load that consists solely of a 5 kilowatt heater. Describe as a function of time: instantaneous voltage, instantaneous current, rms voltage and rms current.

Solution:

For a single phase application,

$$P = I^2 R = VI \text{ watts, where}$$

R = resistance of load (ohms)

V = voltage (rms)

I = current (rms)

$$P = 5000 \text{ (watts)} = (240) I$$

$$I = 5000/240 = 20.83 \text{ amp}$$

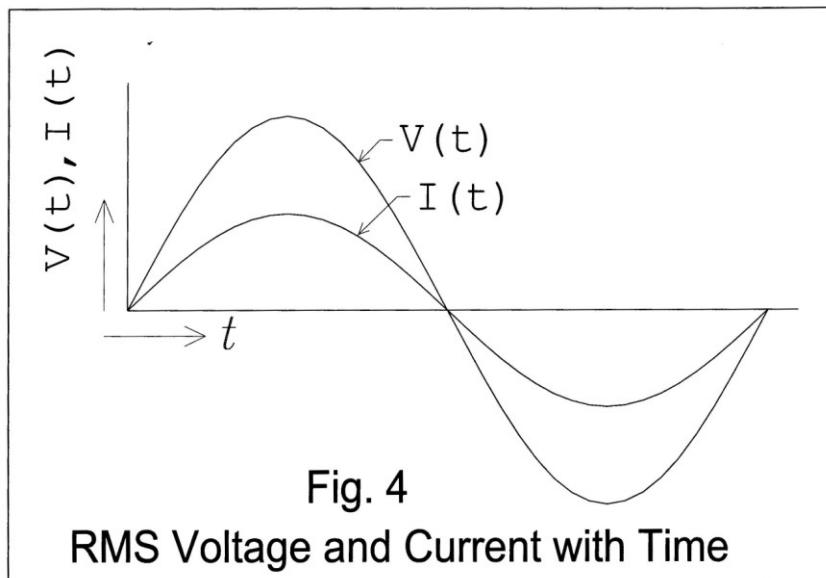


Fig. 4

RMS Voltage and Current with Time

So, the instantaneous voltage and the instantaneous current would be described by the relationships of above Equation 1 and Equation 2.

$$v^i = [(240)(1.414)] \sin \omega t \text{ (volts), or}$$

$$v^i = (339.5) \sin \omega t \text{ (volts), and}$$

$$i^i = [(20.83)(1.414)] \sin \omega t \text{ (amps), or}$$

$$i_i = (29.46) \sin \omega t \text{ (amps)}$$

According to common practice, the expression for the rms value of voltage as a function of time is represented by the expression,

$$V(t) = (240) \sin \omega t \text{ (volts)}$$

Since the load is resistive, $\theta_{SP} = 0$ and per Equation 4 the expression for rms current becomes,

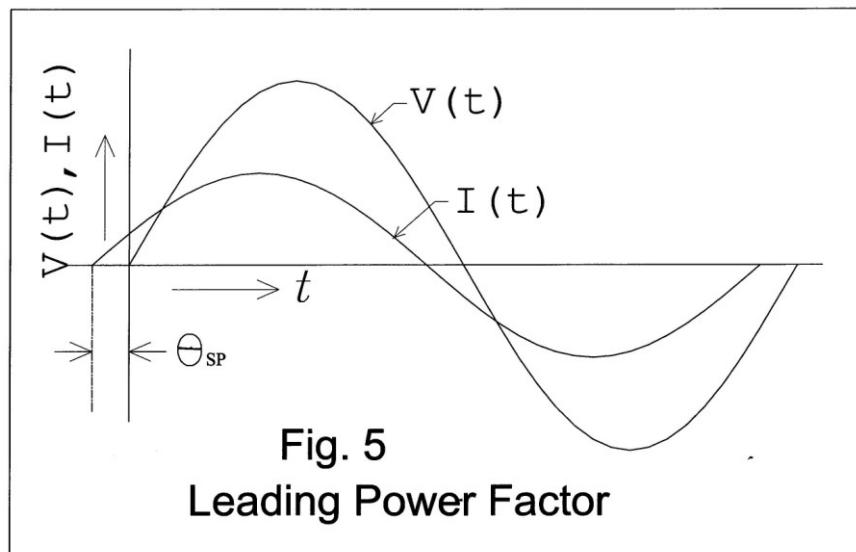
$$I(t) = (20.83) \sin \omega t \text{ (amps)}$$

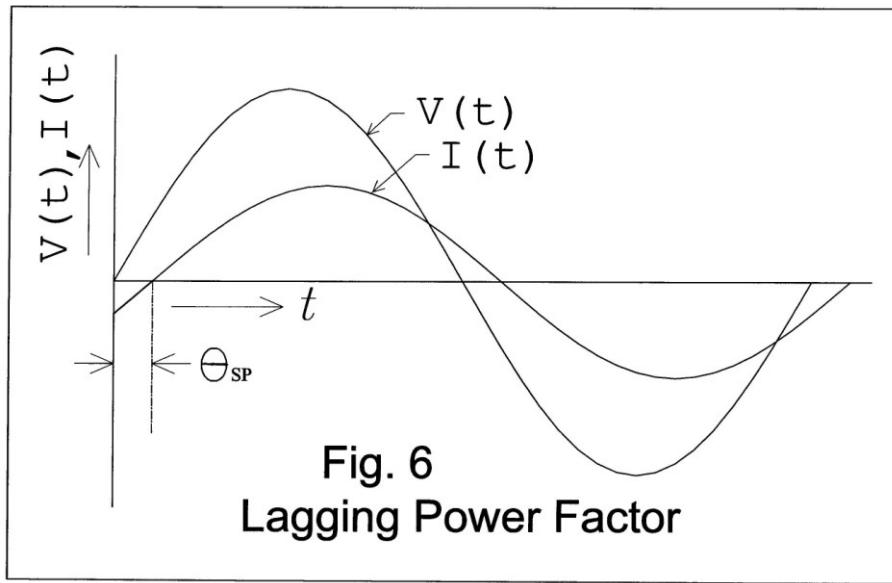
3B. Leading and Lagging Power Factor

The load represented in Fig. 3 could be resistive, inductive, capacitive or any combination of these possibilities. In the case of a resistive load, the current is in phase with the voltage. On the other hand, inductive or capacitive elements may cause the current to either lag or lead the voltage. Common inductive loads would include transformers, relays, motor starters or solenoids. A capacitive load would commonly be capacitors or electrical components that emulate a capacitor.

For a configuration of the type represented in Fig. 3 where the load consists of mostly capacitive elements the current would lead the voltage as shown in Fig. 5. The current leads by the amount θ_{SP} .

For a configuration of the type represented in Fig. 3 where the load consists of mostly inductive elements the current would lag the voltage as shown in Fig. 6. The current lags by the amount θ_{SP} .





Example 3

Problem: Assume that for the trace of voltage and current of Fig. 6, the value of θ_{SP} is -20° (i.e. lagging current). Find the actual value of time lag.

Solution:

$$180^\circ = \pi \text{ radians}$$

$$\theta_{SP} = - (20^\circ / 180^\circ) \pi \text{ radians, or}$$

$$\theta_{SP} = -0.3491 \text{ radians}$$

Then, from Equation 4

$$I(t) = I \sin(\omega t + \theta_{SP})$$

At $I(t) = 0$, $\sin(\omega t + \theta_{SP}) = 0$, and

$$\omega t = 2\pi f t = -\theta_{SP} = -0.3491 \text{ rad}$$

$$t = |-0.3491 / 2\pi f| = 0.000926 \text{ sec}$$

As explained above, for an AC, 60 hz applications, a single cycle (360°) would be 0.0166 second in length. If true, then the trace of i would cross the abscissa at

$$t = (20^\circ / 360^\circ) (0.0166) \text{ sec, or}$$

$t = 0.000926$, which checks!

The computation of “power factor” and power is important in both single phase circuits and three phase circuits.

In a single phase circuit,

$P = VI \cos \theta_{SP}$, where

P = power (watts)

V = voltage (rms)

I = current (rms)

θ_{SP} = angle of lead or angle of lag of current with respect to voltage (radians or degrees) in a single phase circuit

$\cos \theta_{SP}$ = power factor

In single phase circuits, the term “ θ_{SP} ” is a measure of the lead or lag of current with respect to the applied voltage and the value of $\cos \theta_{SP}$ is the power factor. The term “ θ_{SP} ” is negative for a lagging power factor and positive for a leading power factor. Nevertheless, the value of the power factor ($\cos \theta_{SP}$) is always a positive number (i.e. $PF > 0$). This is necessarily the case since the angle of lead or lag in a single phase circuit can be no less than -90° and no more than $+90^\circ$, and within that region the cosine of the angle of lead or lag is only positive. (So, the often-used industry term of a “negative” power factor as applied to a single phase circuit is somewhat of a misnomer. Power and power factor in three phase circuits are discussed in greater detail below.)

3C. Phasor Diagrams of Single Phase Circuits

In the study of AC circuits, and particularly three phase circuits, it is common to use phasor diagrams to depict the relationship between voltages and currents. (References 1, 2) A phasor diagram uses vectors similar to the types used in vector analysis to make a visual representation of the currents and voltages in a circuit. A good visualization of the currents and voltages helps ensure that any determinations of currents will be correct. While more

commonly used in the study of three phase circuits, phasors can also be used to represent the properties of a single phase circuit.

In the way of illustration, consider above Equation 1 which is for a single phase circuit. The circuit can be represented by a vector (or "phasor") as shown in Fig. 7. In Fig. 7 it is apparent that if the value of t or ωt is increased, the vector V would rotate counterclockwise about its base which is at the intersection of the abscissa and the ordinate. The

projection on the y-axis (ordinate) would then equal the term $[V \sin \omega t]$ as defined by Equation 3.

A vector representative of current can likewise be included in a phasor diagram as shown in Fig. 8.

In Fig. 8, the projection on the y-axis (ordinate) would equal the mathematical term $[I \sin \omega t]$ as defined by Equation 4. The configuration of Fig. 8 assumes that the current is in phase with the voltage so that $\theta_{SP} = 0$ in Equation 4.

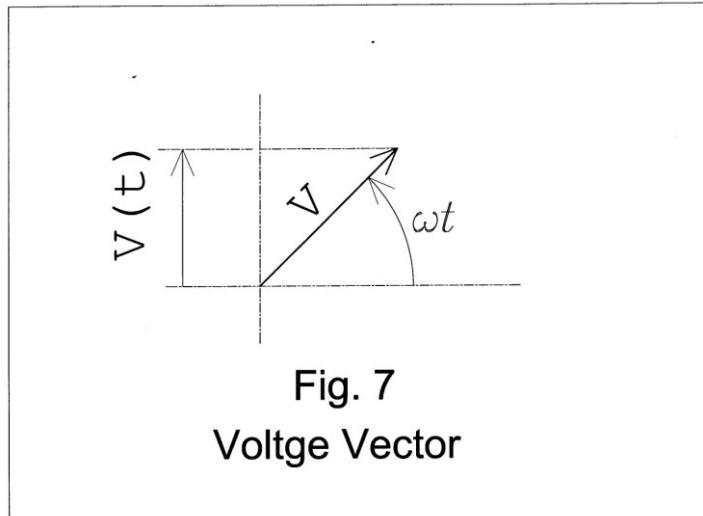


Fig. 7
Voltage Vector

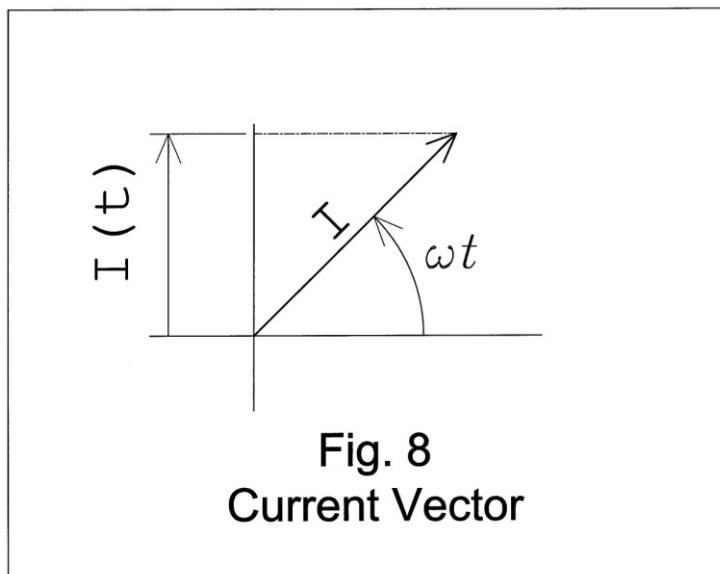


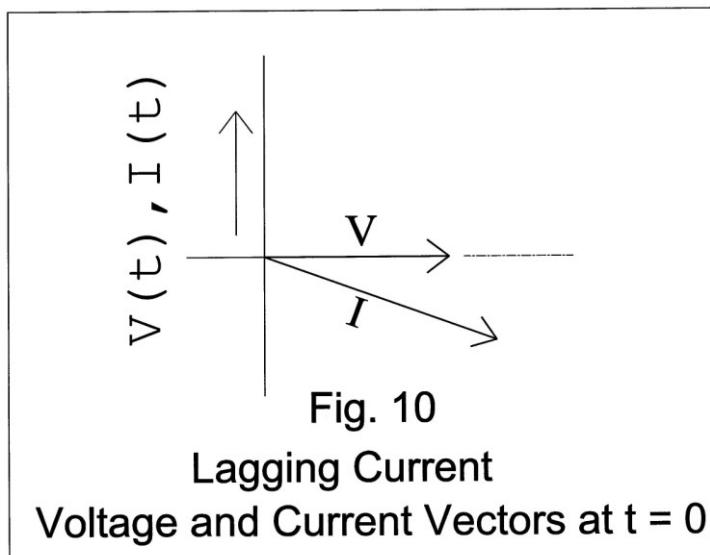
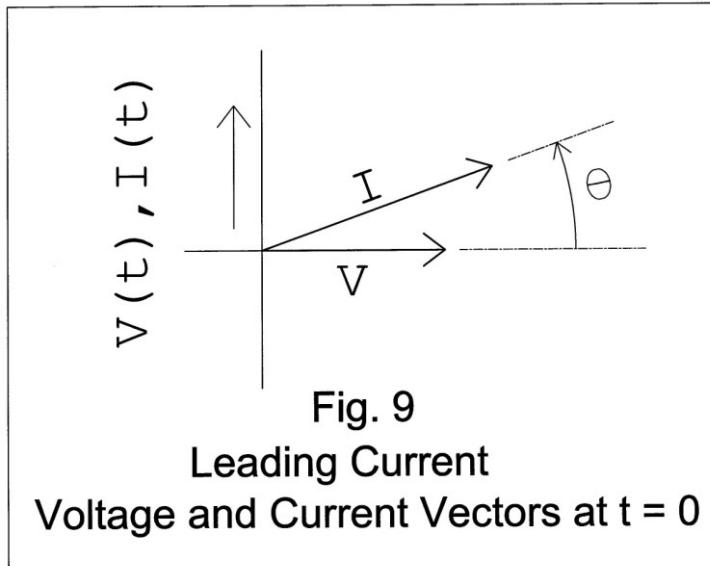
Fig. 8
Current Vector

If current leads the applied voltages described by Equation 4, and which is represented by the plot of Fig. 5, the associated voltage and current vectors would be similar to that shown in Fig. 9. The representation of Fig. 9 is for a particular point in time, namely at $t = 0$. When the current leads the applied voltage, $\theta_{SP} > 0$.

If current lags the voltage as described by Equation 4, and which is represented by the plot of Fig. 6, the associated voltage and current vectors, or phasors, would be similar to that shown in Fig. 10. When the current lags the applied voltage (as represented in Fig. 10) the measure of lag is $\theta_{SP} < 0$.

Above, it is shown that the projections of the rotating vector V with time are the mathematical equivalent of the (rms) current with time. Phasor diagrams are less concerned with the time variable and examine electrical properties at a selected time. So to speak, phasor diagrams look at electrical values with electrical properties frozen in time.

Phasor diagrams are especially helpful in assisting a person to visualize AC



circuits, single phase or three phase, with power factors other than unity. As demonstrated in the below illustrations, phasor diagrams are particularly valuable when analyzing three phase circuits. A phasor diagram not only provides a visual picture of the relationships between the voltage and current in a selected phase, it also helps a person to understand the relationship of current and voltage in one phase to the voltage and current in another phase of a three phase circuit.

3D. Parallel Single Phase Loads

The merits of a phasor diagram become apparent when considering parallel circuits with different power factors. A typical single phase AC circuit with parallel loads is represented in Fig. 11. One of the loads is represented by L_1 and the second, parallel load is represented by L_2 . The current of load L_1 is assumed to be I_1 at power factor $\cos \theta_1$ and the current of load L_2 is I_2 at power factor $\cos \theta_2$. Since both L_1 and L_2 are subject to the same voltage (V_{ab}), the currents may both be referenced to that voltage as shown in Fig. 12.

To construct a phasor diagram, a few of the rules from vector analysis are borrowed for the purpose. This is not to say that an understanding of vector analysis is required. Essentially, only two relatively simple rules need to be observed.

One rule pertains to the addition of vectors. The second rule is that $-I_{XY} = I_{YX}$, (i.e. the negative of vector I_{XY} is a vector that is of equal magnitude but pointing in a direction 180° from that of vector I_{XY} .)

When adding vectors, both magnitude and direction are important. Vectors may be added by adding the abscissa components to determine the abscissa

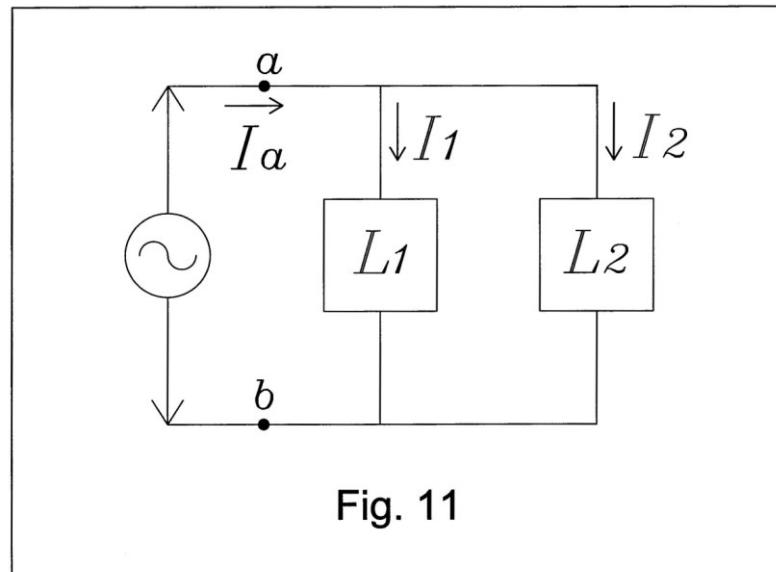


Fig. 11

component of the resultant vector.

Likewise, the ordinate components are added to determine the ordinate component of the resultant vector. A typical vector addition is represented in Fig. 12.

The angle of lag, θ_{SP} , of any vector is shown as positive in the

counterclockwise direction. The vectors I_1 and I_2 are added to determine vector I_a . The abscissa component of I_a is X_a which is the sum of X_1 and X_2 . The ordinate of I_a is Y_a which is the sum of Y_1 and Y_2 . For the vectors of Fig. 12,

$$X_a = I_1 \cos \theta_1 + I_2 \cos \theta_2, \text{ and}$$

$$Y_a = [I_1 \sin \theta_1 + I_2 \sin \theta_2]$$

$$(I_a)^2 = (X_a)^2 + (Y_a)^2, \text{ or}$$

$$I_a = \{(X_a)^2 + (Y_a)^2\}^{1/2} \dots \text{ Equation 5A}$$

$$\text{Also, } \sin \theta_a = Y_a \div I_a, \text{ or}$$

$$\theta_a = \sin^{-1} (Y_a \div I_a)$$

Obviously, if there are more than two currents comprising a line current,

$$X_a = I_1 \cos \theta_1 + I_2 \cos \theta_2 + \dots I_n \cos \theta_n, \text{ and}$$

$$Y_a = [I_1 \sin \theta_1 + I_2 \sin \theta_2 + \dots I_n \sin \theta_n]$$

Example 4

Problem: Assume that for the configuration of Fig. 11,

$I_1 = 10 \text{ amp} @ \text{PF} = 0.60$, leading and

$I_2 = 4 \text{ amp} @ \text{PF} = 0.90$, leading

Determine I_a and its power factor (PF)

Solution:

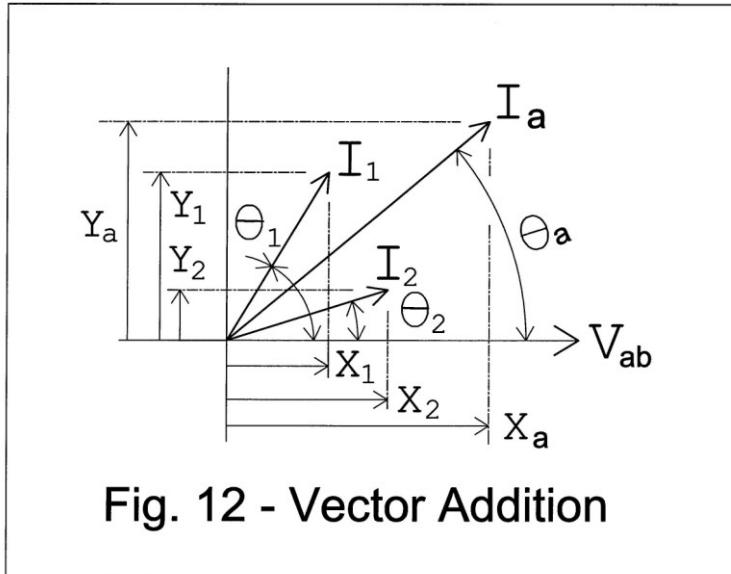


Fig. 12 - Vector Addition

From Equation 5A,

$$I_a = \{(X_a)^2 + (Y_a)^2\}^{1/2}$$

For I_1 , $\cos \theta_1 = 0.60$, $\theta_1 = 53.13^\circ$, and $\sin \theta_1 = 0.800$

For I_2 , $\cos \theta_2 = 0.90$, $\theta_2 = 25.84^\circ$, and $\sin \theta_2 = 0.4358$

I_a is determined by first adding the abscissa and ordinate components.

$$X_a = I_1 \cos \theta_1 + I_2 \cos \theta_2,$$

$$X_a = (10) (0.600) + (4) (0.900) = 6.000 + 3.600 = 9.600, \text{ and}$$

$$Y_a = [I_1 \sin \theta_1 + I_2 \sin \theta_2]$$

$$Y_a = [(10) (0.80) + (4) (0.4358)] = [8.00 + 1.7432] = 9.743$$

$$I_a = \{(X_a)^2 + (Y_a)^2\}^{1/2} = \{(9.600)^2 + (-9.743)^2\}^{1/2} = 13.678 \text{ amp}$$

$$\sin \theta_a = Y_a \div I_a = 9.744 \div 13.678 = .712$$

$$\theta_a = \sin^{-1} (Y_a \div I_a) = \sin^{-1} (9.744 \div 13.678) = 45.43^\circ$$

$$PF = \cos 45.43^\circ = 0.702$$

3E. Polar Notation

As stated in the Introduction, complex numbers are not used in this course to calculate current values. Nevertheless, polar notation is worth mentioning.

