PDHOnline Course G492 (4 PDH)

Vector Mechanics: Statics

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Introduction

Mechanics is the branch of science concerned with the behavior of physical bodies when subjected to forces or displacements, and the effects of the bodies on their environment. Mechanics is a physical science incorporating mathematical concepts directly applicable to many fields of engineering such as mechanical, civil, structural and electrical engineering.

Vector analysis is a mathematical tool used in mechanics to explain and predict physical phenomena. The word “vector” comes from the Latin word vectus (or vehere – meaning to carry). A vector is a depiction or symbol showing movement or a force carried from point A to point B.

Statics (or vector mechanics) is the branch of mechanics that is concerned with the analysis of loads (or forces and moments) on physical systems in static equilibrium. Systems that are in static equilibrium are either at rest or the system's center of mass moves at a constant velocity. Problems involving statics use trigonometry to find a solution.

Newton's First Law states that an object at rest tends to stay at rest or an object in motion tends to stay in motion at a constant velocity, unless acted upon by an external force. In the area of statics Newton's First Law dictates that the sum of all forces, or net force, and net moment on every part of the system are both zero.

The term "static" means still or unchanging. In relation to vector mechanics the terms "still" or "unchanging" pertain to the system under evaluation. The system may be at rest or may be moving at a constant velocity, but all of the components of the system are still or in equilibrium with each other. However, there are forces within the system usually acted upon by gravity, but all of the forces are balanced.

Vectors

Note: vectors in this course will be denoted as a boldface letter: **A**.

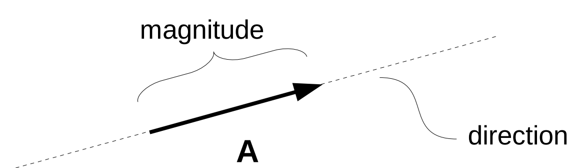


Figure 1 – Illustration of a vector

Vectors play an important role in physics (specifically in kinematics) when discussing velocity and acceleration. A velocity vector contains a scalar (speed) and a given direction. Acceleration, also a vector, is the rate of change of velocity.

Vector Decomposition

A vector can connect two points in space as in Figure 2.

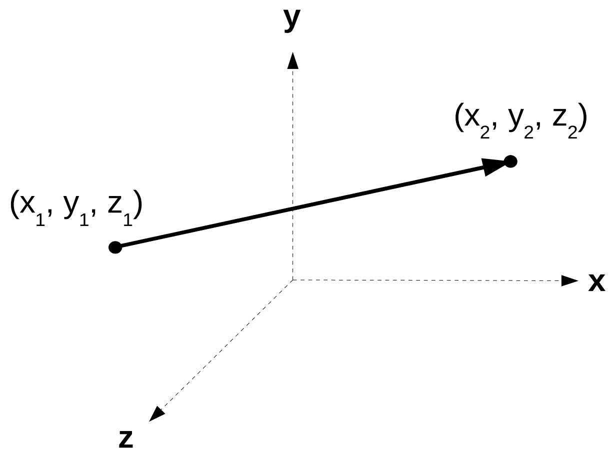


Figure 2 - Vector connecting two points in space

Components of a Vector

In a Cartesian coordinate system the components of a vector are the projections of the vector along the x, y and z axes. Consider the vector **A**. The vector **A** can be broken down into its components along each axis: A_x, A_y and A_z in the following manner:

$$\mathbf{A} = A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\,\mathbf{k}$$

Note that the vectors **i**, **j** and **k** are the unit vectors along each corresponding axis. The unit vectors **i**, **j** and **k** each have a length of one, and the magnitudes along each direction are given by A_x, A_y and A_z.

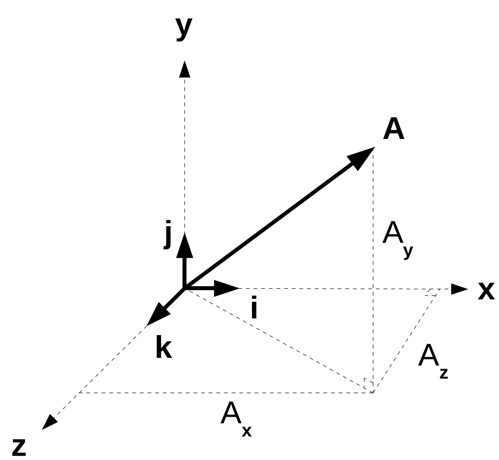


Figure 3 - Vector decomposition showing components along each axis

Trigonometry is utilized to compute the vector components A_x, A_y and A_z. Consider a vector in 2-dimensional space:

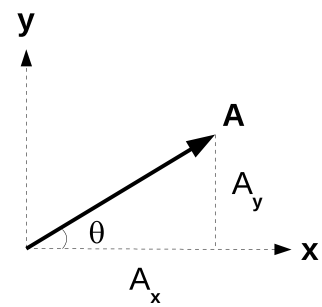


Figure 4 - Vector in 2-dimensional space

The components of this 2-dimensional vector are computed with respect to the angle θ as follows:

$$A_x = A \cos \theta$$

and

$$A_y = A \sin \theta$$

where A is the magnitude of **A** given by

$$A = \sqrt{Ax^2 + Ay^2}$$

Sample Problem:

For example, let

$$A = 5 \text{ and } \theta = 36.8°$$

then

$$A_x = 5\cos(36.8°)$$
$$= 5(0.8)$$
$$= 4$$

and

$$A_y = 5\sin(36.8°)$$
$$= 5(0.6)$$
$$= 3$$

Therefore, the vector (in rectangular form) is

$$\mathbf{A} = 4\mathbf{i} + 3\mathbf{j}$$

As a result of the Pythagorean Theorem from trigonometry the magnitude of a vector may be calculated by

$$A = \sqrt{Ax^2 + Ay^2 + Az^2}$$

For a detailed description of vectors see course G383: Vector Analysis.

Force

A force is either a push or a pull. A force is the action of one body acting on another. A force tends to move a body in the direction of its action. Force is a vector quantity. Its effect depends on the direction as well as the magnitude of the action. Forces are measured in Newtons (N) in the metric system and in pounds (lb) in the English system.

By Newton's Second Law, the acceleration (a) of a body is directly proportional to, and in the same direction as, the net force (F) acting on the body, and inversely proportional to its mass (m):

$$F = ma$$

The acceleration due to gravity is F = mg, in which a = g. The value of g = 9.81 m/s^2 or g = 32.2 ft/s^2 (in English units).

In statics the net result of all forces acting on a system is zero, in other words, the sum of all forces equals zero.

$$\Sigma F = 0$$

To evaluate forces in a system, each force must be decomposed into its individual components. In a Cartesian coordinate system each force will have a component in the x-direction, y-direction and z-direction.

Sample Problem:

For example, consider a force (F = 10 N) in two dimensions at an angle of 30°:

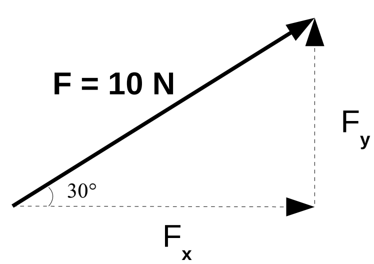


Figure 5 - Force vector components

The components in the x-direction and y-direction are F_x and F_y respectively.

$$F_x = F \cos\theta$$
$$= 10 \cos 30°$$
$$= 8.66 \text{ N}$$

$$F_y = F\sin\theta$$
$$= 10\sin 30°$$
$$= 5\text{ N}$$

or, in vector form

$$\mathbf{F} = 8.66\mathbf{i} + 5\mathbf{j}\ \text{N}$$

Sample Problem:

Now consider a force (F = 1300lb) in two dimensions at an angle of 47°:

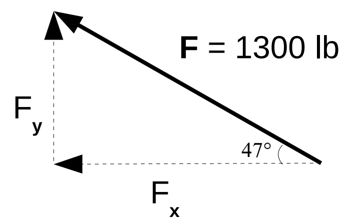


Figure 6 - Force vector components

The components in the x-direction and y-direction are F_x and F_y respectively. Note that the direction of F_x negative, but the magnitude is always positive.

$$F_x = 1300\cos 47°$$
$$= 887\text{ lb}$$
$$F_y = 1300\sin 47°$$
$$= 951\text{ lb}$$

or, in vector form:

$$\mathbf{F} = -877\mathbf{i} + 951\mathbf{j}\ \text{lb}$$

Equilibrium

When all of the forces acting on a body are balanced, then the system is said to be in a state of equilibrium. If an object is in equilibrium then all of the forces and moments are balanced. The sum of all forces equals zero and the sum of all moments equals zero. The conditions for equilibrium are as follows:

$$\Sigma F = 0$$
$$\Sigma M = 0$$

Equilibrium of a Particle

As shown below there are two forces (F_1 and F_2) acting on a particle. The opposing forces are acting in opposite directions and are equal (both 200 N). Therefore, the particle is in a state of equilibrium. It is static, not moving.