Polar notation is found in some texts that treat three phase currents and three phase voltages. The polar form can be used to describe the position of a voltage or a current on a phasor diagram. In polar notation, a current (amperage) or a potential (voltage) is described by the magnitude of the variable and angle in the CCW (Counter Clockwise) direction from the positive abscissa. In the way of illustration consider the currents of above Example 4. Currents I_1 and I_2 are described in the example as:

$$I_1 = 10 \text{ amp @ PF} = 0.60, \text{ leading } (\theta_1 = 53.13^\circ) \text{ and}$$

$$I_2 = 4 \text{ amp @ PF} = 0.90, \text{ leading } (\theta_2 = 25.84^\circ)$$

In polar notation, these two currents would be represented, respectively, as:

$$I_1 = 10 \angle 53.13^\circ$$

$$I_2 = 4 \angle 25.84^\circ$$

If the currents were lagging instead of leading, the representation would be:

$$I_1 = 10 \angle -53.13^\circ$$

$$I_2 = 4 \angle -25.84^\circ$$

Since, $360^\circ - 53.13^\circ = 306.87^\circ$, and $360^\circ - 25.84^\circ = 334.16^\circ$, the lagging

currents could also be expressed as:

$$\underline{I_1} = 10 \angle 306.87^\circ$$

$$\underline{I_2} = 4 \angle 334.16^\circ$$

In Example 4, current $\underline{I_a}$ is determined by adding the abscissa and ordinate components of $\underline{I_1}$ and $\underline{I_2}$. In vector notation the addition is represented by the expression:

$$\underline{I_a} = \underline{I_1} + \underline{I_2}$$

The underscores indicate that the variables are vectors and not algebraic values. Or,

$$\underline{I_a} = 10 \angle 53.13^\circ + 4 \angle 25.84^\circ$$

In Example 4 the voltage source is single phase and the positive abscissa would naturally be taken as the single phase voltage source. The angle of lead or lag would be in reference to that voltage. In three phase delta circuits the phase voltage is the same as the line voltage and the positive abscissa is taken as that voltage. So, in three phase delta circuits, the phase currents or the line currents are stated in reference to the line, or phase, voltage. In the case of wye circuits, there are line voltages as well as phase voltages, i.e. the line-to-neutral voltages. Therefore, when using polar notation to describe phase or line currents, care must be taken to separately indicate that the angle stated in the polar notation is in reference to either the phase voltage or the line voltage.

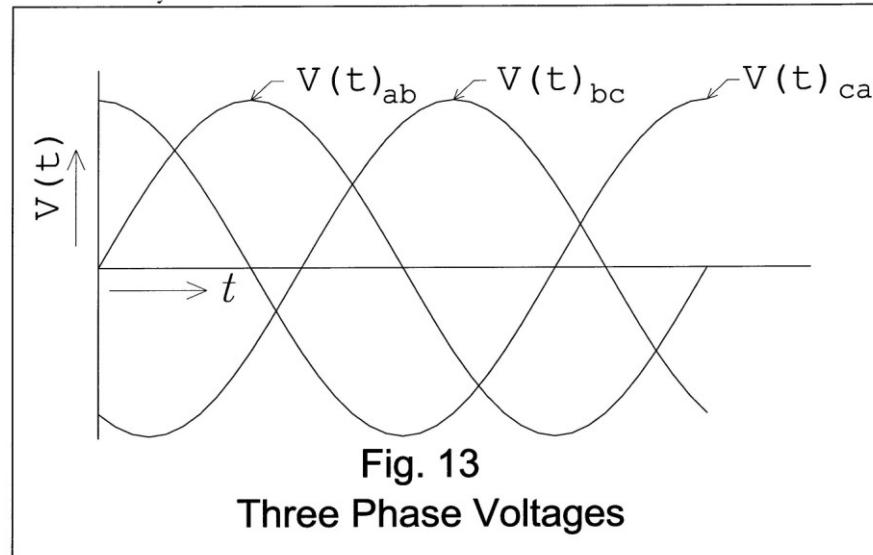
4. Balanced Three Phase Circuits

4A. Voltages in Three Phase Circuits - General

It is fair to say a three phase circuit consists of three separate, single phase voltages. A trace of the three voltages of a three phase circuit with time would be similar to that represented in Fig. 13 where the sequence is assumed to be A-B-C, i.e. $V_{ab} - V_{bc} - V_{ca}$ which is the more popular USA sequence.

Determination of a three phase voltage sequence is of particular importance when large motors are involved. In any installation, a motor is intended to rotate in a predetermined direction. If the sequence of the voltage connected to the motor has been inadvertently reversed, the motor will rotate in an unintended direction. Depending on the application, considerable property damage could result when a motor is started and it rotates in reverse to the

intended direction. A voltage sequence meter is especially valuable in the field to ascertain voltage sequence.



A typical three wire three phase circuit with a delta load is represented in Fig. 14. Of course, single phase loads may be taken from any two of the conductors of a three phase circuit and this is often the case. If there are a large number of single phase users, then a four wire three phase service is more practical.

A typical three phase wye circuit is shown in Fig. 15. Three phase wye circuits could be three wire or four wire. If the circuit is a four wire circuit, point "d" of Fig. 15 would

be connected to ground. A 480 VAC, four wire - three phase service is especially suited for commercial building as the single phase circuits can be taken for fluorescent lighting without imposing a need for a transformer. The lighting circuits would be at 277 VAC. If a separate 120/240 VAC single phase circuit is not brought to the building an in-house transformer would be needed to provide a 120/240 VAC single phase service.

4B. Calculation of Power in a Balanced Three Phase Circuit

For both balanced three wire three phase and balanced four wire or three wire three phase loads, the applicable parameters as amperage and power may be

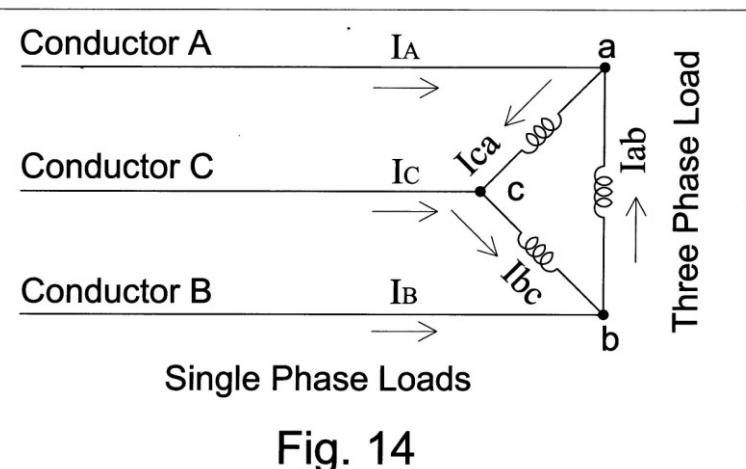


Fig. 14
Three Phase Circuit with Delta Load

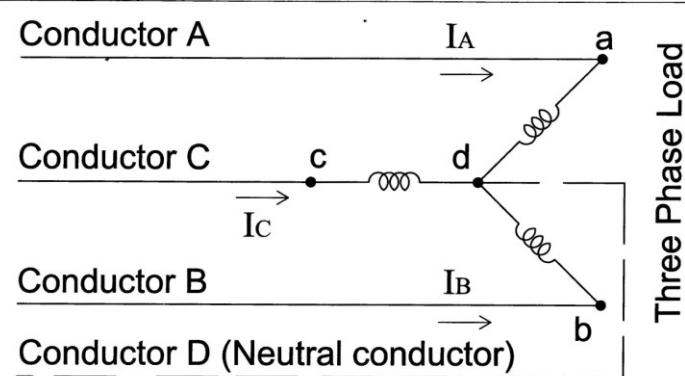


Fig. 15
Three Phase wye Circuit

calculated by relatively simple formulas. For balanced three phase loads power may be calculated by the equation,

$$P = (\sqrt{3}) V_L I_L (\text{PF}) \dots \text{Equation 6}$$

where

P = power (watts)

V_L = voltage (rms voltage)

I_L = current (rms amperage)

PF = power factor = $\cos \theta_P$

θ_P = angle of lead or angle of lag of phase current with respect to phase voltage (degrees or radians) (The subscript "p" designates "phase", i.e. θ_P designates lead/lag in a phase.)

Power Factor is often stated as the ratio of "real power" to "imaginary power". (Note: Imaginary power is also known as "total power.") In those instances where power, line voltage and line current are known, power factor may be computed for balanced three phase loads by the formula,

$$\text{PF} = P \div (V_L \cdot I_L \cdot \sqrt{3}), \text{ also}$$

$$\text{PF} = P \div (\text{volts} \cdot \text{amps})$$

[It is noted that in single phase circuits, the term "volt-amps" is defined as the circuit volts times the circuit amps. In other words, in single phase circuits, "volt-amps" = $(V_L \cdot I_L)$. However, in three phase balanced circuits,

"volt-amps" = $(V_L \cdot I_L \cdot \sqrt{3})$ In balanced three phase circuits, the term "volts-amps" is equal to the volts times the amps times the square root of three. Thus, the term "volt-amps" as commonly used with reference to three phase circuits is a misnomer and can lead to some confusion. If care is not taken, use of the value "volts-amps" in three phase computations can also lead to incorrect computations. In unbalanced three phase circuits, the term "volts-amps" has no significance.]

As with single phase circuits, the phase current with resistive loads is in-phase with phase voltage. Likewise, capacitive elements cause phase current to lead phase voltage, and inductive elements cause phase current to lag phase voltage. Typical resistive loads would be heaters or incandescent lighting. A

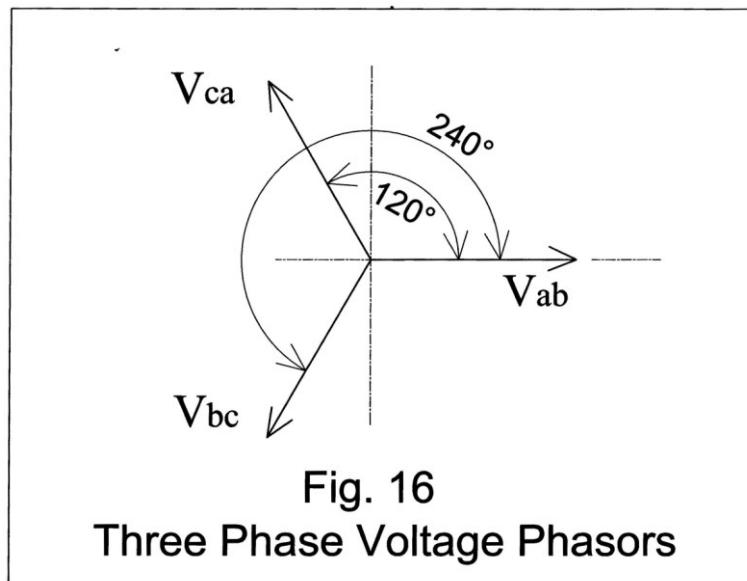
capacitor bank installed to counteract a severely lagging power factor would present a capacitive load. Common inductive loads include induction motors, which represent the most common form of three phase motors, and which always have a lagging power factor.

It is important to note that the equation for determining power in a balanced three phase circuit involves use of the power factor or the cosine of the angle between phase voltage and phase current (and not line current). If the angle of lag or lead is known for the line current with respect to line voltage and the phase lead/lag is desired the angle of line lead/lag in a phase is determined by use of the relationship $\theta_P = \theta_L - 30^\circ$.

4C. Phasor Diagrams of Three Phase Circuits

Construction of a phasor diagram for a three phase circuit essentially follows the rules that are applicable to single phase circuits. The voltage vectors for the three phase circuit of Fig. 13, Fig. 14 and Fig. 15 are shown in Fig. 16. As is the case with the single phase circuit, projections of the three vectors on the ordinate describe the voltages of the three phases. The vectors of Fig. 16, in accordance with the rules for the construction of phasor diagrams, are shown at $t = 0$. Note that the

assumed sequence of voltage as depicted in Fig. 13 is A-B, B-C and C-A which are represented in a clockwise sequence. According to standard practice the phasor for V_{ca} is positioned 120° counterclockwise from V_{ab} , and V_{bc} is positioned 240° counterclockwise from V_{ab} as shown in Fig. 16. Since the sequence is stated as A-B, B-C, C-A, the phasors are represented as shown in Fig. 16.



Line currents in a three phase circuit can be computed by means of the mathematical methodology described below. On the other hand, a phasor diagram alone can be used to determine these values. If high precision is not needed, a graphical depiction with paper and pencil may be used. For higher accuracy, a phasor diagram may be prepared by means of AutoCAD or a similar program to determine current values. In some ways, use of a phasor diagram seems more reliable as it is not susceptible to the types of errors that are commonly encountered when making algebraic calculations.

4D. Calculating Currents in a Balanced Three Phase Delta Circuit – General

A typical three phase circuit with a delta load is represented in Fig. 14. In three phase delta circuits the voltage across the load is the line voltage but the phase current is different from the line current. In Fig. 13, the instantaneous voltage sequence is $V(t)_{ab} - V(t)_{bc} - V(t)_{ca}$. Each of the phase voltages is 120° apart from the adjacent phase as shown in the phasor diagram of Fig. 16. Expressed in rms terms, the rotation would be $V_{ab} - V_{bc} - V_{ca}$. In Fig. 14, the line current in conductor ‘A’ would be I_A . The line current in conductor ‘B’ would be I_B and the line current in conductor ‘C’ would be I_C . The current in phase ‘ab’ would be I_{ab} . The current in phase ‘bc’ would be I_{bc} and the current in phase ‘ca’ would be I_{ca} .

For balanced three phase delta circuits, the line currents are determined by the relationship,

$$I_L = (\sqrt{3}) I_P \dots \text{Equation 7 (Reference 1)}$$

Where,

I_L = line current (“L” designates “line”), and

I_P = phase current (“P” designates “phase”)

4D.1 Resistive Loads

Considered separately, each of the phases of a three phase delta circuit is a single phase circuit. Accordingly, if the three loads in the delta circuit of Fig. 15 are all resistive the phase currents would be in-phase with the phase voltages as represented in the phasor diagram of Fig. 17. Obviously, for each of the three phases, $PF = 1.0$. In Fig. 17 the respective voltage vectors are

represented by the symbols V_{ab} , V_{bc} and V_{ca} . Since the phase voltages for delta circuits are the same as the line voltages, vectors V_{ab} , V_{bc} , and V_{ca} are for both the line and the phase voltages. The current vectors are represented by the symbols I_{ab} , I_{bc} and I_{ca} . As represented in Fig. 14, the current in Conductor B is the current entering point 'b' from phase 'a-b' less the current that flows from phase 'b-c'. Stated in mathematical terms,

$$I_B = I_{ab} - I_{bc}, \text{ where}$$

I_B = current in conductor B

I_{ab} = current in phase a-b

I_{bc} = current in phase b-c

Similarly,

$$I_C = I_{bc} - I_{ca}, \text{ where}$$

I_C = current in conductor C

I_{bc} = current in phase b-c

I_{ca} = current in phase c-a
and

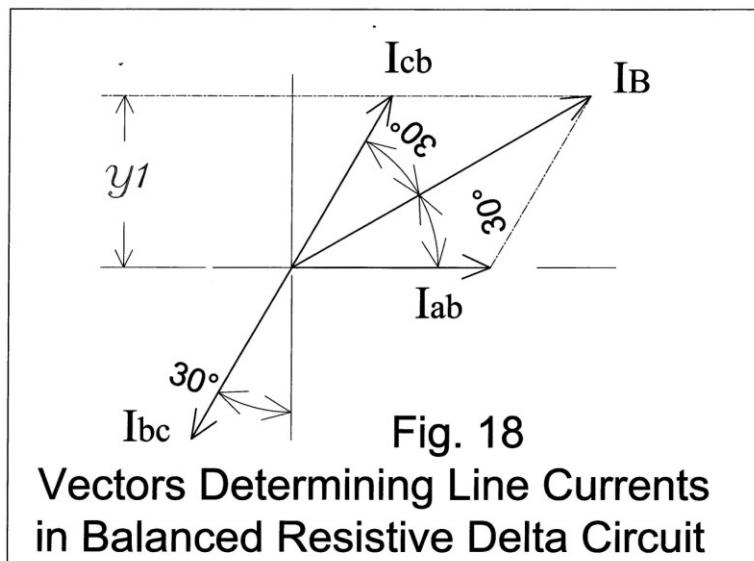
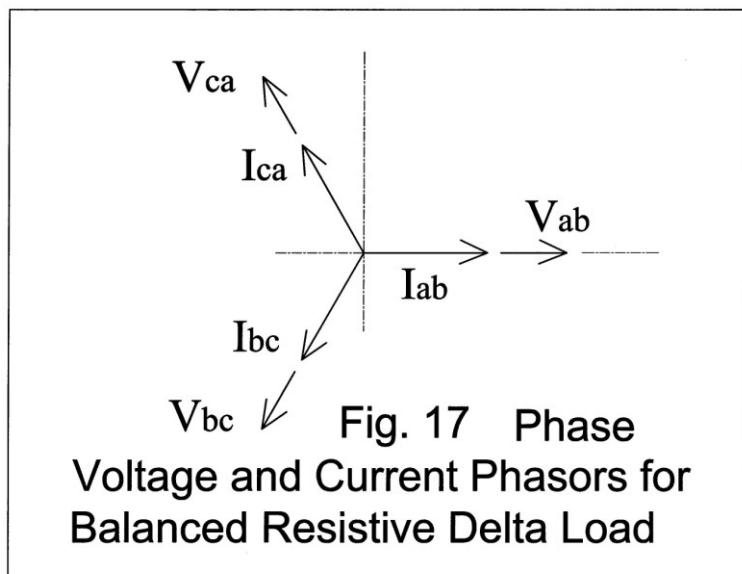
$$I_A = I_{ca} - I_{ab}, \text{ where}$$

I_A = current in conductor A

I_{ca} = current in phase c-a

I_{ab} = current in phase a-b

The phasors for I_{ab} and I_{bc} are shown in Fig. 18. To determine the phasor for the current in line B, the negative vector of I_{bc} is added to vector I_{ab} . The negative vector of I_{bc} (i.e. $-I_{bc}$) is $(+I_{cb})$ as shown in Fig. 18. The addition of vector I_{ab} and vector I_{cb} generates vector I_B .



In Fig. 18,

$$\cos 30^\circ = [y_1 \div I_{cb}]$$

$$y_1 = [I_{cb}] \cos 30^\circ$$

$$\cos 30^\circ = [\sqrt{3}/2]$$

$$y_1 = [I_{cb}] [\sqrt{3}/2]$$

and

$$\cos 60^\circ = y_1 \div I_B$$

$$I_B = y_1 \div \cos 60^\circ$$

$$\cos 60^\circ = 1/2$$

$$I_B = [I_{cb}] \sqrt{3}, \text{ or}$$

$$I_B = [I_{bc}] \sqrt{3}$$

This relatively simple computation for the assumed specific case is in agreement with Equation 7 and the cited reference. Since the circuit was said to be balanced, $I_B = I_A = I_C$ and each current is separated by 120° from the other. With this criteria, a phasor diagram can be generated to show the relationship between

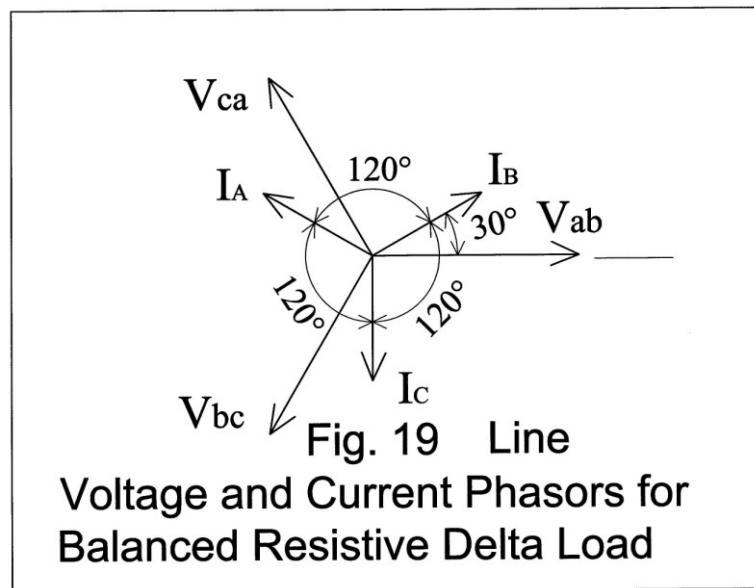
the line currents and the line voltages. This representation is shown in Fig. 19.

Example 5

Problem: Assume that for the three phase delta load of Fig. 14, each of the three loads is a

heater rated 5 kW and the line voltage is 480 VAC. Find the line and phase currents.

Solution: Since all three loads are equal, the circuit is considered balanced. The line currents and the phase currents are all equal.



Let,

phase current a-b = I_{ab} (rms amp)

phase current b-c = I_{bc} (rms amp)

phase current c-a = I_{ca} (rms amp)

phase a-b power = P_{ab}

phase b-c power = P_{bc}

phase c-a power = P_{ca}

line current in Conductor A = I_A (rms)

line current in Conductor B = I_B (rms)

line current in Conductor C = I_C (rms)

For each phase,

$$P = VI$$

$$P_{ab} = 5000 \text{ (watts)} = (480) I$$

$$I_P = I_{ab} = I_{bc} = I_{ca} = 5000/480 = 10.416 \text{ amps}$$

According to Equation 7

$$I_L = I_A = I_B = I_C = (\sqrt{3}) (10.416) = 18.04 \text{ amps}$$

In this relatively simple example line currents, phase currents and the respective powers could be determined by means of well-known and straightforward formulas.

Example 6

Problem: Confirm that for a balanced resistive load in a delta circuit the phasor diagram for line current replicates the mathematical relationship of all three voltages and currents.

Solution: The value of instantaneous current as represented by Vector I_{ab} can be represented by general Equation 4. According to Equation 4,

$$I(t)_{ab} = I_{ab} \sin(\omega t + \theta), \text{ where}$$

θ = lead or lag of current with respect to voltage V_{ab}

Reference is made to Fig. 14. Since the load under consideration is a balanced, resistive, three phase load, $\theta_P = 0$ for all three phases. Let,

$$I_{ab} = I_{bc} = I_{ca} = I_P, \text{ and}$$

$$I(t)_{ab} = I_{ab} \sin(\omega t + 0)$$

$$I(t)_{bc} = I_{bc} \sin(\omega t + 240^\circ)$$

$$I(t)_{ca} = I_{ca} \sin(\omega t + 120^\circ)$$

The expression $I(t)_{bc}$ can be stated as

$$I(t)_{bc} = I_{bc} \sin(\omega t + 240^\circ), \text{ or}$$

$$I(t)_{bc} = I_{bc} [\sin(\omega t + 240^\circ)] = I_p \{ \sin \omega t \cos 240^\circ + \cos \omega t \sin 240^\circ \}$$

$$\cos 240^\circ = -\cos 60^\circ$$

$$\sin 240^\circ = -\sin 60^\circ$$

$$I(t)_{bc} = I_p \{ -\sin \omega t \cos 60^\circ - \cos \omega t \sin 60^\circ \}$$

$$I(t)_{bc} = -I_p \{ \sin \omega t \cos 60^\circ + \cos \omega t \sin 60^\circ \}$$

$$I(t)_{bc} = -I_p \sin(\omega t + 60^\circ) = -I_p \{ (1/2) \sin \omega t + (\sqrt{3}/2) \cos \omega t \}$$

$$I(t)_{ab} = I_p \sin \omega t$$

$$I(t)_{ab} - I(t)_{bc} = I_p \sin \omega t + I_p \{ (1/2) \sin \omega t + (\sqrt{3}/2) \cos \omega t \}$$

$$I(t)_B = I(t)_{ab} - I(t)_{bc} = I_p \{ (3/2) \sin \omega t + (\sqrt{3}/2) \cos \omega t \}$$

$$I(t)_B = I_p (\sqrt{3}) \{ (\sqrt{3}/2) \sin \omega t + (1/2) \cos \omega t \}$$

$$I(t)_B = I_p (\sqrt{3}) \{ \cos 30^\circ \sin \omega t + \sin 30^\circ \cos \omega t \}$$

$$I(t)_B = I_p (\sqrt{3}) \sin(\omega t + 30^\circ)$$

This computation confirms the value of one of the line currents. Since the circuit is balanced, the other two line currents would be equal in magnitude and rotated by 120° from one another. The computation also confirms that the line current in a balanced resistive delta load is $(\sqrt{3})$ times the phase current and, further, that the line current leads the phase currents by 30° . Therefore the model as defined by the vector diagram of Fig. 18 is an accurate representation of the phase and line currents for the described example.

The computations of Example 6 also illustrate the difficulties in mathematically determining currents. These computations also demonstrate the merits and simplicity in using phasor diagrams to determine current values.

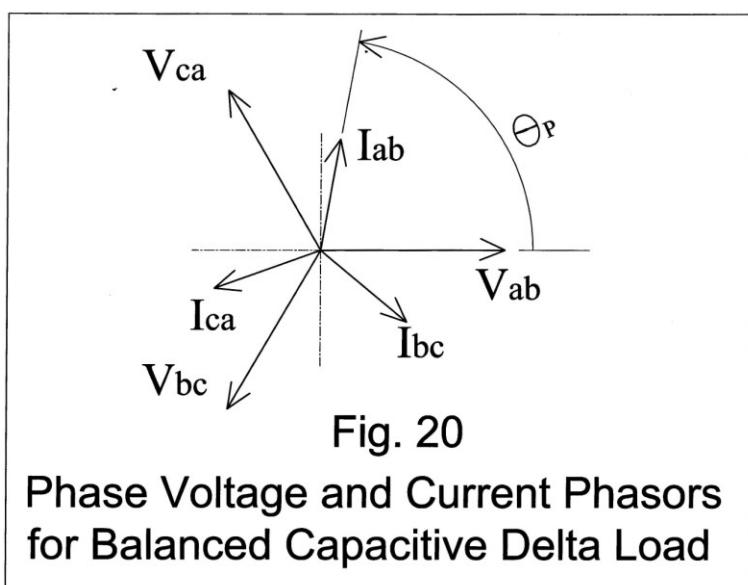
4D.2 Capacitive Loads

Above it was shown how currents in a balanced delta circuit with resistive loads are determined. Unlike currents in a resistive load where the phase

currents are in-phase with the phase voltages, phase currents in a capacitive circuit lead the phase voltages by some amount between 0° and 90° .

Commercial and industrial loads are mostly inductive in large part because the largest part of their power usage is generally attributed to the use of three phase induction motors. And, all induction motors operate with a lagging power factor. Nevertheless capacitive circuits are often found at commercial and industrial users. The most popular use of capacitive circuits in electrical power circuits is in capacitor banks. A capacitor bank can counteract a lagging power factors that results from the use of induction motors. Because some utilities will charge users an extra fee for a lagging power factor, capacitor banks are sometimes installed by customers in-parallel with the normal loads to the service correct power factor. A capacitor bank alone provides a leading power factor without consuming significant power and, when combined with a lagging power factor, it will bring the service lagging power factor more near to unity. A typical example of the use of a capacitor bank to correct power factor is treated below in Section 4D.4, "Two or More Loads" and in Example 9.

A circuit with a typical capacitive load is represented in the phasor diagram of Fig. 20. (Actually, the circuit represented in Fig. 20 would be only partly capacitive. The circuit would necessarily contain resistive and/or inductive element since the angle θ_P is shown as less than 90° degrees. If the circuit were purely capacitive, the angle of lead, θ_P , would be 90° .) Note that the arc indicating the angle θ_P shows the positive direction of θ_P to be CCW from the positive abscissa as indicated by the arrowhead on the arc. This practice is consistent with the polar notation of



complex numbers whereby the positive angle of a vector is likewise considered the CCW direction from the positive abscissa. The vectors determining line current I_B , assuming the phase current vectors of Fig. 20, are shown in Fig. 21. The phasors representative of currents I_A and I_C would be determined in a similar manner. The resultant line currents, I_A , I_B and I_C , are shown in Fig. 22.

4D.3 Inductive Loads

In a balanced three phase inductive circuit, phase currents lag the phase voltages by some amount between 0° and 90° . For a delta circuit, Equation 7 remains applicable so that

for each of the three currents $I_L = (\sqrt{3}) I_P$. Using the specific notation of Fig. 14 for current in conductor B, $I_B = (\sqrt{3}) I_{ab}$. The geometry of the phasors determining I_B indicate that $\theta_P = \theta_L - 30^\circ$. The phase currents for an inductive load lag line voltage and in the phasor diagram the phasor for the phase current is positioned clockwise from the phasor for line voltage. For an inductive load, $\theta_P < 0^\circ$ by definition. It may also be noted that for θ_P greater than -30° the line current leads the line voltage and for θ_P less than -30° the line current lags line voltage.

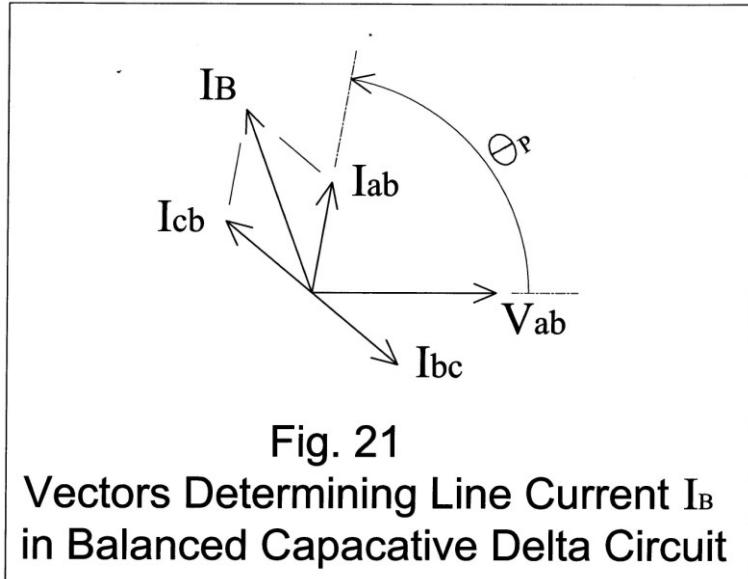


Fig. 21
Vectors Determining Line Current I_B
in Balanced Capacitive Delta Circuit

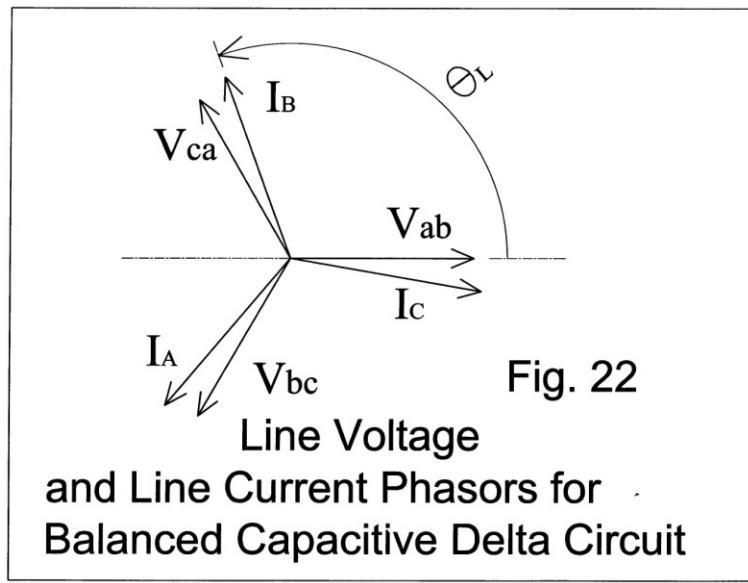


Fig. 22
Line Voltage
and Line Current Phasors for
Balanced Capacitive Delta Circuit

Much as with the phasor diagram for a capacitive load, the phasor for a particular line current is determined by adding component vectors as shown in Fig. 20 for current I_B in a capacitive circuit. Below Example 7 demonstrates the procedure for determining line currents in a specific inductive circuit.

Example 7

Problem: The nameplate on a delta wound induction motor states: “480-3-60”, “FLA” as 10A and “PF” at 0.707. Determine line currents, draw the phasor diagram for line and phase currents and calculate power consumption.

Solution:

The line voltage is 480 VAC, three phase, 60 hz, and the line current 10 amps. Since induction motors have a lagging power factor, the current lags voltage by:

$$\theta_P = \cos^{-1} 0.707, \text{ or}$$

$$\theta_P = -45^\circ$$

according to Equation 12-1,

$$\theta_P = \theta_L - 30^\circ, \text{ or}$$

$$\theta_L = \theta_P + 30^\circ = -15^\circ$$

According to Equation 6,

$$P = [\sqrt{3}] V_L I_L \cos \theta_P$$

$$P = \sqrt{3} (480) (10) \times (0.707)$$

$$= 5,877 \text{ watts}$$

$$I_P = I_L / \sqrt{3} = 10 / \sqrt{3} = 5.77 \text{ A}$$

Expressed in polar notation,

$$I_L = 10 \angle -15^\circ, \text{ and } I_P = 5.77 \angle -45^\circ$$

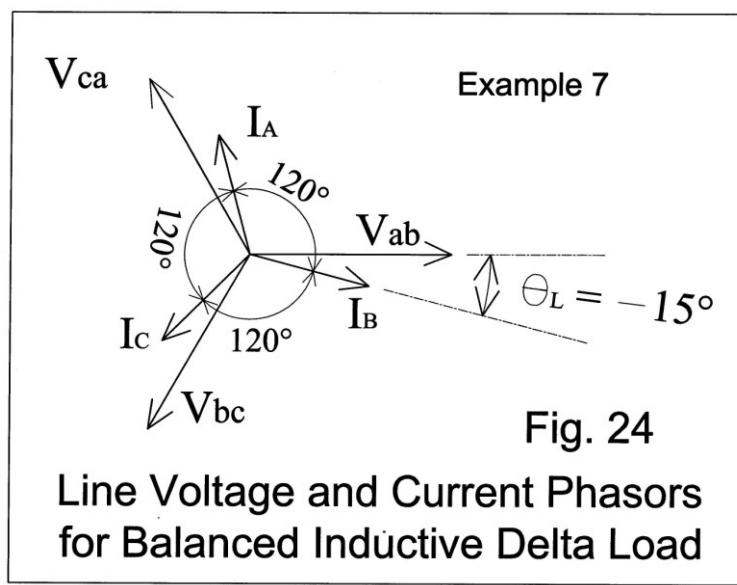
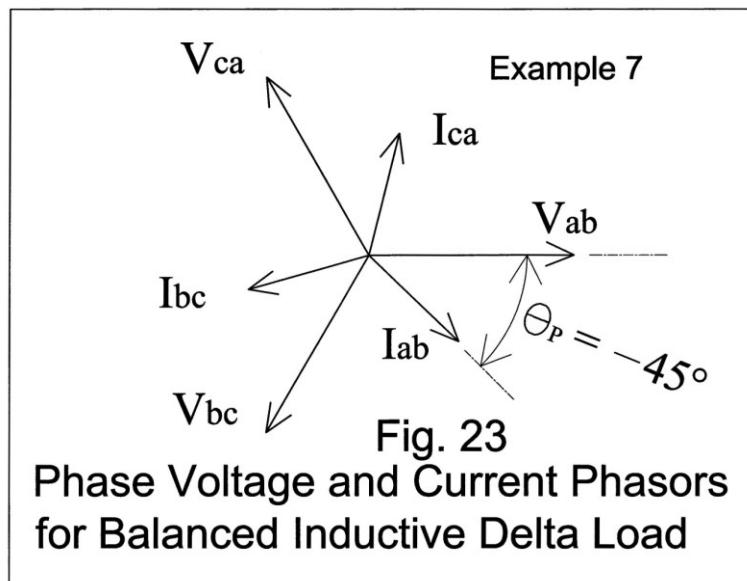
The phasor diagram for the phase currents and voltages is shown in Fig. 23. The notation of Fig. 14, which is for a delta circuit, is followed. The current phasor for Phase a-b (i.e. I_{ab}) is rotated clockwise 45° from phase voltage V_{ab} , thereby indicating that the phase current lags the phase voltage. Each of the line voltages is rotated 120° from one another. Likewise, all three phase currents (I_{ab} , I_{bc} & I_{ca}) are all 120° apart.

The phasor diagram for the line currents and line voltages for Example 7 are shown in Fig. 24 where the line current in conductor “B” is shown as lagging the respective line voltage vector (V_{ab}) by 15° . The three line currents are shown separated from one another by 120° . (Note that in Fig. 23 and Fig. 24 the arcs indicating the positions and values of θ_p and θ_L , respectively, have arrowheads at both ends. This practice is followed since the values are negative and should not be confused with the positive direction of the values.)

The positive direction of the variables are indicated by an arc having a single arrowhead that designates the positive direction of the respective variables.)

4D.4 Two or More Loads

It is very common to have two or more balanced delta loads on a common three phase feed. A typical three phase feed with two balanced delta loads is represented in Fig. 25 where the three phase feed consists of Conductors “A”, “B” and “C”. The common feeder serves two loads, namely Load 1 and Load



2. Conductor A has branches P and S. Conductor B has two branches Q and T, and Conductor C has branches R and U. The most common interest would be the currents in Conductors A, B and C.

Obviously,

$$\underline{I_A} = \underline{I_P} + \underline{I_S}$$

$$\underline{I_B} = \underline{I_Q} + \underline{I_T}, \text{ and}$$

$$\underline{I_C} = \underline{I_R} + \underline{I_U}$$

(The underlined currents indicate the values are vectors and not algebraic values.)

The phasor diagram for the two loads of Fig. 25 would be similar to that represented in Fig. 26. The primary objective of the phasor diagram of Fig. 26 would be to determine the values of I_A , I_B & I_C . Since it was assumed that the loads are balanced, $I_A = I_B = I_C$. So, it becomes necessary to merely determine, say, I_B . (It is preferred to determine current I_B , rather than currents I_A or I_C , because the phasor for the associated reference voltage, V_{ab} , would be positioned along the positive abscissa. Consequently, the associated calculations are more easily performed than those for determining I_A or I_C .) The values of Fig. 26 may be determined by general

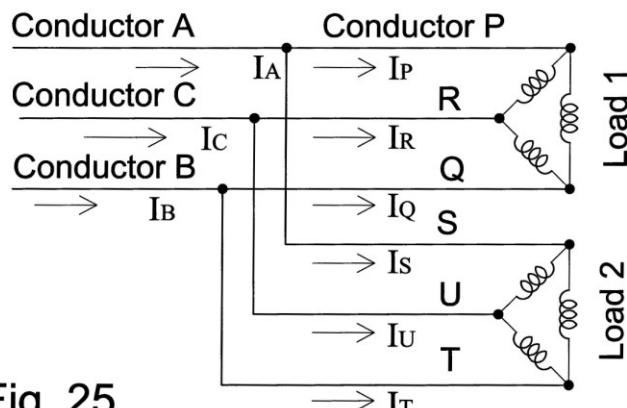


Fig. 25

Two Balanced Delta Loads

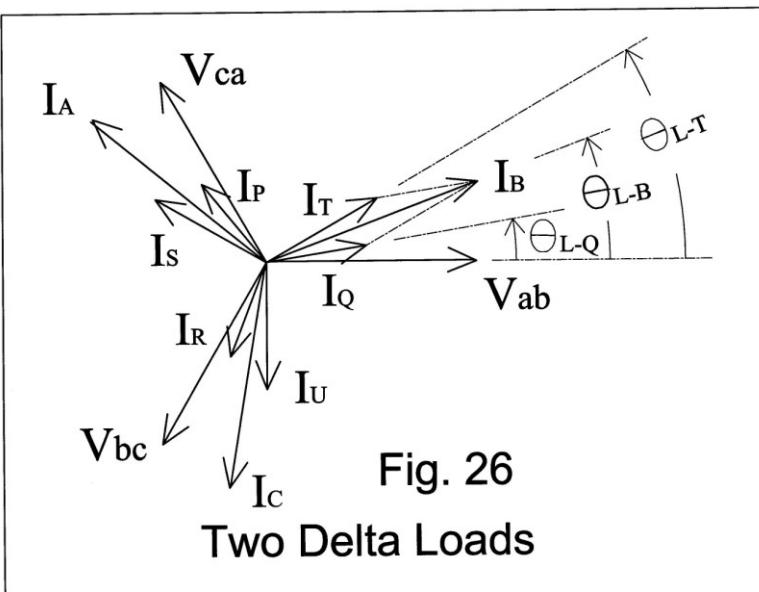


Fig. 26

Two Delta Loads

Equation 5A, which is for a single phase circuit. The values are substituted into Equation 5A as applicable to a three phase circuit and for the purpose of determining current I_B .

Accordingly,

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2} \dots \text{Equation 5B}$$

where,

$$X_B = I_1 \cos \theta_1 + I_2 \cos \theta_2 + \dots I_n \cos \theta_n, \text{ and}$$

$$Y_B = [I_1 \sin \theta_1 + I_2 \sin \theta_2 + \dots I_n \sin \theta_n]$$

$$\theta_B = \sin^{-1} (Y_B \div I_B)$$

Specifically, as applicable to Fig. 26:

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}, \text{ where}$$

$$X_B = I_Q \cos \theta_Q + I_T \cos \theta_T, \text{ and}$$

$$Y_B = I_Q \sin \theta_Q + I_T \sin \theta_T$$

$$\theta_B = \sin^{-1} (Y_B \div I_B)$$

Example 8

Problem:

Assume there are two delta wound induction motors on a common 480 VAC circuit as represented in Fig. 25. For Load 1, FLA = 6 A, PF = 0.90, lagging and for Load 2, FLA = 7 A, PF = 0.70, lagging. Find the currents and power factor in the common three phase feeder (Currents A, B & C in Fig. 25).

Solution:

For Load 1,

$$\cos \theta_{P-1} = 0.90$$

Since the motors are induction motors, the power factors are lagging.

$$\theta_{P-1} = -\cos^{-1} 0.90$$

$$= -25.842^\circ, I_1 = 6$$

$$\theta_{L-1} = \theta_{P-1} + 30^\circ$$

$$= -25.842^\circ + 30^\circ = +4.15^\circ \text{ (line current leads line voltage)}$$

For Load 2,

$$\cos \theta_{P-2} = 0.70$$

$$\theta_{P-2} = -\cos^{-1} 0.70$$

$$= -45.572^\circ, I_2 = 7$$

$$\theta_{L-2} = -45.572^\circ + 30^\circ$$

$$= -15.572^\circ \text{ (line current lags line voltage)}$$

The orientations of the vectors that are representative of the line currents are shown in Fig. 27.

Reference Fig. 25. Let,

$$\theta_{L-A} = \text{lead/lag of}$$

$$(\text{line}) \text{ current "A"}$$

$$\theta_{L-B} = \text{lead/lag of}$$

$$(\text{line}) \text{ current "B"}$$

$$\theta_{L-C} = \text{lead/lag of}$$

$$(\text{line}) \text{ current "C"}$$

$$I_1 = 6 = I_P = I_Q = I_R$$

$$I_2 = 7 = I_S = I_T = I_U$$

Example 8

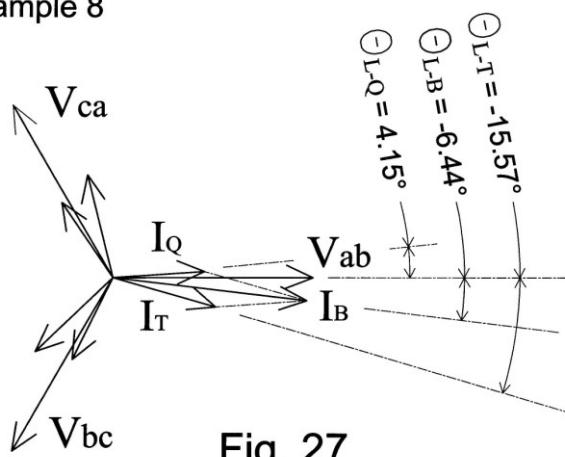


Fig. 27

Two Inductive Delta Loads

Reference Equation 5B which states in general terms:

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}$$

where,

$$X_B = I_1 \cos \theta_I + I_2 \cos \theta_2 + \dots I_n \cos \theta_n, \text{ and}$$

$$Y_B = [I_1 \sin \theta_1 + I_2 \sin \theta_2 + \dots I_n \sin \theta_n]$$

$$\theta_B = \sin^{-1} (Y_B \div I_B)$$

The notation of Equation 5B is amended for two loads. The subscript "1" designates Load 1 and the subscript "2" designates Load 2.

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}, \text{ where}$$

$$X_B = I_1 \cos \theta_1 + I_2 \cos \theta_2, \text{ and}$$

$$Y_B = [I_1 \sin \theta_1 + I_2 \sin \theta_2]$$

$$\theta_{L-B} = \sin^{-1} (Y_B \div I_B)$$

For the line currents in the common feeder (i.e. currents A, B & C)

$$X_B = I_1 \cos \theta_1 + I_2 \cos \theta_2 = (6) (\cos 4.15^\circ) + (7) (\cos -15.57^\circ)$$

$$= (6) (.997) + (7) (.963) = 5.982 + 6.741 = 12.723$$

$$Y_B = [I_1 \sin \theta_1 + I_2 \sin \theta_2] = [(6) (\sin -4.15^\circ) + (7) (\sin +15.57^\circ)]$$

$$= [(6) (.072) - (7) (.268)] = [(.432) - (1.876)] = -1.444$$

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}$$

$$I_B = \{(12.723)^2 + (-1.444)^2\}^{1/2}$$

$$I_B = 12.804 \text{ amps}$$

$$\text{Obviously } I_B = I_A = I_C = 12.804 \text{ amps}$$

$$\theta_{L-B} = \sin^{-1} (Y_B \div I_B)$$

$$\theta_{L-B} = \sin^{-1} (-1.444 \div 12.804) = \sin^{-1} (-.1127)$$

$$\theta_{L-B} = \theta_{L-A} = \theta_{L-C} = -6.47^\circ \text{ (line current lagging line voltage)}$$

Power factor pertains to phase lead/lag and not line lead/lag. So, per Equation 12-1, $\theta_P = \theta_L - 30^\circ$.

$$\theta_P = -6.47^\circ - 30^\circ = -36.47^\circ \text{ & PF} = \cos -36.47^\circ = .804$$

Example 9

Problem: Reference is made to Fig. 25. Assume a utility customer has a net load that would be the equivalent of Load 1, Fig. 25, with: 480 VAC, 100 amps & PF = 0.50, lagging. Find the capacitor bank current size required to bring the line power factor to unity.

Solution:

The capacitor bank would be installed in parallel with the inductive load.