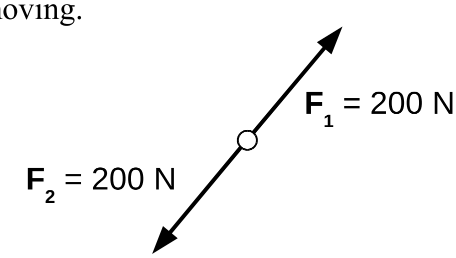


Figure 7 - Equilibrium of a particle

Sample Problem:

As shown below there are several forces (F_1, F_2, F_3 and F_4) acting on a particle. The system is in equilibrium. The forces F_1, F_2 and F_3 are known as well as the angle between F_2 and F_3. F_4 is unknown as well as the angle that it makes to the vertical line. So, in order to determine the force F_4 and its angle, all of the force vectors must sum to zero ($\Sigma F = 0$). Since this problem is in two dimensions (only x and y), this concept can be broken down into equations: $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Also, in this problem the magnitude of a vector will be calculated using the Pythagorean theorem: $F = \sqrt{Fx^2 + Fy^2}$.

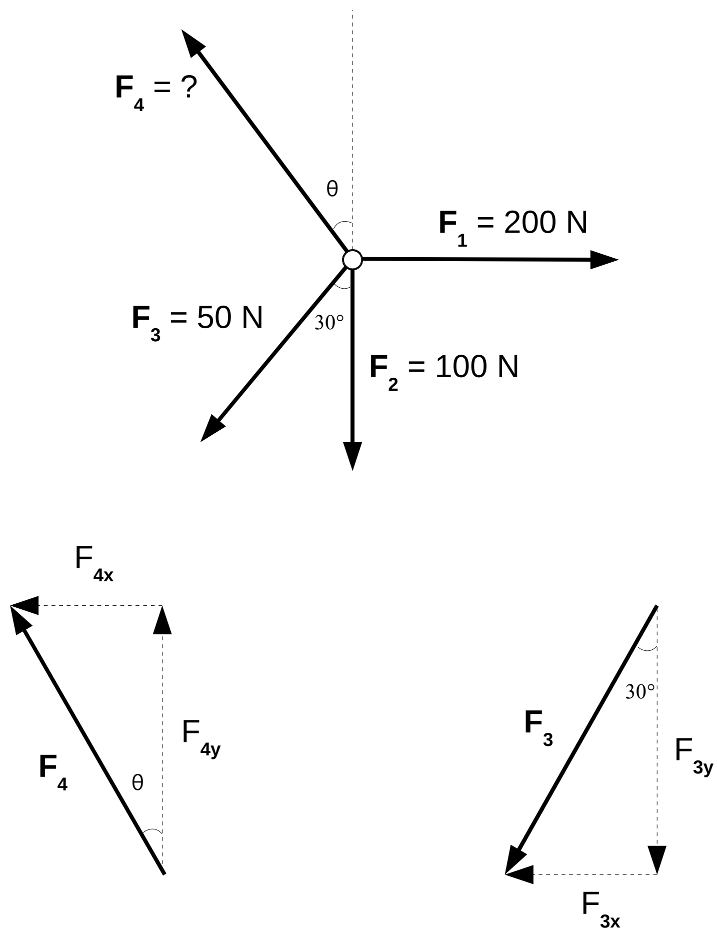


Figure 8 - Equilibrium of a particle

$$\Sigma F_x = 0$$

$$200 - F_{3x} - F_{4x} = 0$$

$$200 - F_3 \sin 30° - F_{4x} = 0$$

$$F_{4x} = 200 - 50 \sin 30°$$

$$F_{4x} = 175 \text{ N}$$

$$\Sigma F_y = 0$$

$$F_{4y} - 100 - F_3 \cos 30° = 0$$

$$F_{4y} = 100 + 50 \cos 30°$$

$$F_{4y} = 143 \text{ N}$$

$$F_4 = \sqrt{F4x^2 + F4y^2}$$
$$= \sqrt{175^2 + 143^2}$$
$$= 226 \text{ N}$$

$$\tan\theta = F_{4x} / F_{4y}$$
$$\theta = \tan^{-1}(F_{4x} / F_{4y})$$
$$= \tan^{-1}(175 / 143)$$
$$= 50.7°$$