Let θ_{L-A} = lead/lag of (line) current "A" of electrical source

Let θ_{L-B} = lead/lag of (line) current "B" of electrical source

Let θ_{L-C} = lead/lag of (line) current "C" of electrical source

Let θ_{L-P} = lead/lag of (line) current "P" of lagging load

Let θ_{L-Q} = lead/lag of (line) current "Q" of lagging load

Let θ_{L-R} = lead/lag of (line) current "R" of lagging load

Let θ_{L-S} = lead/lag of (line) current "S" of capacitor bank

Let θ_{L-T} = lead/lag of (line) current "T" of capacitor bank

Let θ_{L-U} = lead/lag of (line) current "U" of capacitor bank

$$I_1 = 100 \text{ amps} = I_P = I_R = I_Q$$

For Load 1 (the lagging load), $\cos \theta_{P-1} = 0.50$, $\theta_{P-1} = -60^\circ$

In general, from Equation 12-1

$$\theta_P = \theta_L - 30^\circ$$

$$\theta_L = \theta_P + 30^\circ$$

So,

$$\theta_{L-P} = \theta_{L-R} = \theta_{L-Q} = -60^\circ + 30^\circ = -30^\circ$$

Current in the capacitor bank will lead phase voltage by approximately 90° . So, for Load 2,

$$\theta_{P-2} = +90^\circ$$

$$\theta_{L-S} = \theta_{L-T} = \theta_{L-U}$$

$$= \theta_{P-2} + 30^\circ = 120^\circ$$

The phasor diagram for this example is shown in Fig. 28. The objective is to have I_A , I_B and I_C at $\theta_L = +30^\circ$, i.e. to have the line currents leading line voltage by 30° . This would be the position of the line currents

corresponding to a pure resistive load which would correspond to a unity power factor.

Reference Equation 5B which states in general terms:

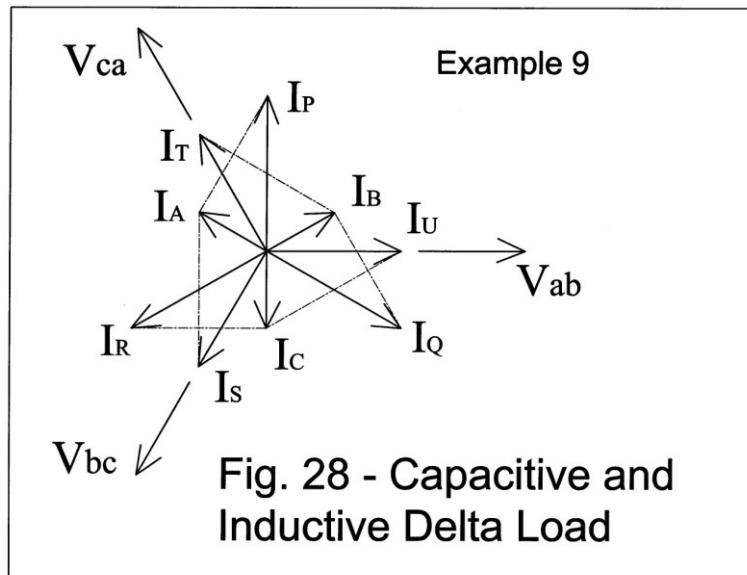
$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}$$

where,

$$X_B = I_1 \cos \theta_I + I_2 \cos \theta_2 + \dots + I_n \cos \theta_n, \text{ and}$$

$$Y_B = [I_1 \sin \theta_1 + I_2 \sin \theta_2 + \dots + I_n \sin \theta_n]$$

$$\theta_B = \sin^{-1} (Y_B / I_B)$$



With notation corrected to use the notation of Fig. 25 for two loads,

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}, \text{ where}$$

$$X_B = I_1 \cos \theta_1 + I_2 \cos \theta_2, \text{ and}$$

$$Y_B = [I_1 \sin \theta_1 + I_2 \sin \theta_2]$$

$$\theta_{L-B} = \sin^{-1} (Y_B \div I_B)$$

By trial and error computations, it was determined that a capacitor bank current of 86.61 amp will satisfy the required criteria.

Check!

Try $I_2 = 86.61$ amp; $I_1 = 100$ amp; $\theta_1 = -30^\circ$; $\theta_2 = +120^\circ$

$$\begin{aligned} X_B &= I_1 \cos \theta_1 + I_2 \cos \theta_2 = (100) (\cos -30^\circ) + (86.61) (\cos 120^\circ) \\ &= (100) (.8660) + (86.61) (-.5) = 86.60 - 43.309 = 43.291 \end{aligned}$$

$$Y_B = [I_1 \sin \theta_1 + I_2 \sin \theta_2]$$

$$\begin{aligned} Y_B &= [(100) (\sin -30^\circ) + (86.61) (\sin 120^\circ)] \\ &= [-50 + (75.00)] = 25.00 \end{aligned}$$

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}$$

$$I_B = \{(43.291)^2 + (25.00)^2\}^{\frac{1}{2}} = 50.00 \text{ amps}$$

$$\begin{aligned} \theta_{L-B} &= \sin^{-1} (Y_B \div I_B) = \sin^{-1} (25.00 \div 50.00) \\ &= \sin^{-1} (.500) = 30^\circ \text{ (i.e. line current leads line voltage by } 30^\circ) \end{aligned}$$

It may be noted that if, say, a wattmeter were installed in the line to the motor, the power would be measured at 100 amps and a power factor of 0.5, resulting in a power calculation of 41.569 kW. If a capacitor bank were installed and power measured in the common feeder upstream of the capacitor bank and the motor, the meter would indicate the same power, namely 41.569 kW, although at 50.00 amps and unity power factor.

In summary: Installation of a capacitor bank with current of 86.61 amps will return service line current to unity power factor (from a lagging power factor of 0.50) and reduce service current (I_A , I_B and I_C of Fig. 25) from 100 amps to 50.00 amps. Power consumption would remain unaltered.

4E. Calculating Currents in a Balanced Three Phase Wye Circuit - General

It was noted earlier that in a delta circuit the phase voltage is identical in magnitude to the line voltage but the phase current has a magnitude and lead or lag that is different from the line current. In some ways, the wye circuit is the opposite of a delta circuit. In a wye circuit the phase voltage has a magnitude and lead or lag that is different from the line voltage, but the phase current is identical in magnitude to the line current. A typical three phase wye circuit is represented in Fig. 15. In Fig. 15, the voltage sequence is assumed to be $V_{ab} - V_{bc} - V_{ca}$ as was the case for the delta circuit of Fig. 14. Each of the line voltages of Fig. 14 is 120° apart from the adjacent phase as shown in Fig. 16 for the assumed delta circuit. In Fig. 14, the line current in conductor 'A' is I_A . The line current in conductor 'B' is I_B , and the line current in conductor 'C' is I_C . In a wye circuit there is a fourth point, namely Point "d" of Fig. 15. In a four-wire wye circuit, point d would be connected to ground. A fourth or ground wire is needed if it is anticipated that the three phases may not be balanced. However, all wye circuits do not have or need a neutral wire. A wye connected induction motor, for example, would have no neutral wire since the currents in all three phase would be very nearly equal. In Fig. 15, the current in phase 'ad' is I_{ad} . The current in phase 'bd' is I_{bd} , and the current in phase 'cd' is I_{cd} . If point "d" is connected to ground by Conductor D, current in Conductor D would be nil in a balanced wye circuit. For a balanced three phase wye circuit, the phase voltages are related to the line voltages by the equation,

$$V_L = (\sqrt{3}) V_P \dots \text{Equation 8 (Reference 2)}$$

Where,

V_L = line voltage, and

V_P = phase voltage

A typical phasor diagram for a balanced wye circuit is shown in Fig. 29. (Reference 2) It may be seen that voltage V_{db} leads voltage V_{ab} by 30° . Voltage V_{da} leads voltage V_{ca} by 30° and voltage V_{dc} leads voltage V_{bc} by 30° . It is also apparent that for any wye circuit, balanced or unbalanced, $\theta_P = \theta_L - 30^\circ$, as was also noted earlier to be applicable to a balanced delta circuit.

4E.1 Resistive Loads

Resistive loads in single phase circuits are necessarily in-phase with the applied voltage. If all three phases of a three phase circuit have

resistive loads of equal magnitude, the phase current would be in-phase with the respective phase voltage as represented in Fig. 30. The phase currents are equal in magnitude to the line currents but at angle of 30° , to the line current.

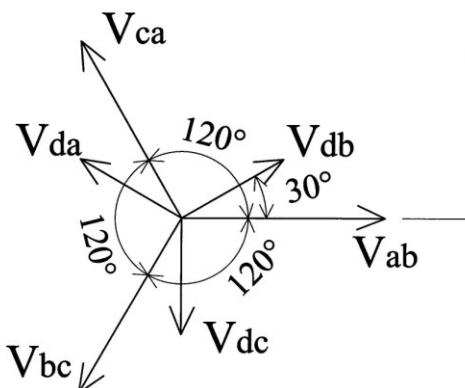


Fig. 29 - Voltages in a
Balanced wye Circuit

Example 10

Problem: Assume that for the wye load of Fig. 15 each of the three loads is a heater rated 5kW (as in Example 5) and the line voltage is 480 VAC (also as in Example 5). Find the line and phase currents.

Solution:

The applicable phasors are as represented in Fig. 30.

Let,

phase current a-d = I_{ad} (rms amp)

phase current b-d = I_{bd} (rms amp)

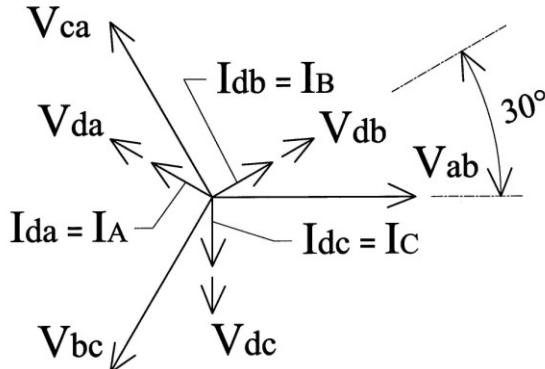


Fig. 30 - Voltage and Current
Phasors for Balanced Resistive
Wye Load

phase current c-d = I_{cd} (rms amp)

phase a-d power = P_{ad}

phase b-d power = P_{bd}

phase c-d power = P_{cd}

line current in Conductor A = I_A (rms)

line current in Conductor B = I_B (rms)

line current in Conductor C = I_C (rms)

From Equation 8, $V_{ad} = V_{bd} = V_{cd} = 1/(\sqrt{3}) (480) = 277.13$ volts

For each phase, $P = VI$

$I_P = P/V = 5000/277.13 = 18.04$ amps

The phase current is in-phase with the phase voltage. So,

$\theta_P = 0$

$\cos \theta_P = 1.0$

Check!

From Equation 6,

$P = (\sqrt{3}) V_L I_L \cos \theta_P$

The phase current equals the line current.

The total power of all three phases is:

$P = (\sqrt{3}) (480) (18.04) (1.0) = 15,000$ watts

Checks!

$\theta_L = \theta_P + 30^\circ = 0 + 30^\circ = 30^\circ$

In summary,

$I_P = 18.04$ A @ 0° to phase voltage, and

$I_L = 18.04$ A @ 30° to line voltage (leading)

4E.2 Inductive Loads

With an inductive load, the phase current lags the phase voltage by angle θ_P which would be between 0° and -90° . If the phase current lags by less than 30° the line current leads line voltage. On the other hand, if phase current lags phase voltage by more than 30° the line current would also lag line voltage.

4E.3 Capacitive Loads

For capacitive loads, the phase current leads the phase voltage by angle θ_P which would be between 0° and $+90^\circ$. Currents in capacitive circuits always lead line voltage.

5. Unbalanced Three Phase Circuits

5A. Unbalanced Three Phase Circuits - General

Once a person understands balanced three phase circuits and the use of phasor diagrams to visualize the voltages and currents in those circuits, it is an easy transition into the realm of unbalanced three phase circuits. As with balanced circuits, phasor diagrams can be used to present a clear picture of the voltages and currents that are involved. Below, delta circuits are considered first. The wye circuits are considered after the delta circuits. Because of the very large number of possible combinations of loads and phase angles, all of these combinations cannot be treated individually. Rather, the method for calculating line currents is described. With that methodology, a person may then readily calculate currents in any possible, specific combination of unbalanced delta and wye circuits.

By definition, an unbalanced circuit has at least one phase current that is not equal to the other phase currents. Of course, all three phase currents could be of unequal magnitude. In all cases, line voltages are assumed to be of equal magnitude, separated by 120° of rotation and in the sequence A-B, B-C, C-A. In wye circuits all three phase voltages are assumed to be equal and separated by 120° of rotation.

5B Unbalanced Three Phase Delta Circuits with Resistive, Inductive or Capacitive Loads - General

When all three loads of a delta circuit are resistive, the phase currents are all in-phase with the line voltages. Also, the power factors of all three phases are unity ($PF = 1.0$) and the lead/lag angle in the phases are all zero ($\theta_P = 0$). As explained earlier, in a balanced resistive delta circuit the line currents all lead the line voltages by 30° . If the loads are resistive and unbalanced, the line currents could be at various angles to the line voltages. Much the same may be

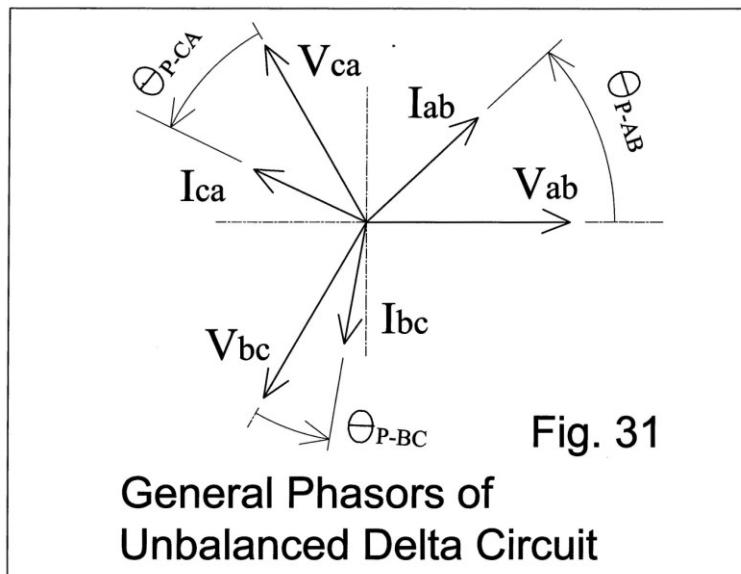
said of a three phase circuit that contains a mix of delta and wye circuits with leading and lagging currents.

5B.1 Unbalanced Three Phase Delta Circuits with Resistive, Inductive or Capacitive Loads

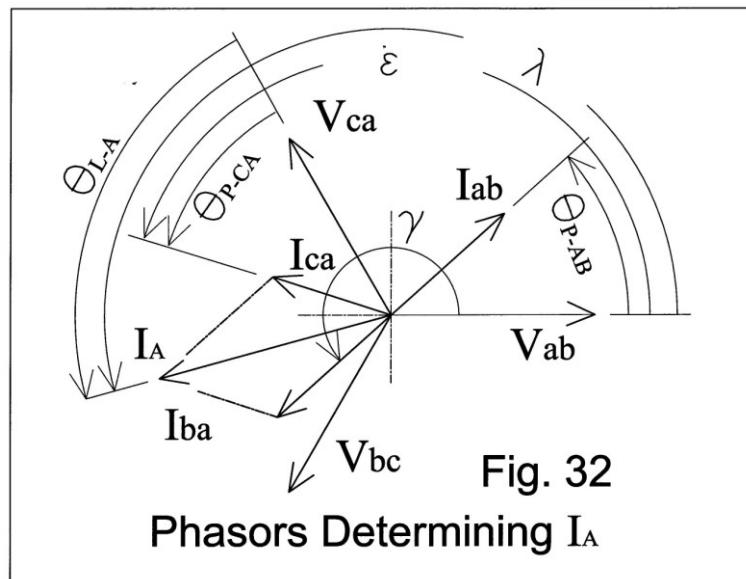
As with balanced three phase circuits, phasor diagrams can be used to determine line currents in unbalanced three phase circuits. A determination of line currents is necessary if, say, the currents in the conductors of a common feeder circuit are to be calculated.

A generalized view of a delta circuit would assume that the current in each phase is at some angle, θ_p , to the phase voltage (which in the case of a delta circuit is also the line voltage). In other words, if the current in a phase is in-phase with the phase voltage (as would be the case with resistive loads) then $\theta_p = 0$. If the load is capacitive, $\theta_p > 0$ and the current is said to be “leading” and the power factor “negative”. If the load is inductive $\theta_p < 0$ and the current is said to be “lagging” and the power factor is “positive”. A typical delta circuit is represented in Fig. 14 and a generalized summary of the phase voltages and phase currents are represented in Fig. 31. Above it was shown how the line currents are determined. For example, the phasor of line current I_A is determined by adding the phasor for I_{ca} and the negative phasor of I_{ab} , namely I_{ba} . The procedure for adding the vectors that determine I_A is illustrated below. For the purposes of analysis, it is assumed that no two currents are necessarily of equal magnitude or necessarily at the same lead/lag angle.

In order to develop the relationship for line current I_A , let,
 X_{ab} = the abscissa component of vector I_{ab}
 X_{ca} = the abscissa component of vector I_{ca}
 X_A = the abscissa component of vector I_A



Y_{ab} = the ordinate component of vector I_{ab}
 Y_{ca} = the ordinate component of vector I_{ca}
 Y_A = the ordinate component of vector I_A
 Current I_A is determined as shown in Fig. 32.



In Fig. 32,

$$X_{ba} = I_{ba} \cos \gamma$$

$$\gamma = 180^\circ + \theta_{P-AB}$$

$$\cos \gamma = -\cos \theta_{P-AB}$$

$$X_{ba} = I_{ba} (-\cos \theta_{P-AB})$$

$$X_{ba} = -I_{ba} \cos \theta_{P-AB}$$

$$Y_{ba} = I_{ba} \sin \gamma$$

$$\sin \gamma = -\sin \theta_{P-AB}$$

$$Y_{ba} = -I_{ba} \sin \theta_{P-AB}$$

$$X_{ca} = I_{ca} \cos \epsilon$$

$$\epsilon = 120^\circ + \theta_{P-CA}$$

$$\cos \epsilon = \cos (120^\circ + \theta_{P-CA})$$

$$= \cos 120^\circ \cos \theta_{P-CA} - \sin 120^\circ \sin \theta_{P-CA}$$

$$\cos 120^\circ = -1/2$$

$$\sin 120^\circ = (\sqrt{3}/2)$$

$$X_{ca} = I_{ca} [(-1/2) (\cos \theta_{P-CA}) - (\sqrt{3}/2) (\sin \theta_{P-CA})]$$

$$X_{ca} = -I_{ca} (1/2) [(\sqrt{3}/2) \sin \theta_{P-CA} + \cos \theta_{P-CA}]$$

$$Y_{ca} = I_{ca} \sin \epsilon$$

$$\sin \epsilon = \sin (120^\circ + \theta_{P-CA}) = [(\sqrt{3}/2) \cos \theta_{P-CA} + (-1/2) \sin \theta_{P-CA}]$$

$$Y_{ca} = I_{ca} (1/2) [(\sqrt{3}/2) \cos \theta_{P-CA} - \sin \theta_{P-CA}]$$

$$X_A = X_{ba} + X_{ca}$$

$$Y_A = Y_{ba} + Y_{ca}$$

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2}$$

$$\lambda = \sin^{-1} (Y_A \div I_A)$$

$$\theta_{L-A} = (\lambda - 120^\circ)$$

To develop the relationship for line current I_B , let,

X_{ab} = the abscissa component of vector I_{ab}

X_{cb} = the abscissa component of vector I_{cb}

X_B = the abscissa component of vector I_B

Y_{ab} = the ordinate component of vector I_{ab}

Y_{cb} = the ordinate

component of vector I_{cb}

Y_B = the ordinate

component of vector I_B

Current I_B is determined as shown in Fig. 33.

With reference to Fig. 31 and Fig. 33, it may be noted that,

$$X_{ab} = I_{ab} \cos \theta_{P-AB}$$

$$Y_{ab} = I_{ab} \sin \omega$$

$$Y_{ab} = I_{ab} \sin \theta_{P-AB}$$

$$X_{cb} = I_{cb} \cos \beta$$

$$\beta = 60^\circ + \theta_{P-BC}$$

$$X_{cb} = I_{cb} \cos (60^\circ + \theta_{P-BC})$$

$$X_{cb} = I_{cb} [\cos 60^\circ \cos \theta_{P-BC} - \sin 60^\circ \sin \theta_{P-BC}]$$

$$\cos 60^\circ = 1/2$$

$$\sin 60^\circ = \sqrt{3}/2$$

$$= I_{cb} [(1/2) \cos \theta_{P-BC} - (\sqrt{3}/2) \sin \theta_{P-BC}]$$

$$X_{cb} = -I_{cb} (1/2) [(\sqrt{3}) \sin \theta_{P-BC} - \cos \theta_{P-BC}]$$

$$Y_{cb} = I_{cb} \sin \beta$$

$$Y_{cb} = I_{cb} \sin (60^\circ + \theta_{P-BC})$$

$$= I_{cb} [\sin 60^\circ \cos \theta_{P-BC} + \cos 60^\circ \sin \theta_{P-BC}]$$

$$= I_{cb} [(\sqrt{3}/2) \cos \theta_{P-BC} + (1/2) \sin \theta_{P-BC}]$$

$$Y_{cb} = I_{cb} (1/2) [(\sqrt{3}) \cos \theta_{P-BC} + \sin \theta_{P-BC}]$$

Thus,

$$X_B = X_{ab} + X_{cb}$$

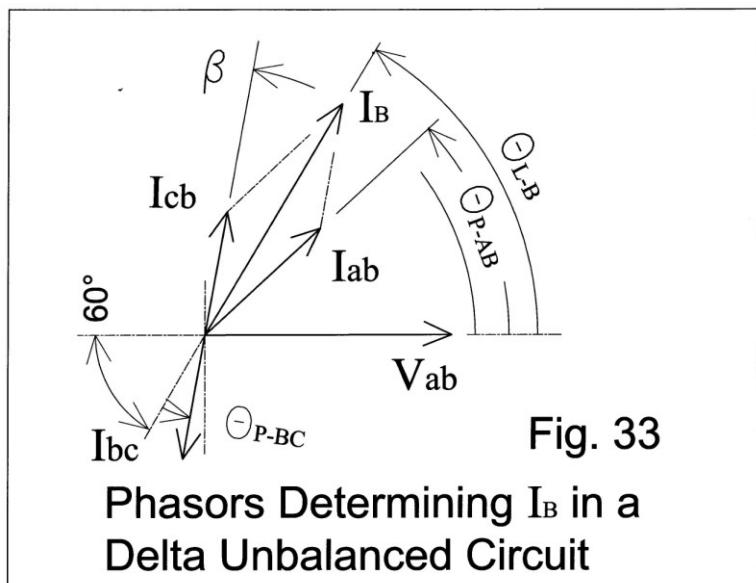


Fig. 33

Phasors Determining I_B in a Delta Unbalanced Circuit

$$Y_B = Y_{ab} + Y_{cb}$$

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}$$

$$\theta_{L-B} = \sin^{-1} (Y_B \div I_B)$$

Current I_C is determined as shown in Fig. 34.

Let,

X_{bc} = the abscissa component of vector I_{bc}

I_{bc}

X_{ac} = the abscissa component of vector I_{ac}

I_{ac}

X_C = the abscissa component of vector I_C

I_C

Y_{bc} = the ordinate component of vector I_{bc}

I_{bc}

Y_{ac} = the ordinate component of vector I_{ac}

Y_C = the ordinate component of vector I_C

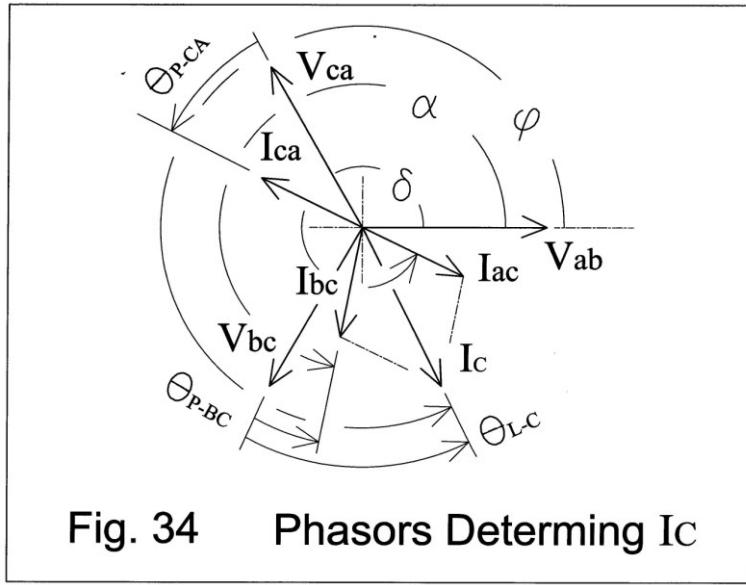


Fig. 34 Phasors Determining I_C

In Fig. 34,

$$X_{bc} = I_{bc} \cos \alpha$$

$$\alpha = 240^\circ + \theta_{P-BC}$$

$$\cos \alpha = \cos (240^\circ + \theta_{P-BC})$$

$$\cos (240^\circ + \theta_{P-BC}) = \cos 240^\circ \cos \theta_{P-BC} - \sin 240^\circ \sin \theta_{P-BC}$$

$$\cos 240^\circ = -1/2$$

$$\sin 240^\circ = -(\sqrt{3}/2)$$

$$\cos (240^\circ + \theta_{P-BC}) = (-1/2) \cos \theta_{P-BC} - [-(\sqrt{3}/2)] \sin \theta_{P-BC}$$

$$= (-1/2) [\cos \theta_{P-BC} + (\sqrt{3}/2) \sin \theta_{P-BC}]$$

$$= (1/2) [(\sqrt{3}/2) \sin \theta_{P-BC} - \cos \theta_{P-BC}]$$

$$X_{bc} = I_{bc} (1/2) [(\sqrt{3}/2) \sin \theta_{P-BC} - \cos \theta_{P-BC}]$$

$$Y_{bc} = I_{bc} \sin \alpha$$

$$\sin \alpha = \sin (240^\circ - \theta_{P-BC})$$

$$\begin{aligned}\sin(240^\circ + \theta_{P-BC}) &= \sin 240^\circ \cos \theta_{P-BC} + \cos 240^\circ \sin \theta_{P-BC} \\ &= -(\sqrt{3}/2) \cos \theta_{P-BC} + (-1/2) \sin \theta_{P-BC} \\ &= (-1/2) [(\sqrt{3}) \cos \theta_{P-BC} + \sin \theta_{P-BC}]\end{aligned}$$

$$Y_{bc} = -I_{bc} (1/2) [(\sqrt{3}) \cos \theta_{P-BC} + \sin \theta_{P-BC}]$$

$$X_{ac} = I_{ac} \cos \delta$$

$$\delta = (300^\circ + \theta_{P-AC})$$

$$X_{ac} = I_{ac} \cos (300^\circ + \theta_{P-AC})$$

$$\cos(300^\circ + \theta_{P-AC}) = \cos 300^\circ \cos \theta_{P-AC} - \sin 300^\circ \sin \theta_{P-AC}$$

$$\cos 300^\circ = 1/2$$

$$\sin 300^\circ = -(\sqrt{3}/2)$$

$$\begin{aligned}\cos(300^\circ + \theta_{P-AC}) &= 1/2 \cos \theta_{P-AC} - [-(\sqrt{3}/2)] \sin \theta_{P-AC} \\ &= (1/2) [(\sqrt{3}) \sin \theta_{P-AC} + \cos \theta_{P-AC}]\end{aligned}$$

$$X_{ac} = I_{ac} (1/2) [(\sqrt{3}) \sin \theta_{P-AC} + \cos \theta_{P-AC}]$$

$$Y_{ac} = I_{ac} \sin \delta$$

$$Y_{ac} = I_{ac} \sin (300^\circ + \theta_{P-AC})$$

$$\sin(300^\circ + \theta_{P-AC}) = \sin 300^\circ \cos \theta_{P-AC} + \cos 300^\circ \sin \theta_{P-AC}$$

$$= -(\sqrt{3}/2) \cos \theta_{P-AC} + (1/2) \sin \theta_{P-AC}$$

$$= -(1/2) [(\sqrt{3}) \cos \theta_{P-AC} - \sin \theta_{P-AC}]$$

$$Y_{ac} = -I_{ac} (1/2) [(\sqrt{3}) \cos \theta_{P-AC} - \sin \theta_{P-AC}]$$

$$X_C = X_{bc} + X_{ac}$$

$$Y_C = Y_{bc} + Y_{ac}$$

$$I_C = \{(X_C)^2 + (Y_C)^2\}^{1/2}$$

$$\phi = \sin^{-1}(Y_C \div I_C)$$

$$\theta_{L-C} = (\phi - 240^\circ)$$

Recognizing,

$$|I_{ba}| = |I_{ab}|, |I_{cb}| = |I_{bc}| \text{ and } |I_{ac}| = |I_{ca}|, \text{ and}$$

$\theta_{P-BA} = \theta_{P-AB}$, $\theta_{P-CB} = \theta_{P-BC}$ and $\theta_{P-AC} = \theta_{P-CA}$, the algebraic equations may be rewritten to exclude the terms I_{ba} , I_{cb} , I_{ac} , θ_{P-BA} , θ_{P-CB} and θ_{P-AC} .

In summary, for an unbalanced delta circuit:

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2} \dots \text{Equation 9-1}$$

$$\theta_{L-A} = (\lambda - 120^\circ) \dots \text{Equation 9-2}$$

where,

I_A = current in line A

θ_{L-A} = lead (lag) of current I_A with respect to line voltage V_{ca}

$$X_{ba} = -I_{ab} \cos \theta_{P-AB}$$

$$X_{ca} = -I_{ca} (1/2) [(\sqrt{3}) \sin \theta_{P-CA} + \cos \theta_{P-CA}]$$

$$X_A = X_{ba} + X_{ca}$$

$$Y_{ba} = -I_{ab} \sin \theta_{P-AB}$$

$$Y_{ca} = I_{ca} (1/2) [(\sqrt{3}) \cos \theta_{P-CA} - \sin \theta_{P-CA}]$$

$$Y_A = Y_{ba} + Y_{ca}$$

$$\lambda = \sin^{-1} (Y_A \div I_A)$$

Valid range of θ_{P-AB} & θ_{P-CA} : $\pm 90^\circ$; valid range of θ_{L-A} : $+120^\circ$ to -60°

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2} \dots \text{Equation 9-3}$$

$$\theta_{L-B} = \sin^{-1} (Y_B \div I_B) \dots \text{Equation 9-4}$$

where,

I_B = current in line B

θ_{L-B} = lead (lag) of current I_B with respect to line voltage V_{ab}

$$X_{ab} = I_{ab} \cos \theta_{P-AB}$$

$$X_{cb} = -I_{bc} (1/2) [(\sqrt{3}) \sin \theta_{P-BC} - \cos \theta_{P-BC}]$$

$$X_B = X_{ab} + X_{cb}$$

$$Y_{ab} = I_{ab} \sin \theta_{P-AB}$$

$$Y_{cb} = I_{bc} (1/2) [(\sqrt{3}) \cos \theta_{P-BC} + \sin \theta_{P-BC}]$$

$$Y_B = Y_{ab} + Y_{cb}$$

Valid range of θ_{P-AB} & θ_{P-BC} : $\pm 90^\circ$; valid range of θ_{L-B} : $+120^\circ$ to -60°

$$I_C = \{(X_C)^2 + (Y_C)^2\}^{1/2} \dots \text{Equation 9-5}$$

$$\theta_{L-C} = (\varphi - 240^\circ) \dots \text{Equation 9-6}$$

where,

I_C = current in line C

θ_{L-C} = lead (lag) of current I_C with respect to line voltage V_{bc}

$$X_{bc} = I_{bc} (1/2) [(\sqrt{3}) \sin \theta_{P-BC} - \cos \theta_{P-BC}]$$

$$X_{ac} = I_{ca} (1/2) [(\sqrt{3}) \sin \theta_{P-CA} + \cos \theta_{P-CA}]$$

$$X_C = X_{bc} + X_{ac}$$

$$Y_{bc} = -I_{bc} (1/2) [(\sqrt{3}) \cos \theta_{P-BC} + \sin \theta_{P-BC}]$$

$$Y_{ac} = -I_{ca} (1/2) [(\sqrt{3}) \cos \theta_{P-CA} - \sin \theta_{P-CA}]$$

$$Y_C = Y_{bc} + Y_{ac}$$

$$\phi = \sin^{-1} (Y_C \div I_C)$$

Valid range of θ_{P-BC} & θ_{P-AC} : $\pm 90^\circ$; valid range of θ_{L-C} : $+120^\circ$ to -60°

Example 11

Problem: With reference to Fig. 14 and Fig. 31, assume the following conditions for an unbalanced delta circuit:

$I_{ab} = 5$ amps @ PF = 1.0

$I_{bc} = 10$ amps @ PF = 0.9 lagging

$I_{ca} = 15$ amps @ PF = 0.8 leading

Find line currents in conductors A, B and C.

Solution:

$$\theta_{P-AB} = \cos^{-1} 1.0 = 0$$

$$\theta_{P-BC} = \cos^{-1} 0.9 = -25.84^\circ$$

$$\theta_{P-CA} = -\cos^{-1} 0.8 = +36.86^\circ$$

The respective currents may also be stated in polar notation. The lead/lag of current I_{ab} is with regard to voltage V_{ab} . Voltage V_{ab} is assumed to be in alignment with the positive abscissa. Therefore the polar notation of current I_{ab} is:

$$I_{ab} = 5/0^\circ$$

The lead/lag of current I_{bc} with regard to voltage V_{bc} is 240° CCW from the positive abscissa. Current I_{bc} lags V_{bc} . Therefore the measure of angle to current I_{bc} in the CCW direction from the positive abscissa is: $240^\circ - 25.84^\circ = 214.16^\circ$. The polar notation of I_{bc} is:

$$I_{bc} = 10/-214.16^\circ$$

The lead/lag of current I_{ca} is with regard to voltage V_{ca} which is 120° CCW from the positive abscissa. Current I_{ca} leads V_{ca} . Therefore the measure of angle to current I_{ca} in the CCW direction from the positive abscissa is: $120^\circ + 36.86^\circ = 156.86^\circ$. The polar notation, then, is:

$$I_{ca} = 10/156.86^\circ$$

From Equation 9-1 and Equation 9-2,

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2}$$

$$\theta_{L-A} = (\lambda - 120^\circ)$$

where,

I_A = current in line A

θ_{L-A} = lead (lag) of current I_A with respect to line voltage V_{ca}

$$X_{ba} = -I_{ab} \cos \theta_{P-AB} = -(5) \cos 0 = -5$$

$$Y_{ba} = -I_{ab} \sin \theta_{P-AB} = I_{ba}(0) = 0$$

$$\begin{aligned} X_{ca} &= -I_{ca}(1/2) [(\sqrt{3}) \sin \theta_{P-CA} + \cos \theta_{P-CA}] \\ &= -(15)(1/2) [(\sqrt{3}) \sin \theta_{P-CA} + \cos \theta_{P-CA}] \\ &= -(7.5) [(\sqrt{3}) \sin(36.86^\circ) + \cos(36.86^\circ)] \\ &= -(7.5) [(\sqrt{3})(0.6) + (0.8)] = -(7.5) [1.039 + 0.8] = -13.794 \end{aligned}$$

$$Y_{ca} = I_{ca}(1/2) [(\sqrt{3}) \cos \theta_{P-CA} - \sin \theta_{P-CA}]$$

$$\begin{aligned} Y_{ca} &= (15)(1/2) [(\sqrt{3}) \cos(36.86^\circ) - \sin(36.86^\circ)] \\ &= (7.5) [(\sqrt{3})(0.8) + (0.6)] = (7.5) [(1.385) - (0.6)] = 5.89 \end{aligned}$$

$$X_A = X_{ba} + X_{ca} = -5 + (-13.79) = -18.79$$

$$Y_A = Y_{ba} + Y_{ca} = 0 + 5.89 = 5.89$$

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2} = \{(-18.79)^2 + (5.89)^2\}^{1/2} = 19.69 \text{ amps}$$

$$\lambda = \sin^{-1}(Y_A \div I_A) = \sin^{-1}[5.89 \div 19.69] = \sin^{-1}0.299 = 162.59^\circ$$

$$\theta_{L-A} = (\lambda - 120^\circ) = (162.59^\circ - 120^\circ) = 42.59^\circ$$

$$\text{In polar notation: } I_A = 19.69 / (120^\circ + 42.59^\circ) = 19.69 / 162.59^\circ$$

From Equation 9-3 and Equation 9-4,

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}$$

$$\theta_{L-B} = \tan^{-1}(Y_B \div X_B)$$

where,

I_B = current in line B

θ_{L-B} = lead (lag) of current I_B with respect to line voltage V_{ab}

$$X_B = X_{ab} + X_{cb}$$

$$Y_B = Y_{ab} + Y_{cb}$$

$$X_{ab} = I_{ab} \cos \theta_{P-AB} = (5) \cos 0 = 5$$

$$Y_{ab} = -I_{ab} \sin \theta_{P-AB} = -(5) \sin 0 = 0$$

$$\begin{aligned} X_{cb} &= -I_{bc}(1/2) [(\sqrt{3}) \sin \theta_{P-BC} - \cos \theta_{P-BC}] \\ &= -(10)(1/2) [(\sqrt{3}) \sin -25.84^\circ - \cos -25.84^\circ] \end{aligned}$$

$$\begin{aligned}
 &= -(5) [(\sqrt{3})(-0.4358) - (.9)] = -(5) [(-.754) - (.9)] = 8.27 \\
 Y_{cb} &= I_{cb} (1/2) [(\sqrt{3}) \cos \theta_{P-BC} + \sin \theta_{P-BC}] \\
 &= (10) (1/2) [(\sqrt{3}) \cos -25.84^\circ + \sin -25.84^\circ] \\
 &= (5) [(\sqrt{3})(.9) + (-0.4358)] = 5.61 \\
 X_B &= X_{ab} + X_{cb} = 5 + 8.27 = 13.27 \\
 Y_B &= Y_{ab} + Y_{cb} = 0 + 5.61 = 5.61 \\
 I_B &= \{(X_B)^2 + (Y_B)^2\}^{1/2} = \{(13.27)^2 + (5.61)^2\}^{1/2} = 14.407 \text{ amps} \\
 \theta_{L-B} &= \sin^{-1} (Y_B \div I_B) = \sin^{-1} (5.61 \div 14.407) = 22.91^\circ \\
 \text{In polar notation: } I_B &= 14.40 / 22.91^\circ
 \end{aligned}$$

From Equation 9-5 and Equation 9-6,

$$I_C = \{(X_C)^2 + (Y_C)^2\}^{1/2}$$

where,

I_C = current in line C

θ_{L-C} = lead (lag) of current I_C with respect to line voltage V_{bc}

$$\begin{aligned}
 X_{bc} &= I_{bc} (1/2) [(\sqrt{3}) \sin \theta_{P-BC} - \cos \theta_{P-BC}] \\
 &= (10) (1/2) [(\sqrt{3}) \sin -25.84^\circ - \cos -25.84^\circ] \\
 &= (5) [(\sqrt{3})(-0.435) - (0.9)] = -(5)(1.6549) = -8.27 \\
 Y_{bc} &= -I_{bc} (1/2) [(\sqrt{3}) \cos \theta_{P-BC} + \sin \theta_{P-BC}] \\
 &= -(10) (1/2) [(\sqrt{3}) \cos -25.84^\circ + \sin -25.84^\circ] \\
 &= -(5) [(\sqrt{3})(.9) - (.435)] = -(5)(1.123) = -5.61
 \end{aligned}$$

$$\begin{aligned}
 X_{ac} &= I_{ca} (1/2) [(\sqrt{3}) \sin \theta_{P-CA} + \cos \theta_{P-CA}] \\
 &= (15) (1/2) [(\sqrt{3}) \sin 36.86^\circ + \cos 36.86^\circ] \\
 &= (7.5) [(\sqrt{3})(.6) + (.8)] = (7.5)(1.83) = 13.79
 \end{aligned}$$

$$\begin{aligned}
 Y_{ac} &= -I_{ca} (1/2) [(\sqrt{3}) \cos \theta_{P-CA} - \sin \theta_{P-CA}] \\
 &= -(15) (1/2) [(\sqrt{3}) \cos 36.86^\circ - \sin 36.86^\circ] \\
 &= -(7.5) [(\sqrt{3})(.8) - (0.6)] = -(7.5)(.786) = -5.89
 \end{aligned}$$

$$X_C = X_{bc} + X_{ac} = -8.27 + 13.79 = 5.52$$

$$Y_C = Y_{bc} + Y_{ac} = -5.61 + (-5.89) = -11.50$$

$$\begin{aligned}
 I_C &= \{(X_C)^2 + (Y_C)^2\}^{1/2} = \{(5.52)^2 + (-11.50)^2\}^{1/2} = 12.76 \text{ amps} \\
 \theta_{L-C} &= (\varphi - 240^\circ)
 \end{aligned}$$

I_C is in Quadrant IV

$$\phi = \sin^{-1} (Y_C \div I_C) = \sin^{-1} (-11.500 \div 12.756) = 295.64^\circ$$

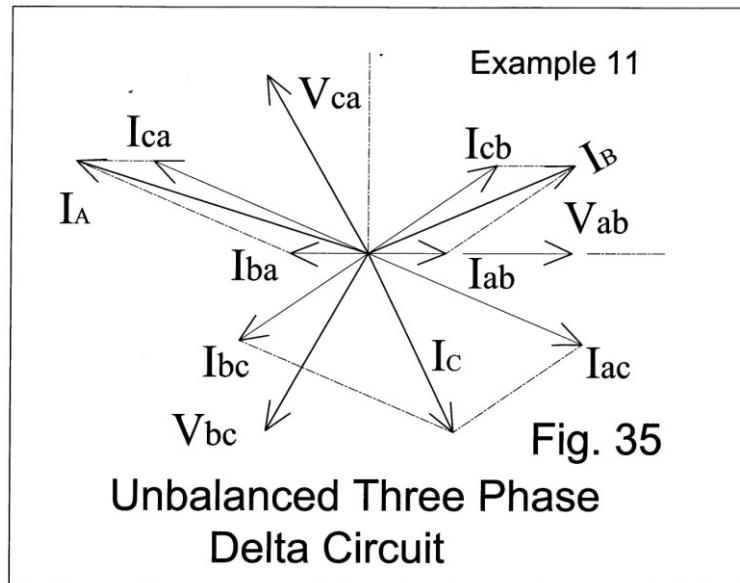
$$\theta_{L-C} = (\phi - 240^\circ) = 295.64^\circ - 240^\circ = 55.64^\circ$$

In polar notation:

$$I_C = 12.756 / (240 + 55.64^\circ)$$

$$= 12.756 / 295.64^\circ$$

The values calculated in Example 11 are shown in Fig. 35 where the phasors and the associated angles are drawn to-scale.



5B.2 Unbalanced Three Phase Delta Circuit with Only Resistive Loads

A typical three phase delta circuit is represented in Fig. 14. With a balanced resistive load, the delta phasor showing phase currents would be as represented in Fig. 17. The line currents for the balanced circuit would be determined as shown in Fig. 18. For an unbalanced delta circuit with resistive loads, a similar phasor diagram can be constructed. A typical phasor diagram for a resistive circuit is represented in Fig. 36. In Fig. 36, two of the phasors representative of phase currents I_{bc} and I_{ca} are shown as equal whereas the third current, I_{ab} , is assumed to be smaller.

The phasor diagram for determining line current I_B is shown in Fig. 37. As was the procedure with a balanced delta circuit, the vector I_B is determined by adding the vector of I_{ab} and the negative vector of I_{bc} , which is I_{cb} . According to the rules for adding vectors, the “x” (or abscissa) component of vector I_B is determined by adding the “x” components of I_{ab} and I_{cb} . Likewise, the “y” (or ordinate) component of vector I_B is determined by adding the y components of vectors I_{ab} and I_{cb} . Since I_{ab} is in-phase with reference voltage V_{ab} , the vector for current I_{ab} has no “y” component.

Essentially, the equations for a delta circuit with all resistive loads becomes a special case of Equations 9-1 through 9-6 wherein $\theta_{L-A} = 0$, $\theta_{L-B} = 0$ &

$\theta_{L-C} = 0$. Under these circumstances, Equations 9-1 through 9-6 simplify to the equations shown below as Equations 10-1 through 10-6.

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2} \dots \text{Equation 10-1}$$

$$\theta_{L-A} = (\lambda - 120^\circ) \dots \text{Equation 10-2}$$

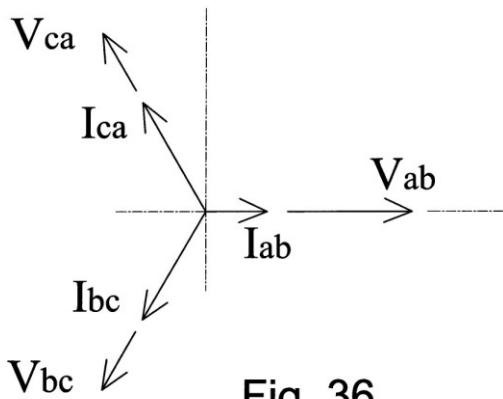


Fig. 36
Phasors for Unbalanced Delta Circuit with Resistive Load

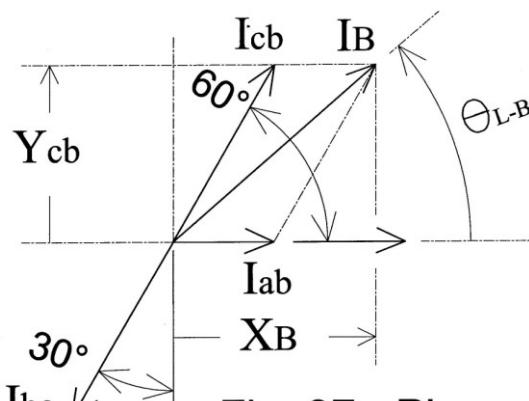


Fig. 37 - Phasors Determining I_B in an Unbalanced Resistive Delta Circuit

where,

I_A = current in line A

θ_{L-A} = lead (lag) of current I_A with respect to line voltage V_{ca}

$$X_A = -I_{ab} - (1/2) I_{ca}$$

$$Y_A = I_{ca} (\sqrt{3}/2)$$

$$\lambda = \sin^{-1} (Y_A \div I_A)$$

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2} \dots \text{Equation 10-3}$$

$$\theta_{L-B} = \sin^{-1} (Y_B \div I_B) \dots \text{Equation 10-4}$$

where,

I_B = current in line B

θ_{L-B} = lead (lag) of current I_B with respect to line voltage V_{ab}

$$X_B = I_{ab} + (1/2) I_{bc}$$

$$Y_B = I_{bc} (\sqrt{3}/2)$$

$$I_C = \{(X_C)^2 + (Y_C)^2\}^{1/2} \dots \text{Equation 10-5}$$

$$\theta_{L-C} = (\varphi - 240^\circ) \dots \text{Equation 10-6}$$

where,

I_C = current in line C

θ_{L-C} = lead (lag) of current I_C with respect to line voltage V_{bc}

$$X_C = (1/2) I_{ca} - (1/2) I_{bc}$$

$$Y_C = -I_{bc} (\sqrt{3}/2) - I_{ca} (\sqrt{3}/2)$$

$$\varphi = \sin^{-1} (Y_C \div I_C)$$

Example 12

Problem: With reference to Fig. 14 (a delta circuit), given that I_{ab} is 5 amps, resistive, and both I_{bc} and I_{ca} are each 10 amps, resistive. Find line currents I_A , I_B and I_C and the respective lead/lag with respect to line voltages.

Solution:

Reference is made to Equations 10-1 through 10-6.

$$X_A = -I_{ab} - (1/2) I_{ca} = -(5) - (1/2)(10) = -10$$

$$Y_A = I_{ca} (\sqrt{3}/2) = (10) (\sqrt{3}/2) = 5\sqrt{3} = 8.66$$

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2} = \{(-10)^2 + (8.66)^2\}^{1/2} = 13.22 \text{ amps}$$

$$\lambda = \sin^{-1} (Y_A \div I_A) = \sin^{-1} (8.66 \div 13.22) = \sin^{-1} (.655)$$

I_A is in Quadrant II

$$\lambda = 139.08^\circ$$

$$\theta_{L-A} = (\lambda - 120^\circ) = (139.08^\circ - 120^\circ) = 19.10^\circ$$

$$X_B = I_{ab} + (1/2) I_{bc} = 5 + (1/2)(10) = 10$$

$$Y_B = I_{bc} (\sqrt{3}/2) = (10) (\sqrt{3}/2) = 5\sqrt{3}$$

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2} = \{(10)^2 + (5\sqrt{3})^2\}^{1/2} = 13.228 \text{ amps}$$

$$\theta_{L-B} = \sin^{-1} (Y_B \div I_B) = \sin^{-1} (5\sqrt{3} \div 13.228) = \sin^{-1} (.655)$$

I_B is in Quadrant I

$$\theta_{L-B} = 40.89^\circ$$

$$X_C = (1/2) I_{ca} - (1/2) I_{bc} = (1/2)(10) - (1/2)(10) = 0$$

$$Y_C = -I_{bc} (\sqrt{3}/2) - I_{ca} (\sqrt{3}/2) = -(10) (\sqrt{3}/2) - (10)(\sqrt{3}/2) = -(10)\sqrt{3} = -17.32$$

$$I_C = \{(X_C)^2 + (Y_C)^2\}^{1/2}$$

$$= \{(0)^2 + (-10\sqrt{3})^2\}^{1/2}$$

$$= (10)\sqrt{3} = 17.32 \text{ amps}$$

$$\phi = \sin^{-1} (Y_C \div I_C)$$

$$= \sin^{-1} (-17.32 \div 17.32)$$

$$= \sin^{-1} (-1)$$

I_C is on the negative ordinate.

$$\phi = 270^\circ$$

$$\theta_{L-C} = \theta_{L-C} = (\phi - 240^\circ) = (270^\circ - 240^\circ) = 30^\circ$$

5C. Unbalanced Three Phase Wye Circuit

A typical three phase wye circuit is represented in Fig. 15, and the phasors for a typical unbalanced wye load are represented in Fig. 38. For the purposes of analysis, it is assumed that no two phase currents are necessarily of equal value or necessarily at the same lead/lag angle to the respective phase voltage. In a resistive load $\theta_P = 0$, i.e. the phase current is in-phase with the phase voltage. If the load is capacitive, $\theta_P > 0$ (and the current is said to be “leading”). If the load is inductive $\theta_P < 0$ (and the current is said to be “lagging”). As with a delta three phase load, let,

I_A = current in Conductor A

I_B = current in Conductor B

I_C = current in Conductor C

I_D = current in Conductor D

θ_{L-A} = lead/lag angle of vector I_A with respect to voltage V_{ca}

θ_{L-B} = lead/lag angle of vector I_B with respect to voltage V_{ab}

θ_{L-C} = lead/lag angle of vector I_B with respect to voltage V_{bc}

[Note that in the case of an unbalanced wye circuit, the current in the neutral conductor (Conductor D) is of importance and the size of the neutral conductor is likewise important. Conductor D must be sized to carry the largest combination of currents that may result from the combination of currents in Conductors A, B and C. It may be shown that the largest possible current in Conductor D will be twice the maximum current in conductors A, B or C.]

In comparison to the delta circuit, determining line currents in a wye circuit is a simple process since the phase currents are equal in magnitude to the line currents. Furthermore, the phase voltages lead line voltages by 30° .

Consequently, the phase currents lead line voltages by 30° plus or minus the respective lead/lag angle. More specifically,

$$\theta_{P-DA} = \theta_{L-A} - 30^\circ$$

$$\theta_{P-DB} = \theta_{L-B} - 30^\circ$$

$$\theta_{P-DC} = \theta_{L-C} - 30^\circ$$

Example 13

Problem: Assume wye circuit parameters similar to those selected in Example 12. (Example 12 was for a delta circuit). With reference to Fig.

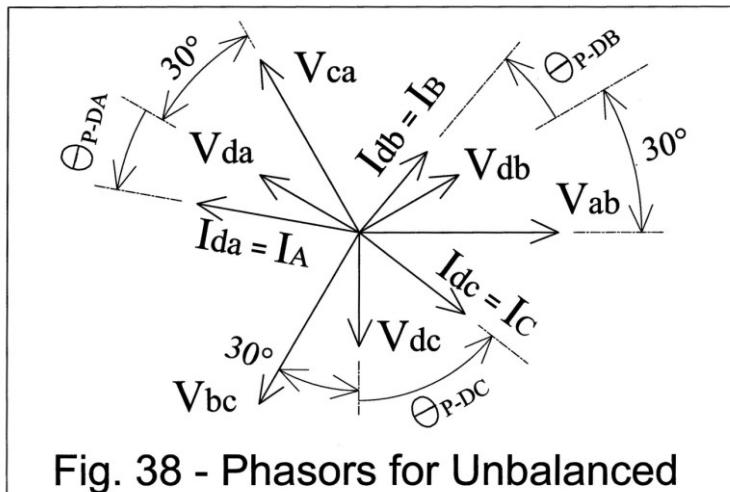


Fig. 38 - Phasors for Unbalanced wye Circuit

15 and Fig. 38, also assume the following conditions for an unbalanced wye circuit:

$$I_{db} = 5 \text{ amps} @ \text{PF} = 1.0$$

$$I_{dc} = 10 \text{ amps} @ \text{PF} = 0.9 \text{ lagging}$$

$$I_{da} = 15 \text{ amps} @ \text{PF} = 0.8 \text{ leading}$$

Find line currents in conductors A, B and C.

Solution:

The phase currents are the line currents and are repeated since the magnitude of the phase currents is the same as that of the line currents.

$$I_{da} = I_A = 15 \text{ amps}$$

$$I_{db} = I_B = 5 \text{ amps}$$

$$I_{dc} = I_C = 10 \text{ amps}$$

$$\theta_{P-DA} = \cos^{-1} 0.8 = 36.86^\circ \text{ (leading)},$$

$$\theta_{P-DB} = \cos^{-1} 1.0 = 0$$

$$\theta_{P-DC} = \cos^{-1} 0.9 = -25.84^\circ \text{ (lagging)}$$

From Equation 12-1,

$$\theta_P = \theta_L - 30^\circ$$

$$\theta_{L-A} = \theta_{P-DA} + 30^\circ = 36.86^\circ + 30^\circ = +66.86^\circ$$

$$\theta_{L-B} = \theta_{P-DB} + 30^\circ = 0^\circ + 30^\circ = +30^\circ$$

$$\theta_{L-C} = \theta_{P-DC} + 30^\circ = -25.84^\circ + 30^\circ = +4.16^\circ$$

The computations of both Example 12 and Example 13 demonstrate that, given phase currents, it is much easier to determine line currents for a wye circuit than for a delta circuit.

5D. Combined Unbalanced Three Phase Circuits

As may often happen, a three phase circuit will serve a mixture of delta and wye loads as well as both balanced and unbalanced loads. A common problem, then, is to calculate line currents in the conductors of a three phase circuit that serves more than a single load and loads that could be balanced or unbalanced.

Often, calculations are not required for three phase loads as the line currents may already be provided. For example, motors always have the full load currents (which would be the line current) stamped on the nameplate, and often the full load power factor (which describes the phase current lead/lag with respect to phase voltage – and not lead/lag of current with respect to line voltage) is included on the motor nameplate. In such cases, the motor data can be converted to lead/lag of currents with respect to line voltages (by vector addition) with other applicable data. The problem then is to correctly add the line currents in branch circuits to determine the net currents in a common feeder.

Above it was shown how to calculate line currents for delta or wye loads whether the phases are balanced or unbalanced. Using the techniques demonstrated, it becomes an easy task to calculate currents in each of the three phases. Essentially, the net current in a selected phase merely requires the addition of the “x” and “y” components of the contributing phasors to determine, respectively, the “x” and “y” components that determine the net currents in each of the common feeder phases. In the way of illustration, a three phase feeder is shown in Fig. 39 serving three loads which are designated as Load 1, Load 2 and Load 3. The mentioned loads, Load 1, Load 2 and Load 3, could be delta, wye or single phase and balanced or unbalanced.

In Fig. 39:
The current in conductor A is the sum of the currents in conductors E, H & X.
The current in conductor B is the sum of the currents in conductors F, J & Y.
The current in conductor C is the sum of the currents in conductors G, K & Z.

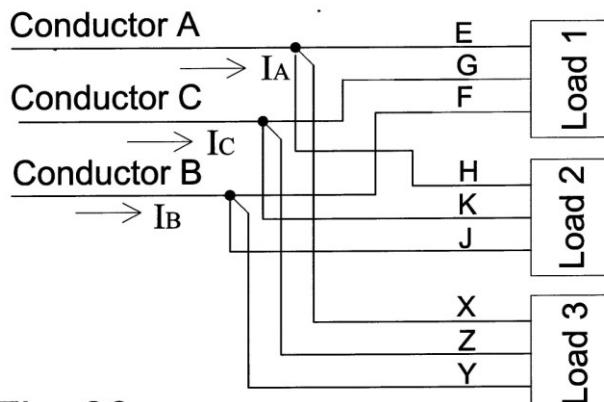


Fig. 39

Three - Three Phase Loads

The current in conductor C is the sum of the currents in conductors G, K & Z.

The currents in conductors A, B and C can be obtained by the vector additions of the components that determine the currents in conductors A, B & C. To determine the (line) current in Conductor B, the vectors representing the (line) currents in conductors F, J and Y are added as shown in Fig. 40.

(Current I_B is determined first for the reason that the computations for I_B are somewhat easier than those for currents I_A and I_C .) In Fig. 40 arbitrary phasor values are shown as representative of currents F, J and Y.

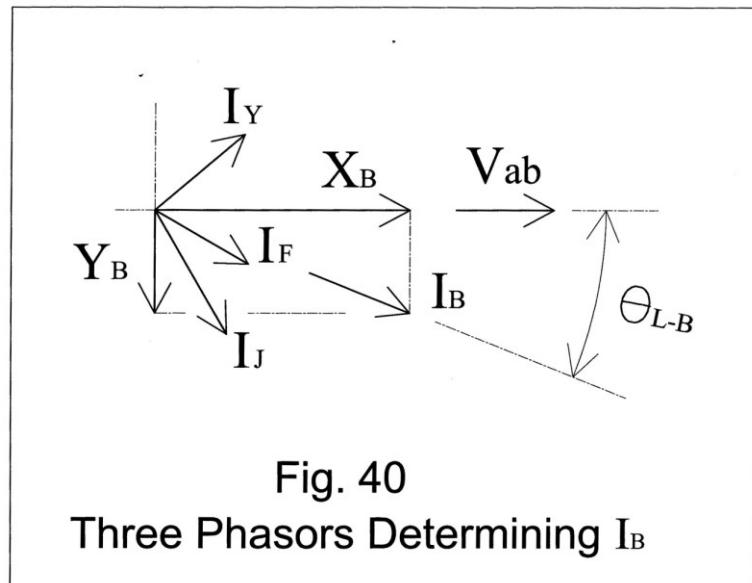


Fig. 40
Three Phasors Determining I_B

For the circuit configuration of Fig. 39,

$$X_B = X_F + X_J + X_Y, \text{ and}$$

$$Y_B = Y_F + Y_J + Y_Y, \text{ where}$$

X_B = abscissa component of current

Y_B = ordinate component of current B

$$X_F = I_F \cos \theta_{L-F}$$

$$X_J = I_J \cos \theta_{L-J}$$

$$X_Y = I_Y \cos \theta_{L-Y}$$

$$Y_F = I_F \sin \theta_{L-F}$$

$$Y_J = I_J \sin \theta_{L-J}$$

$$Y_Y = I_Y \sin \theta_{L-Y}$$

θ_{L-F} = lead/lag of line current F with respect to line voltage V_{ab}

θ_{L-J} = lead/lag of line current J with respect to line voltage V_{bc}

θ_{L-Y} = lead/lag of line current F with respect to line voltage V_{ca}

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}$$

$$\theta_{L-B} = \sin^{-1} (Y_B \div I_B)$$

Example 14

Problem: For the configuration of Fig. 39, assume:

$$I_F = 12 \text{ amps} @ \theta_{L-F} = -30^\circ \text{ (lagging with respect to } V_{ab})$$

$$I_J = 16 \text{ amps} @ \theta_{L-J} = -60^\circ \text{ (lagging with respect to } V_{ab})$$

$$I_Y = 13 \text{ amps} @ \theta_{L-Y} = +40^\circ \text{ (leading with respect to } V_{ab})$$

Find current I_B and its lead/lag with respect to voltage V_{ab} .

Solution:

$$X_F = 12 \cos -30^\circ = (12) (.866) = 10.392$$

$$X_J = 16 \cos -60^\circ = (16) (.500) = 8.000$$

$$X_Y = 13 \cos 40^\circ = (13) (.766) = 9.958$$

$$X_B = X_F + X_J + X_Y = 10.392 + 8.000 + 9.958 = 28.350$$

$$Y_F = (12 \sin -30^\circ) = (12) (-.500) = -6.000$$

$$Y_J = (16 \sin -60^\circ) = (16) (-.866) = -13.856$$

$$Y_Y = (13 \sin 40^\circ) = (13) (.642) = 8.356$$

$$Y_B = Y_F + Y_J + Y_Y = -6.000 - 13.856 + 8.356 = -11.500$$

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2} = \{(28.350)^2 + (-11.500)^2\}^{1/2} = 30.593 \text{ amps}$$

$$\theta_{L-B} = \sin^{-1}(Y_B \div I_B) = \sin^{-1}(-11.500 \div 30.593)$$

I_B is in Quadrant IV

$$\sin^{-1}(-11.500 \div 30.593) = \sin^{-1}(-.376) = -22.080^\circ$$

$$\theta_{L-B} = -22.080^\circ$$

The given values of X_F , X_J , X_Y , Y_F , Y_J , Y_Y , θ_{L-F} , θ_{L-J} , θ_{L-Y} and the computed values of X_B , Y_B , I_B , and θ_{L-B} are shown, drawn to scale, in Fig. 40. (The arc designating the value and position of θ_{L-B} in Fig. 40 is shown with arrowheads at both ends to indicate it is either a negative value or a parameter without a positive direction indication.)

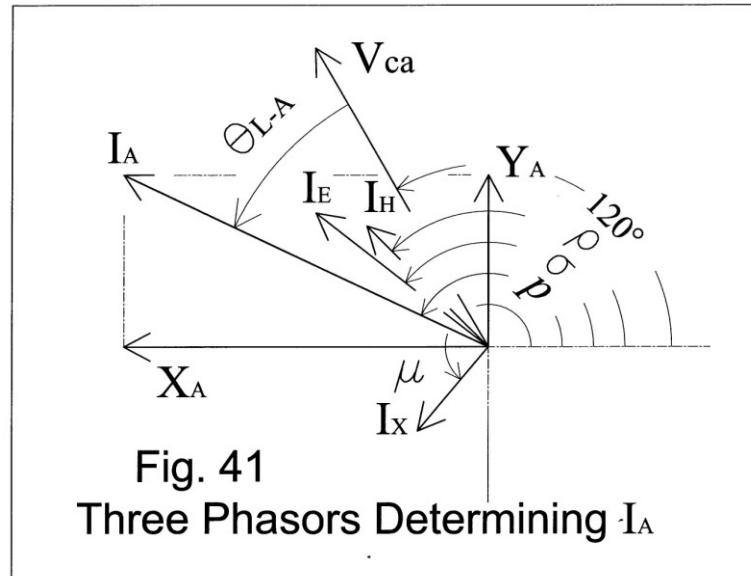
In order to arrive at the value of I_A , the phasors determining I_A are arranged as represented in Fig. 41.

In Fig. 41:

$$\sigma = 120^\circ + \theta_{L-E}$$

$$\rho = 120^\circ + \theta_{L-H}$$

$$\mu = 120^\circ + \theta_{L-X}$$



$$\begin{aligned}
 X_E &= I_E \cos \sigma \\
 &= I_E \cos (120^\circ + \theta_{L-E}) \\
 &= I_E [\cos 120^\circ \cos \theta_{L-E} \\
 &\quad - \sin 120^\circ \sin \theta_{L-E}] \\
 \cos 120^\circ &= -1/2 \\
 \sin 120^\circ &= (\sqrt{3}/2)
 \end{aligned}$$

$$X_E = I_E [(-1/2) \cos \theta_{L-E} - (\sqrt{3}/2) \sin \theta_{L-E}]$$

$$X_E = -I_E [(\sqrt{3}/2) \sin \theta_{L-E} + (1/2) \cos \theta_{L-E}], \text{ and}$$

$$X_H = -I_H [(\sqrt{3}/2) \sin \theta_{L-H} + (1/2) \cos \theta_{L-H}]$$

$$X_X = -I_X [(\sqrt{3}/2) \sin \theta_{L-X} + (1/2) \cos \theta_{L-X}]$$

$$Y_E = I_E \sin \sigma = I_E \sin (120^\circ + \theta_{L-E})$$

$$\begin{aligned}
 Y_E &= I_E [\sin 120^\circ \cos \theta_{L-E} + \cos 120^\circ \sin \theta_{L-E}] \\
 &= I_E [(\sqrt{3}/2) \cos \theta_{L-E} + (-1/2) \sin \theta_{L-E}]
 \end{aligned}$$

$$Y_E = I_E [(\sqrt{3}/2) \cos \theta_{L-E} - (1/2) \sin \theta_{L-E}], \text{ and}$$

$$Y_H = I_H [(\sqrt{3}/2) \cos \theta_{L-H} - (1/2) \sin \theta_{L-H}]$$

$$Y_X = I_X [(\sqrt{3}/2) \cos \theta_{L-X} - (1/2) \sin \theta_{L-X}]$$

$$X_A = X_E + X_H + X_X$$

$$Y_A = Y_E + Y_H + Y_X$$

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2}$$

Let,

$$\kappa = \sin^{-1} (Y_A \div I_A)$$

(Note: For suggestions on foolproof methods to calculate \sin^{-1} values to determine λ , φ , κ and ζ See Equation 13-1 in Section 7B.

$$\theta_{L-A} = \kappa - 120^\circ$$

Example 15

Problem:

Let,

$$I_E = 14 \text{ amps } @ +22^\circ \text{ (leading with respect to } V_{ca})$$

$$I_H = 11 \text{ amps } @ +15^\circ \text{ (leading with respect to } V_{ca})$$

$$I_X = 7 \text{ amps } @ +110^\circ \text{ (leading with respect to } V_{ca})$$

Determine I_A

Solution:

$$\begin{aligned} X_E &= -I_E [(\sqrt{3}/2) \sin \theta_{L-E} + (1/2) \cos \theta_{L-E}] \\ &= -(14) [(\sqrt{3}/2) \sin (22^\circ) + (1/2) \cos (22^\circ)] \\ &= -(14) [(\sqrt{3}/2) (.3746) + (1/2) (.9271)] \\ &= -(14) [(.3244) + (.4635)] = -(14) (.7879) = -11.030 \end{aligned}$$

$$\begin{aligned} X_H &= -I_H [(\sqrt{3}/2) \sin \theta_{L-H} + (1/2) \cos \theta_{L-H}] \\ &= -(11) [(\sqrt{3}/2) \sin (15^\circ) + (1/2) \cos (15^\circ)] \\ &= -(11) [(\sqrt{3}/2) (.2588) + (1/2) (.9659)] \\ &= -(11) [(.2241) + (.4829)] = -(11) (.7070) = -7.777 \end{aligned}$$

$$\begin{aligned} X_X &= -I_X [(\sqrt{3}/2) \sin \theta_{L-X} + (1/2) \cos \theta_{L-X}] \\ &= -(7) [(\sqrt{3}/2) \sin (110^\circ) + (1/2) \cos (110^\circ)] \\ &= -(7) [(\sqrt{3}/2) (-.9396) + (1/2) (-.3420)] \\ &= -(7) [-(.8137) - (.1710)] = -(7) (.6427) = -4.499 \end{aligned}$$

$$X_A = X_E + X_H + X_X$$

$$X_A = -11.030 + (-7.777) + (-4.499) = -23.306$$

$$\begin{aligned} Y_E &= I_E [(\sqrt{3}/2) \cos \theta_{L-E} - (1/2) \sin \theta_{L-E}] \\ &= (14) [(\sqrt{3}/2) \cos (22^\circ) - (1/2) \sin (22^\circ)] \\ &= (14) [(\sqrt{3}/2) (.9271) - (1/2) (.3746)] \end{aligned}$$

$$Y_E = (14) [(.8029) - (.1873)] = (14) (.6155) = 8.617$$

$$\begin{aligned} Y_H &= I_H [(\sqrt{3}/2) \cos \theta_{L-H} - (1/2) \sin \theta_{L-H}] \\ &= (11) [(\sqrt{3}/2) \cos (15^\circ) - (1/2) \sin (15^\circ)] \\ &= (11) [(\sqrt{3}/2) (.9659) - (1/2) (.2588)] \\ &= (11) [(.8364) - (.1294)] = (11) (.7070) = 7.777 \end{aligned}$$

$$\begin{aligned} Y_X &= I_X [(\sqrt{3}/2) \cos \theta_{L-X} - (1/2) \sin \theta_{L-X}] \\ &= (7) [(\sqrt{3}/2) \cos (110^\circ) - (1/2) \sin (110^\circ)] \end{aligned}$$

$$\begin{aligned}
 &= (7) [(\sqrt{3}/2)(-.3420) - (1/2)(.9396)] \\
 &= (7) [(-.2961) - (.4698)] = (7)(-.7659) = -5.361
 \end{aligned}$$

$$Y_A = Y_E + Y_H + Y_X$$

$$Y_A = 8.617 + 7.777 + (-5.361) = 11.033$$

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2}$$

$$I_A = \{(-23.306)^2 + (11.033)^2\}^{1/2} = 25.785$$

$$\kappa = \sin^{-1}(Y_A \div I_A) = \sin^{-1}(11.033 \div 25.785) = \sin^{-1}.4278$$

I_A is in Quadrant II

$$p = 154.66^\circ$$

$$\theta_{L-A} = 120^\circ - p = 120^\circ - 154.66^\circ = -34.66^\circ$$

The given values of X_E , X_H , X_X , Y_E , Y_H , Y_X , θ_{L-E} , θ_{L-H} , θ_{L-X} and the computed values of X_A , Y_A , I_A and θ_{L-A} are shown, drawn to scale, in Fig. 41.

In order to arrive at the value of I_C , the phasors determining I_C are arranged as represented in Fig. 42.

In Fig. 42:

$$q = 240^\circ + \theta_{L-G}$$

$$r = 240^\circ + \theta_{L-K}$$

$$s = 240^\circ + \theta_{L-Z}$$

$$X_G = I_G \cos q$$

$$= I_G \cos(240^\circ + \theta_{L-G})$$

$$X_G = I_G [\cos 240^\circ \cos \theta_{L-G} - \sin 240^\circ \sin \theta_{L-G}]$$

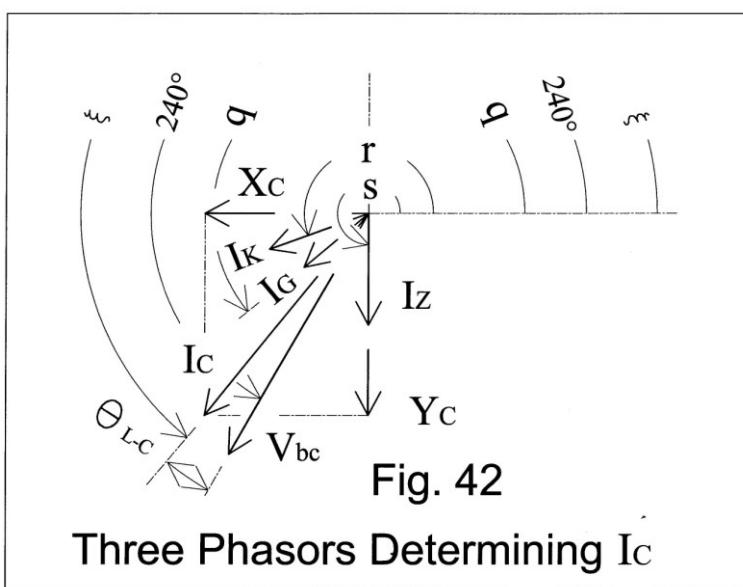
$$\cos 240^\circ = -\cos 60^\circ = -(1/2)$$

$$\sin 240^\circ = -\sin 60^\circ = -(\sqrt{3}/2)$$

$$X_G = I_G [-(1/2) \cos \theta_{L-G} - (-\sqrt{3}/2) \sin \theta_{L-G}]$$

$$X_G = I_G [(\sqrt{3}/2) \sin \theta_{L-G} - (1/2) \cos \theta_{L-G}], \text{ and}$$

$$X_K = I_K [(\sqrt{3}/2) \sin \theta_{L-K} - (1/2) \cos \theta_{L-K}]$$



$$X_Z = -I_Z [(\sqrt{3}/2) \sin \theta_{L-Z} - (1/2) \cos \theta_{L-Z}]$$

$$Y_G = I_G \sin q = I_G \sin (240^\circ + \theta_{L-G})$$

$$= I_G [\sin 240^\circ \cos \theta_{L-G} + \cos 240^\circ \sin \theta_{L-G}]$$

$$= I_G [-(\sqrt{3}/2) \cos \theta_{L-G} + (-1/2) \sin \theta_{L-G}]$$

$$Y_G = -I_G [(\sqrt{3}/2) \cos \theta_{L-G} + (1/2) \sin \theta_{L-G}], \text{ and}$$

$$Y_K = -I_K [(\sqrt{3}/2) \cos \theta_{L-K} + (1/2) \sin \theta_{L-K}]$$

$$Y_Z = -I_Z [(\sqrt{3}/2) \cos \theta_{L-Z} + (1/2) \sin \theta_{L-Z}]$$

$$X_C = X_G + X_K + X_Z$$

$$Y_C = Y_G + Y_K + Y_Z$$

$$I_C = \{(X_C)^2 + (Y_C)^2\}^{1/2}$$

$$\zeta = \sin^{-1} (Y_C \div I_C)$$

$$\theta_{L-C} = \zeta - 240^\circ$$

Example 16

Problem:

Reference Fig. 39. Let,

$$I_G = 12 \text{ amps } @ -20^\circ \text{ (lagging with respect to } V_{bc})$$

$$I_K = 15 \text{ amps } @ -40^\circ \text{ (lagging with respect to } V_{bc})$$

$$I_Z = 16 \text{ amps } @ +30^\circ \text{ (leading with respect to } V_{bc})$$

Determine I_C

Solution:

$$\begin{aligned} X_G &= I_G [(\sqrt{3}/2) \sin \theta_{L-G} - (1/2) \cos \theta_{L-G}] \\ &= (12) [(\sqrt{3}/2) \sin -20^\circ - (1/2) \cos -20^\circ] \\ &= (12) [(\sqrt{3}/2) (-.3420) - (1/2) (.9396)] \\ &= (12) [(-.2961) - (.4698)] = -9.1908 \end{aligned}$$

$$\begin{aligned} X_K &= I_K [(\sqrt{3}/2) \sin \theta_{L-K} - (1/2) \cos \theta_{L-K}] \\ &= (15) [(\sqrt{3}/2) \sin -40^\circ - (1/2) \cos -40^\circ] \\ &= (15) [(\sqrt{3}/2) (-.6427) - (1/2) (.7660)] \\ &= (15) [-.5565 - .3830] = -14.0925 \end{aligned}$$

$$\begin{aligned} X_Z &= I_Z [(\sqrt{3}/2) \sin \theta_{L-Z} - (1/2) \cos \theta_{L-Z}] \\ &= (16) [(\sqrt{3}/2) \sin 30^\circ - (1/2) \cos 30^\circ] \\ &= (16) [(\sqrt{3}/2) (.5) - (1/2) (.8660)] \end{aligned}$$

$$= (16) [.4330 - (.4330)] = 0$$

$$X_C = X_G + X_K + X_Z$$

$$= -9.1908 - 14.0925 - 0 = -23.2833$$

$$Y_G = -I_G [(\sqrt{3}/2) \cos \theta_{L-G} + (1/2) \sin \theta_{L-G}]$$

$$= -(12) [(\sqrt{3}/2) \cos -20^\circ + (1/2) \sin -20^\circ]$$

$$= -(12) [(\sqrt{3}/2) (.9396) + (1/2) (-.3420)]$$

$$= -(12) [.8137 - .1710] = -7.7124$$

$$Y_K = -I_K [(\sqrt{3}/2) \cos \theta_{L-K} + (1/2) \sin \theta_{L-K}]$$

$$= -(15) [(\sqrt{3}/2) \cos -40^\circ + (1/2) \sin -40^\circ]$$

$$= -(15) [(\sqrt{3}/2) (.7660) + (1/2) (-.6427)]$$

$$= -(15) [.6633 - .3213] = -5.130$$

$$Y_Z = -I_Z [(\sqrt{3}/2) \cos \theta_{L-Z} + (1/2) \sin \theta_{L-Z}]$$

$$= -(16) [(\sqrt{3}/2) \cos 30^\circ + (1/2) \sin 30^\circ]$$

$$= -(16) [(\sqrt{3}/2) (.8660) + (1/2) (.500)]$$

$$= -(16) [.7500 + .2500] = -16$$

$$Y_C = Y_G + Y_K + Y_Z$$

$$= -7.7124 - 5.130 - 16.000 = -28.842$$

$$I_C = \{(X_C)^2 + (Y_C)^2\}^{1/2}$$

$$= \{(-23.2833)^2 + (-28.842)^2\}^{1/2} = 37.067 \text{ amps}$$

$$\zeta = \sin^{-1} (Y_C \div I_C) = \sin^{-1} (-28.842 \div 37.067) = \sin^{-1} (-.778)$$

I_C is in Quadrant III

$$\zeta = 231.087^\circ$$

$$\theta_{L-C} = \zeta - 240^\circ = 231.087^\circ - 240^\circ = -8.913^\circ$$

The given values of I_G , I_K , I_Z and the computed values of X_C , Y_C , I_C and θ_{L-C} are shown, drawn to scale, in Fig. 42.

It becomes apparent that when computing the net value of line currents in conductors that deliver power to a variety of three phase loads (which could be balanced or unbalanced), the parameters may be represented as shown in below Equations 11-1 to 11-6. The parameters of the generalized Equations of 11-1 to 11-6 are shown in Fig. 43.

Given Loads 1, 2 and 3, the applicable equations for determining the currents in conductors A, B and C of the representation of Fig. 43 then become:

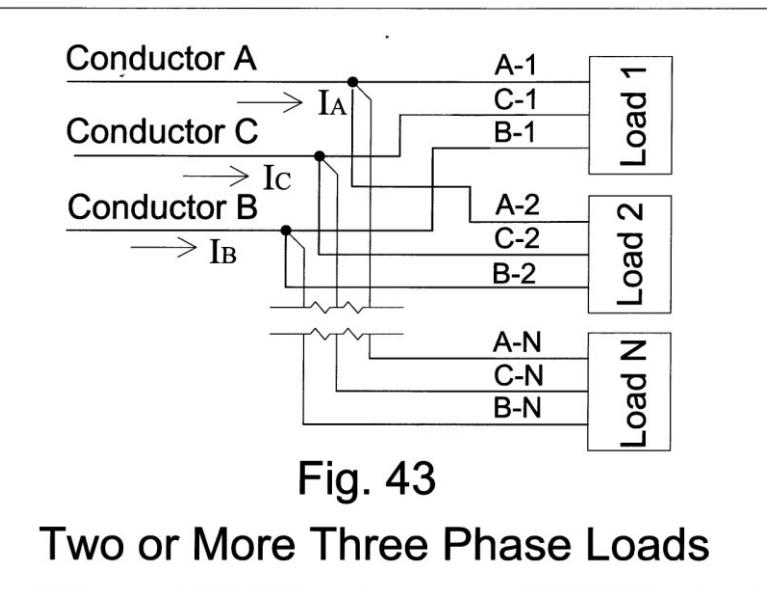


Fig. 43

Two or More Three Phase Loads

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2} \dots \text{Equation 11-1, and}$$

$$\theta_{L-A} = (\kappa - 120^\circ) \dots \text{Equation 11-2, where}$$

$$\kappa = \sin^{-1} (Y_A \div I_A), \text{ and}$$

$$X_A = \sum (X_{A-1} + X_{A-2} + X_{A-3} \dots + X_{A-N})$$

$$Y_A = \sum (Y_{A-1} + Y_{A-2} + Y_{A-3} \dots + Y_{A-N})$$

$$X_{A-1} = -I_{A-1} (1/2) [(\sqrt{3}) \sin \theta_{A-1} + \cos \theta_{A-1}]$$

$$X_{A-2} = -I_{A-2} (1/2) [(\sqrt{3}) \sin \theta_{A-2} + \cos \theta_{A-2}] \dots \text{to}$$

$$X_{A-N} = -I_{A-N} (1/2) [(\sqrt{3}) \sin \theta_{A-N} + \cos \theta_{A-N}]$$

$$Y_{A-1} = I_{A-1} (1/2) [(\sqrt{3}) \cos \theta_{A-1} - \sin \theta_{A-1}] \dots \text{to}$$

$$Y_{A-N} = I_{A-N} (1/2) [(\sqrt{3}) \cos \theta_{A-N} - \sin \theta_{A-N}]$$

I_{A-1} = line current in branch of conductor A to load #1

I_{A-2} = line current in branch of conductor A to load #2 ... to

I_{A-N} = line current in branch of conductor A to load #N

θ_{A-1} = lead/lag of line current I_{A-1} with respect to line voltage V_{ca}

θ_{A-2} = lead/lag of line current I_{A-2} with respect to line voltage V_{ca} ... to

θ_{A-N} = lead/lag of line current I_{A-N} with respect to line voltage V_{ca}

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}, \text{ and } \dots \text{Equation 11-3}$$

$$\theta_{L-B} = \sin^{-1} (Y_B \div I_B), \text{ where } \dots \text{Equation 11-4}$$

$$X_B = \sum (X_{B-1} + X_{B-2} + X_{B-3} \dots + X_{B-N})$$

$$Y_B = \sum (Y_{B-1} + Y_{B-2} + Y_{B-3} \dots + Y_{B-N})$$

$X_{B-1} = I_{B-1} \cos \theta_{B-1}$... to

$X_{B-N} = I_{B-N} \cos \theta_{B-N}$

$Y_{B-1} = I_{B-1} \sin \theta_{B-1}$... to

$Y_{B-N} = I_{B-N} \sin \theta_{B-N}$

I_{B-1} = line current in branch of conductor B to load #1

I_{B-2} = line current in branch of conductor B to load #2 ... to

I_{B-N} = line current in branch of conductor B to load #N

θ_{B-1} = lead/lag of line current I_{B-1} with respect to line voltage V_{ca}

θ_{B-2} = lead/lag of line current I_{B-2} with respect to line voltage V_{ca} ... to

θ_{B-N} = lead/lag of line current I_{B-N} with respect to line voltage V_{ca}

I_{B-1} = line current in branch of conductor B to load #1

I_{B-2} = line current in branch of conductor B to load #2 ... to

I_{B-N} = line current in branch of conductor B to load #N

θ_{B-1} = lead/lag of line current I_{B-1} with respect to line voltage V_{ab}

θ_{B-2} = lead/lag of line current I_{B-2} with respect to line voltage V_{ab} ... to

θ_{B-N} = lead/lag of line current I_{B-N} with respect to line voltage V_{ab}

$$I_C = \{(X_C)^2 + (Y_C)^2\}^{1/2}, \text{ and } \dots \quad \text{Equation 11-5}$$

$$\theta_{L-C} = \zeta - 240^\circ \dots \quad \text{Equation 11-6}$$

$$\zeta = \sin^{-1} (Y_C \div I_C)$$

$$X_C = \sum (X_{C-1} + X_{C-2} + X_{C-3} \dots + X_{C-N})$$

$$Y_C = \sum (Y_{C-1} + Y_{C-2} + Y_{C-3} \dots + Y_{C-N})$$

I_{C-1} = current in branch of conductor C to load #1

I_{C-2} = current in branch of conductor C to load #2 ... to

I_{C-N} = current in branch of conductor C to load #N

$X_{C-1} = I_{C-1} (1/2) [(\sqrt{3}) \sin \theta_{C-1} - \cos \theta_{C-1}]$

$X_{C-2} = I_{C-2} (1/2) [(\sqrt{3}) \sin \theta_{C-2} - \cos \theta_{C-2}] \dots \text{ to}$

$X_{C-N} = I_{C-N} (1/2) [(\sqrt{3}) \sin \theta_{C-N} - \cos \theta_{C-N}]$

$Y_{C-1} = -I_{C-1} (1/2) [(\sqrt{3}) \cos \theta_{C-1} + \sin \theta_{C-1}]$

$Y_{C-2} = -I_{C-2} (1/2) [(\sqrt{3}) \cos \theta_{C-2} + \sin \theta_{C-2}] \dots \text{ to}$

$Y_{C-N} = -I_{C-N} (1/2) [(\sqrt{3}) \cos \theta_{C-N} + \sin \theta_{C-N}]$

θ_{C-1} = lead/lag of line current I_{C-1} with respect to line voltage V_{bc}

θ_{C-2} = lead/lag of line current I_{C-2} with respect to line voltage V_{bc} ... to

θ_{C-N} = lead/lag of line current I_{C-N} with respect to line voltage V_{bc}

5E. Power Computation and Power Factor

While the subject of this course is primarily related to determining electrical currents, the subject of power is indirectly related to the subject of electrical currents. This is so since current values are often needed to determine power and power factor. The equation for calculating power consumption in a single phase circuit is shown in above Section 2C and the equation for a balanced three phase circuit is shown above in Section 4B. The computations are relatively straightforward since it merely requires insertion of the applicable parameters for line voltage, line current and power factor into the applicable formula.

Power consumption of an unbalanced three phase circuit can be calculated by one of several methods. If the current and power factor of each phase of a delta circuit are known then each phase can be treated as a single phase circuit. The power of each phase can be readily calculated and the net power is the sum of the separately-determined values. Likewise, the power consumption of a wye circuit can be calculated if the current and power factor in each phase is known. Again, the net power of a wye circuit is the sum of the three phase values.

The power delivered by any balanced or unbalanced three phase circuit to downstream users can also be calculated if the current and the respective lead/lag in each of the common line conductors serving those users are known. To do so, merely involves assuming that the downstream users can be represented as a single wye circuit. Power consumption is calculated by applying the formula for single phase power to each of the three phases. This rule is applicable regardless of the type of users downstream, whether the users are a single delta circuit, a single wye circuit or any mix of delta and wye users. If, say, line current in conductor A leads line voltage by θ_{L-A} , then the effective power factor is equal to $\cos(\theta_{L-A} - 30^\circ)$. This method of calculation is treated further in below Example 17.

The lead or lag of current in the three phase service lines is of interest in part because a penalty fee may be charged by the utility for a lagging power factor. Lagging currents are the most common problem since a large percentage of

electrical usage at most users is generally attributed to induction motors which always operate at a lagging power factor.

Example 17

Problem: Calculate the power consumption of the three phase delta circuit of Example 11 from its line properties, assuming a line voltage of 480-3-60:

Solution:

The given phase properties of Example 11 are repeated here:

$$I_{ab} = 5 \text{ amps } @ \text{PF} = 1.0$$

$$I_{bc} = 10 \text{ amps } @ \text{PF} = 0.9 \text{ lagging}$$

$$I_{ca} = 15 \text{ amps } @ \text{PF} = 0.8 \text{ leading}$$

The corresponding phase angles are computed as:

$$\theta_{P-AB} = \cos^{-1} 1.0 = 0$$

$$\theta_{P-BC} = \cos^{-1} 0.9 = -25.84^\circ$$

$$\theta_{P-CA} = -\cos^{-1} 0.8 = +36.86^\circ$$

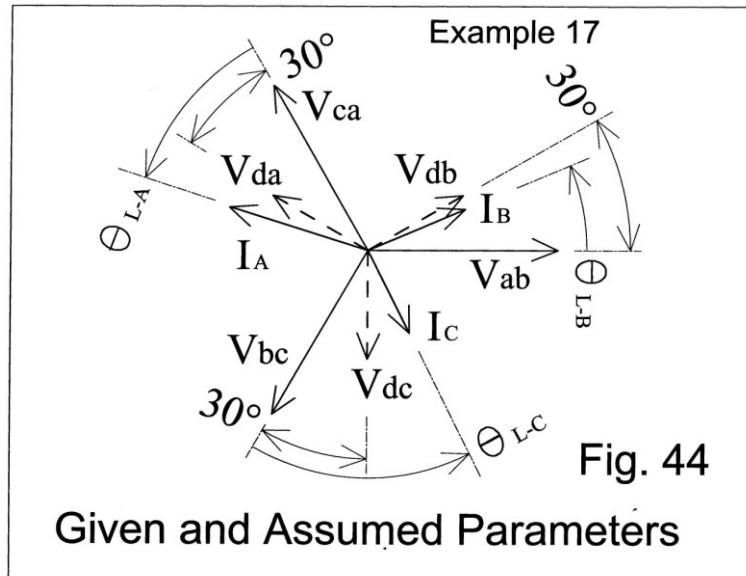
The following line properties were determined in the computations of Example 11:

$$I_A = 19.69624 \text{ amps}; \theta_{L-A} = 42.59291^\circ \text{ (with respect to } V_{ca})$$

$$I_B = 14.41350 \text{ amps}; \theta_{L-B} = 22.926519^\circ \text{ (with respect to } V_{ab})$$

$$I_C = 12.762952 \text{ amps}; \theta_{L-C} = 55.6230^\circ \text{ (with respect to } V_{bc})$$

The involved parameters are shown in Fig. 44. The line voltages are V_{ab} , V_{bc} and V_{ca} . According to the rule explained in Section 5E, the downstream is assumed to be a wye circuit which becomes the basis for the calculation. The phase voltages would be V_{db} , V_{da} and V_{dc} . As explained earlier, the phase voltages for the wye circuit are rotated 30° from the line voltages. The given line currents are I_A , I_B and I_C . The lead/lag of the line currents with respect to line voltages are θ_{L-A} , θ_{L-B} and θ_{L-C} . The line currents are positioned in Fig. 44 as determined in Example 11 and are drawn to-scale. The power for each of the would-be wye circuit are computed using the power factor of the angle between the line currents and the assumed wye phase. The respective power factors are taken as $\cos(\theta_{L-A} - 30^\circ)$, $\cos(\theta_{L-B} - 30^\circ)$ and $\cos(\theta_{L-C} - 30^\circ)$



The calculation of power assuming a wye circuit are as follows.

If the user were a wye circuit, the phase voltage would be:

$$V_{db} = V_{da} = V_{dc} = (480 / (\sqrt{3})) = 277.128 \text{ volts}$$

Power of Phase a-n:

$$P = VI \cos \theta_{P-AN} = (277.128) (19.696) \cos (42.592^\circ - 30^\circ) = 5327.079 \text{ w}$$

Power of Phase b-n:

$$P = VI \cos \theta_{P-BN} = (277.128) (14.413) \cos (22.926^\circ - 30^\circ) = 3963.985 \text{ w}$$

Power of Phase c-n:

$$P = VI \cos \theta_{P-CN} = (277.128) (12.762) \cos (55.6244^\circ - 30^\circ) = 3188.9366 \text{ w}$$

Net power: 12,480.00 watts

(Note: The power computations were conducted using more significant places than what is shown here.)

Check!

For the delta circuit the power of the three phases are:

$$\text{Phase a-b: } P = VI \cos \theta_{P-AB} = (480) (5) (1) = 2,400 \text{ w}$$

$$\text{Phase b-c: } P = VI \cos \theta_{P-BC} = (480) (10) (.9) = 4,320 \text{ w}$$

$$\text{Phase c-a: } P = VI \cos \theta_{P-CA} = (480) (15) (.8) = 5,760 \text{ w}$$

$$\text{Net power: } 2,400 + 4,320 + 5,760 = 12,480.000 \text{ w}$$

Thus, the calculation corroborates the method postulated in Section 5E for computing power in a three phase circuit consisting of any combination of delta or wye loads.

6. Summary of Course Content

The first part of this course includes a review of some applicable elementary principles needed to understand the somewhat more intricate equations and

phasor diagrams used to analyze three phase circuits. Computations pertinent to balanced circuits are reviewed in the first part of the course mostly because many three phase circuits will include distributions to three phase motors which are usually a balanced load. The course proceeds from that point to describe the methods for calculating currents in both unbalanced delta and unbalanced wye circuits.

To a large extent the course assumes that with regard to most three phase circuits, the more frequent need for computations involves the determination of line currents for those loads where the line currents are not given. Once line currents are determined, these values may be combined as described in the course to calculate net currents in, say, a three phase service or a three phase feeder. That data may then be used to meet a number of needs, including the sizing of electrical apparatus.

7. Summary of Symbols and Equations

7A. Symbols

Following is a summary of the symbols used in this course:

Δ = symbol for (three phase) delta loads

v^i = instantaneous voltage (volts)

v_X^i = value of instantaneous voltage v^i at time "X" (volts)

v_{XY}^i = instantaneous value of voltage measured from "X" to "Y" (volts)

v_{PK} = peak value of instantaneous voltage v^i (volts)

$V(t)$ = voltage expressed as a function of time (rms volts)

$V(t)_{ab}$ = voltage A-B expressed as a function of time (rms volts)

V = (absolute) numerical value of DC voltage or (rms) AC voltage (volts)

V_L = line voltage (rms volts - used in reference to a three phase source)

\underline{V}_{XY} = vector V with positive direction X to Y

V_{L-XY} = line voltage (rms volts – measured from X to Y)

V_p = voltage (rms volts - used in reference to a phase of a three phase load)

V_{p-XY} = phase voltage (rms volts – measured from X to Y)

V_{XY} = AC voltage with positive direction measured from "X" to "Y" (rms volts or voltage vector)

i^i = instantaneous current (amps)

i_X^i = value of instantaneous current i^i at time "X" (amps)

i_{PK} = peak value of instantaneous current i^i (amps)

I = (absolute) numerical value of DC current or (rms) AC current (amps)

$I(t)$ = current expressed as a function of time (rms amps)

$I(t)_{ab}$ = current A-B expressed as a function of time (rms amps)

I_L = line current (rms amps) (used in reference to a three phase source)

I_{XY} = vector I with positive direction X to Y

I_{L-X} = line current in conductor X (rms amps)

I_P = phase current (rms amps) (used in reference to a three phase load)

I_{P-XY} = current in Phase X-Y (rms amps)

I_{XY} = AC current with positive direction measured from "X" to "Y" (rms amps or current vector)

L = general representation of an electrical load (which could be resistive, capacitive, inductive or any combination thereof)

R = electrical resistance (ohms)

P = electrical power (watts)

P_{XY} = electrical power in circuit "X-Y" (watts)

t = time (seconds)

t_X = time at "X" (seconds)

f = frequency (hz)

θ_{SP} = single phase lead/lag angle between current and voltage (degrees)

θ_L = general representation of line lead/lag angle between line current and line voltage (degrees or radians)

θ_{L-A} = lead/lag angle of line Current A with respect to line voltage V_{ca}

θ_{L-B} = lead/lag angle of line Current B with respect to line voltage V_{ab}

θ_{L-C} = lead/lag angle of line Current C with respect to line voltage V_{bc}

θ_P = phase lead/lag angle between phase current and phase voltage

(degrees or radians) (for leading current, $\theta_P > 0$; for lagging current, $\theta_P < 0$)

PF = power factor = $\cos \theta_P$ (for balanced delta or balanced wye loads)

$\omega = 2\pi f$ (radians)

7B. Equations

Following is a summary of the equations used in this course:

Equation 1

Instantaneous voltage expressed as a function of time:

$$v^i = (V_{PK}) \sin \omega t \quad \dots \quad \text{Equation 1}$$

Equation 2

Instantaneous current expressed as a function of time:

$$i^i = i_{PK} \sin (\omega t + \theta_{SP}) \quad \dots \quad \text{Equation 2}$$

Equation 3

Single phase RMS voltage expressed as a function of time:

$$V(t) = V \sin (\omega t) \quad \dots \quad \text{Equation 3}$$

Equation 4

$$I(t) = I \sin (\omega t + \theta_{SP}) \quad \dots \quad \text{Equation 4}$$

where,

$I(t)$ = current expressed as a function of time (rms amps)

I = numerical value of current (rms)

θ_{SP} = angle of lead or angle of lag (radians) (current with respect to voltage in a single phase circuit)

for a lagging power factor, $\theta_{SP} < 0$

for a leading power factor, $\theta_{SP} > 0$

Equation 5A

For single phase circuits (Reference Fig. 11) : The magnitude and lead/lag of line current I_a resulting from the addition of line current I_1 at lag/lead angle θ_1 (to line voltage) and line current I_2 at lead/lag angle θ_2 (to line voltage) and ...line current I_n at lead/lag angle θ_n (to line voltage):

$$I_a = \{(X_a)^2 + (Y_a)^2\}^{1/2} \quad \dots \quad \text{Equation 5A}$$

where

$$X_a = I_1 \cos \theta_1 + I_2 \cos \theta_2 + \dots + I_n \cos \theta_n, \text{ and}$$

$$Y_a = [I_1 \sin \theta_1 + I_2 \sin \theta_2 + \dots + I_n \sin \theta_n]$$

$$\theta_a = \sin^{-1}(Y_a \div I_a)$$

Equation 5B

For balanced three phase circuits: The magnitude and lead/lag of line current I_B resulting from the addition of line current I_1 at lag/lead angle θ_1 (to line voltage) and line current I_2 at lead/lag angle θ_2 (to line voltage) and up to ...line current I_n at lead/lag angle θ_n (to line voltage):

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2} \dots \text{Equation 5B}$$

where,

$$X_B = I_1 \cos \theta_1 + I_2 \cos \theta_2 + \dots I_n \cos \theta_n, \text{ and}$$

$$Y_B = [I_1 \sin \theta_1 + I_2 \sin \theta_2 + \dots I_n \sin \theta_n]$$

$$\theta_B = \sin^{-1}(Y_B \div I_B)$$

(Note: For a balanced three phase circuit, the three line currents, I_A , I_B & I_C , are equal in absolute value.)

$$I_B = I_A = I_C$$

Equation 6

Power consumption of a balanced three phase wye or balanced three phase delta load:

$$P = (\sqrt{3}) V_L I_L \cos \theta_P \dots \text{Equation 6}$$

Equation 7

Line current of a balanced three phase delta load:

$$I_L = (\sqrt{3}) I_P \dots \text{Equation 7}$$

Equation 8

Phase voltage in a balanced three phase wye load or an unbalanced three phase load with a grounded neutral:

$$V_L = (\sqrt{3}) V_P \dots \text{Equation 8}$$

Equations 9-1 to 9-6

Equations for calculating line currents when the phase currents and respective leads/lags in an unbalanced delta circuit are known.

With reference to Fig. 14 – where I_{ab} is the current in phase a-b, I_{bc} is the current in phase b-c, I_{ac} is the current in phase a-c, θ_{P-AB} is the lead/lag of current in phase A-B, θ_{P-BC} is the lead/lag of current in phase B-C, θ_{P-CA} is the lead/lag of current in phase C-A, I_A , I_B and I_C are the line currents in, respectively, conductors A, B and C, θ_{L-A} , is the lead/lag of the line current in phase A, θ_{L-B} is the lead/lag of the line current in phase B, and θ_{L-C} is the lead/lag of the line current in phase C.

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2} \quad \dots \text{Equation 9-1}$$

$$\theta_{L-A} = (\lambda - 120^\circ) \quad \dots \text{Equation 9-2}$$

where,

I_A = current in line A

θ_{L-A} = lead (lag) of current I_A with respect to line voltage V_{ca}

$$X_{ba} = -I_{ab} \cos \theta_{P-AB}$$

$$X_{ca} = -I_{ca}(1/2) [(\sqrt{3}) \sin \theta_{P-CA} + \cos \theta_{P-CA}]$$

$$X_A = X_{ba} + X_{ca}$$

$$Y_{ba} = -I_{ab} \sin \theta_{P-AB}$$

$$Y_{ca} = I_{ca}(1/2) [(\sqrt{3}) \cos \theta_{P-CA} - \sin \theta_{P-CA}]$$

$$Y_A = Y_{ba} + Y_{ca}$$

$$\lambda = \sin^{-1} (Y_A \div I_A)$$

Valid range of θ_{P-AB} & θ_{P-CA} : $\pm 90^\circ$; valid range of θ_{L-A} : $+120^\circ$ to -60°

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2} \quad \dots \text{Equation 9-3}$$

$$\theta_{L-B} = \sin^{-1} (Y_B \div I_B) \quad \dots \text{Equation 9-4}$$

where,

I_B = current in line B

θ_{L-B} = lead (lag) of current I_B with respect to line voltage V_{ab}

$$X_{ab} = I_{ab} \cos \theta_{P-AB}$$

$$X_{cb} = -I_{bc}(1/2) [(\sqrt{3}) \sin \theta_{P-BC} - \cos \theta_{P-BC}]$$

$$X_B = X_{ab} + X_{cb}$$

$$Y_{ab} = I_{ab} \sin \theta_{P-AB}$$

$$Y_{cb} = I_{bc}(1/2) [(\sqrt{3}) \cos \theta_{P-BC} + \sin \theta_{P-BC}]$$

$$Y_B = Y_{ab} + Y_{cb}$$

Valid range of θ_{P-AB} & θ_{P-CB} : $\pm 90^\circ$; valid range of θ_{L-B} : $+120^\circ$ to -60°

$$I_C = \{(X_C)^2 + (Y_C)^2\}^{1/2} \dots \text{Equation 9-5}$$

$$\theta_{L-C} = (\varphi - 240^\circ) \dots \text{Equation 9-6}$$

where,

I_C = current in line C

θ_{L-C} = lead (lag) of current I_C with respect to line voltage V_{bc}

$$X_{bc} = I_{bc} (1/2) [(\sqrt{3}) \sin \theta_{P-BC} - \cos \theta_{P-BC}]$$

$$X_{ac} = I_{ca} (1/2) [(\sqrt{3}) \sin \theta_{P-CA} + \cos \theta_{P-CA}]$$

$$X_C = X_{bc} + X_{ac}$$

$$Y_{bc} = -I_{bc} (1/2) [(\sqrt{3}) \cos \theta_{P-BC} + \sin \theta_{P-BC}]$$

$$Y_{ac} = -I_{ca} (1/2) [(\sqrt{3}) \cos \theta_{P-CA} - \sin \theta_{P-CA}]$$

$$Y_C = Y_{bc} + Y_{ac}$$

$$\varphi = \sin^{-1} (Y_C \div I_C)$$

Valid range of θ_{P-BC} & θ_{P-AC} : $\pm 90^\circ$; valid range of θ_{L-C} : $+120^\circ$ to -60°

Equations 10-1 to 10-6

Equations for determining line currents in an unbalanced delta circuit when the loads on all three phases are resistive.

With reference to Fig. 14 – where I_{ab} is the current in phase a-b, I_{bc} is the current in phase b-c, I_{ac} is the current in phase a-c, θ_{P-AB} is the lead/lag of current in phase A-B, θ_{P-BC} is the lead/lag of current in phase B-C, θ_{P-CA} is the lead/lag of current in phase C-A, I_A , I_B and I_C are the line currents in, respectively, conductors A, B and C, θ_{L-A} is the lead/lag of the line current in phase A, θ_{L-B} is the lead/lag of the line current in phase B, and θ_{L-C} is the lead/lag of the line current in phase C.

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2} \dots \text{Equation 10-1}$$

$$\theta_{L-A} = (\lambda - 120^\circ) \dots \text{Equation 10-2}$$

where,

I_A = current in line A

θ_{L-A} = lead (lag) of current I_A with respect to line voltage V_{ca}

$$X_A = -I_{ab} - (1/2) I_{ca}$$

$$Y_A = I_{ca} (\sqrt{3}/2)$$

$$\lambda = \sin^{-1} (Y_A / I_A)$$

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2} \dots \text{Equation 10-3}$$

$$\theta_{L-B} = \sin^{-1} (Y_B / I_B) \dots \text{Equation 10-4}$$

where,

I_B = current in line B

θ_{L-B} = lead (lag) of current I_B with respect to line voltage V_{ab}

$X_B = I_{ab} + (1/2) I_{bc}$

$Y_B = I_{bc} (\sqrt{3}/2)$

$$I_C = \{(X_C)^2 + (Y_C)^2\}^{1/2} \dots \text{Equation 10-5}$$

$$\theta_{L-C} = (\varphi - 240^\circ) \dots \text{Equation 10-6}$$

where,

I_C = current in line C

θ_{L-C} = lead (lag) of current I_C with respect to line voltage V_{bc}

$X_C = (1/2) I_{ca} - (1/2) I_{bc}$

$Y_C = -I_{bc} (\sqrt{3}/2) - I_{ca} (\sqrt{3}/2)$

$\varphi = \sin^{-1} (Y_C / I_C)$

Equations 11-1 to 11-6

Equations for determining the currents in a feeder that delivers power to two or more three phase loads. Reference is made to Fig. 43. Conductor A-1 is the conductor connecting common conductor A to phase A of load #1, A-2 is the conductor connecting conductor A to phase A of load #2, etc. I_A , I_B and I_C are the line currents in, respectively, feeder conductors A, B and C, θ_{L-A} , is the lead/lag of the line current in phase A, θ_{L-B} is the lead/lag of the line current in phase B, and θ_{L-C} is the lead/lag of the line current in phase C.

$$I_A = \{(X_A)^2 + (Y_A)^2\}^{1/2} \dots \text{Equation 11-1, and}$$

$$\theta_{L-A} = (\kappa - 120^\circ) \dots \text{Equation 11-2, where}$$

$$\kappa = \sin^{-1} (Y_A / I_A), \text{ and}$$

$$X_A = \sum (X_{A-1} + X_{A-2} + X_{A-3} \dots + X_{A-N})$$

$$Y_A = \sum (Y_{A-1} + Y_{A-2} + Y_{A-3} \dots + Y_{A-N})$$

$$X_{A-1} = -I_{A-1} (1/2) [(\sqrt{3}) \sin \theta_{A-1} + \cos \theta_{A-1}]$$

$$X_{A-2} = -I_{A-2} (1/2) [(\sqrt{3}) \sin \theta_{A-2} + \cos \theta_{A-2}] \dots \text{ to}$$

$$X_{A-N} = -I_{A-N} (1/2) [(\sqrt{3}) \sin \theta_{A-N} + \cos \theta_{A-N}]$$

$$Y_{A-1} = I_{A-1} (1/2) [(\sqrt{3}) \cos \theta_{A-1} - \sin \theta_{A-1}] \dots \text{ to}$$

$$Y_{A-N} = I_{A-N} (1/2) [(\sqrt{3}) \cos \theta_{A-N} - \sin \theta_{A-N}]$$

I_{A-1} = line current in branch of conductor A to load #1

I_{A-2} = line current in branch of conductor A to load #2 ... to

I_{A-N} = line current in branch of conductor A to load #N

θ_{A-1} = lead/lag of line current I_{A-1} with respect to line voltage V_{ca}

θ_{A-2} = lead/lag of line current I_{A-2} with respect to line voltage V_{ca} ... to

θ_{A-N} = lead/lag of line current I_{A-N} with respect to line voltage V_{ca}

$$I_B = \{(X_B)^2 + (Y_B)^2\}^{1/2}, \text{ and } \dots \text{ Equation 11-3}$$

$$\theta_{L-B} = \sin^{-1} (Y_B \div I_B), \text{ where } \dots \text{ Equation 11-4}$$

$$X_B = \sum (X_{B-1} + X_{B-2} + X_{B-3} \dots + X_{B-N})$$

$$Y_B = \sum (Y_{B-1} + Y_{B-2} + Y_{B-3} \dots + Y_{B-N})$$

$X_{B-1} = I_{B-1} \cos \theta_{B-1}$... to

$X_{B-N} = I_{B-N} \cos \theta_{B-N}$

$Y_{B-1} = I_{B-1} \sin \theta_{B-1}$... to

$Y_{B-N} = I_{B-N} \sin \theta_{B-N}$

I_{B-1} = line current in branch of conductor B to load #1

I_{B-2} = line current in branch of conductor B to load #2 ... to

I_{B-N} = line current in branch of conductor B to load #N

θ_{B-1} = lead/lag of line current I_{B-1} with respect to line voltage V_{ca}

θ_{B-2} = lead/lag of line current I_{B-2} with respect to line voltage V_{ca} ... to

θ_{B-N} = lead/lag of line current I_{B-N} with respect to line voltage V_{ca}

I_{B-1} = line current in branch of conductor B to load #1

I_{B-2} = line current in branch of conductor B to load #2 ... to

I_{B-N} = line current in branch of conductor B to load #N

θ_{B-1} = lead/lag of line current I_{B-1} with respect to line voltage V_{ab}

θ_{B-2} = lead/lag of line current I_{B-2} with respect to line voltage V_{ab} ... to

θ_{B-N} = lead/lag of line current I_{B-N} with respect to line voltage V_{ab}

$$I_C = \{(X_C)^2 + (Y_C)^2\}^{1/2}, \text{ and } \dots \text{ Equation 11-5}$$

$$\theta_{L-C} = \zeta - 240^\circ \dots \text{ Equation 11-6}$$

$$\zeta = \sin^{-1} (Y_C \div I_C)$$

$$X_C = \sum (X_{C-1} + X_{C-2} + X_{C-3} \dots + X_{C-N})$$

$$Y_C = \sum (Y_{C-1} + Y_{C-2} + Y_{C-3} \dots + Y_{C-N})$$

I_{C-1} = current in branch of conductor C to load #1

I_{C-2} = current in branch of conductor C to load #2 ... to

I_{C-N} = current in branch of conductor C to load #N

$$X_{C-1} = I_{C-1} (1/2) [(\sqrt{3}) \sin \theta_{C-1} - \cos \theta_{C-1}]$$

$$X_{C-2} = I_{C-2} (1/2) [(\sqrt{3}) \sin \theta_{C-2} - \cos \theta_{C-2}] \dots \text{ to}$$

$$X_{C-N} = I_{C-N} (1/2) [(\sqrt{3}) \sin \theta_{C-N} - \cos \theta_{C-N}]$$

$$Y_{C-1} = -I_{C-1} (1/2) [(\sqrt{3}) \cos \theta_{C-1} + \sin \theta_{C-1}]$$

$$Y_{C-2} = -I_{C-2} (1/2) [(\sqrt{3}) \cos \theta_{C-2} + \sin \theta_{C-2}] \dots \text{ to}$$

$$Y_{C-N} = -I_{C-N} (1/2) [(\sqrt{3}) \cos \theta_{C-N} + \sin \theta_{C-N}]$$

θ_{C-1} = lead/lag of line current I_{C-1} with respect to line voltage V_{bc}

θ_{C-2} = lead/lag of line current I_{C-2} with respect to line voltage V_{bc} ... to

θ_{C-N} = lead/lag of line current I_{C-N} with respect to line voltage V_{bc}

Equation 12-1: Equation for the relationship of phase current and line current lead/lag angle in a balanced delta and a balanced or unbalanced wye circuit

$$\theta_P = \theta_L - 30^\circ \dots \text{ Equation 12-1}$$

where

θ_L = line lead/lag angle between line current and line voltage (degrees or radians)(for lagging current, $\theta_L < 0$; for leading current, $\theta_L > 0$)

θ_P = phase lead/lag angle between phase current and phase voltage (degrees or radians) (for lagging current, $\theta_P < 0$; for leading current, $\theta_P > 0$)

Specifically,

For balanced delta circuits:

$$\theta_{P-CA} = \theta_{L-A/CA} - 30^\circ$$

(The lead/lag of current in phase C-A with respect to voltage C-A equals the lead/lag of current in line conductor A with respect to voltage C-A less thirty degrees.)

$$\theta_{P-AB} = \theta_{L-B/AB} - 30^\circ$$

(The lead/lag of current in phase A-B with respect to voltage A-B equals the lead/lag of current in line conductor B with respect to voltage A-B less thirty degrees.)

$$\theta_{P-BC} = \theta_{L-C/BC} - 30^\circ$$

(The lead/lag of current in phase B-C with respect to voltage B-C equals the lead/lag of current in line conductor C with respect to voltage B-C less thirty degrees.)

For balanced or unbalanced wye circuits:

$$\theta_{P-A/AD} = \theta_{L-A/CA} - 30^\circ$$

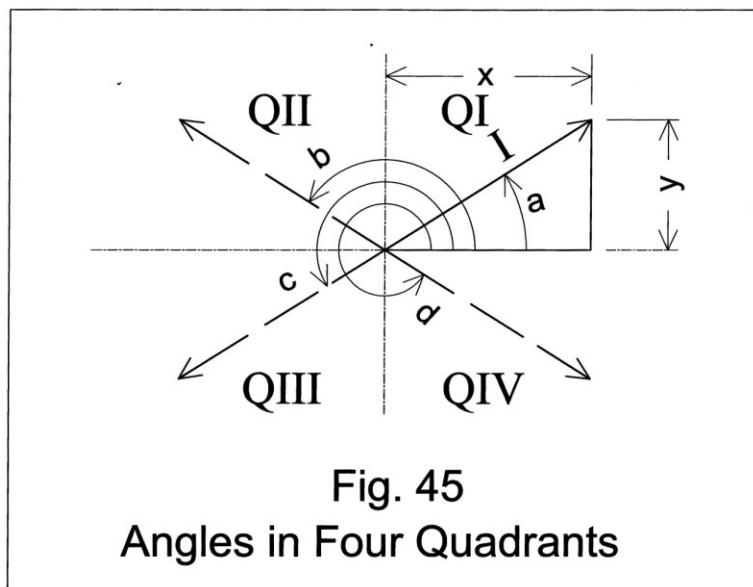
$$\theta_{P-B/BD} = \theta_{L-B/AB} - 30^\circ$$

$$\theta_{P-C/CD} = \theta_{L-C/BC} - 30^\circ$$

Equation 13-1: Guidelines for readily determining the arc sin (\sin^{-1}) of angles λ , φ , K or ζ .

Some of the computations of the course require the calculation of the arc sin (\sin^{-1}) of angles λ , φ , K and ζ in order to determine the displacement angle of current vectors in the CCW direction from the positive abscissa. Unless a person is well experienced and well practiced in calculating the \sin^{-1} of angles it is easy to make errors in the performance of this exercise. Presented here are simple guidelines that may be used as an assist to correctly and simply determine the value of the \sin^{-1} of an angle in any quadrant.

Reference is made to Fig. 45 which depicts a vector "I" in Quadrant I. Vector



I is also the hypotenuse of a triangle with vertical component “y” and horizontal component “x” and vector I is at an angle “a” from the positive abscissa. By definition, the sin of angle a is y/I and the arc sin of y/I ($\sin^{-1} y/I$) is the angle a. While dealing with angles and values pertinent to Quadrant I, little confusion usually ensues. Confusion usually comes about when computing values in Quadrant II, Quadrant III and Quadrant IV. In Quadrant II, the sin of I is the sin of angle “b.” Similarly in Quadrant III the sin of vector I is the sin of angle “c” and in quadrant IV the sin of I is the sin of angle “d.”

If, say, angle b in Quadrant II is 120° and a hand held calculator is used to compute the sin of b, the calculator would correctly indicate the true sin of b to be 0.866. However, if the same calculator is used to compute the \sin^{-1} of 0.866, the calculator indicates the angle to be 60° since the calculator does not “know” that the angle under consideration is in Quadrant II. Therein is the potential for mistaken calculations. Following are some simple rules of thumb that will prove helpful to accurately determine the values of the \sin^{-1} of any angle in any quadrant.

1. Given: The value of \sin^{-1} of vector I which is in Quadrant I, II, III or IV.
2. Determine: The angle of I measured CCW from the positive abscissa (i.e. λ , φ , κ or ζ)
3. Let $M = |\sin^{-1} \lambda|, |\sin^{-1} \varphi|, |\sin^{-1} \kappa|$ or $|\sin^{-1} \zeta|$
4. Compute M by calculator. (The value of M will be between 0° and 90°)
5. From Table 13-1 determine quadrant in which I is located.
6. Calculate value of λ , φ , κ or ζ per appropriate row of Table 13-1

Table 13-1

Value of x	Value of y	Quadrant	Value of λ, φ, κ or ζ
≥ 0	≥ 0	I	M
≤ 0	≥ 0	II	$180^\circ - M$
≤ 0	≤ 0	III	$180^\circ + M$
≥ 0	≥ 0	IV	$360^\circ - M$

8. References

1. Electrical Engineering Fundamentals, 2nd Ed., V. Del Toro, Prentice-Hall, pp 305-314
2. Electrical Fundamentals. 2nd Ed., J.J. DeFrance, Prentice-Hall, pp 431, 604-621, 630-637
3. Electrical Circuit Theory and Technology, 2nd Ed, J Bird, pp 200, 421
4. PDH Course E344, Calculating and Measuring Three Phase Power