All of the vectors in this example can also be expressed in vector form:

$$F_1 = 200\mathbf{i} \text{ N}$$
$$F_2 = -100\mathbf{j} \text{ N}$$
$$F_3 = -50 \sin30°\mathbf{i} - 50 \cos30°\mathbf{j} \text{ N}$$
$$= -25\mathbf{i} - 43.3\mathbf{j} \text{ N}$$
$$F_4 = -175\mathbf{i} + 143\mathbf{j} \text{ N}$$

In order for the system to be in equilibrium, F_4 = 226 N at an angle of θ = 50.7° to the vertical. All of the force vectors must sum to zero ($\Sigma F = 0$), $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

Sample Problem:

Now consider a similar problem with three forces acting on a particle. The system is also in equilibrium. Again all of the force vectors must sum to zero ($\Sigma F = 0$). This problem is also in two dimensions, so the following two equations will be used: $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

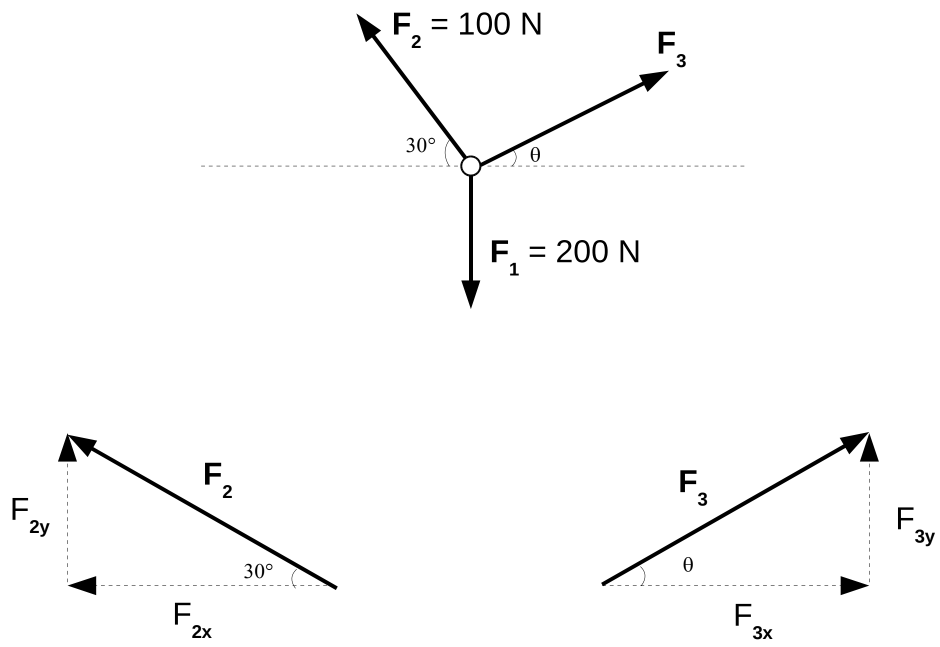


Figure 9 – Equilibrium of a Particle

$$\Sigma F_x = 0$$
$$F_{3x} - F_{2x} = 0$$
$$F_3 \cos\theta - F_2 \cos 30° = 0$$
$$F_3 \cos\theta - 100 \cos 30° = 0$$
$$F_3 \cos\theta = 100 \cos 30°$$
$$F_3 = 100 \cos 30° / \cos\theta$$
$$F_3 = 86.6 / \cos\theta$$

$$\Sigma F_y = 0$$
$$F_{3y} + F_{2y} - F_{1y} = 0$$
$$F_3 \sin\theta + 100 \sin 30° - 200 = 0$$
$$F_3 \sin\theta = 200 - 100 \sin 30°$$
$$F_3 = 150 / \sin\theta$$

from $\Sigma F_x = 0$,

$$F_3 = 86.6 / \cos\theta$$

so,

$$86.6 / \cos\theta = 150 / \sin\theta$$

$$\sin\theta / \cos\theta = 150 / 86.6$$

$$\tan\theta = 150 / 86.6$$

$$\theta = 60.0°$$

and

$$F_3 = 86.6 / \cos 60°$$

$$F_3 = 173 \text{ N}$$

or, in vector form

$$F_1 = -200\mathbf{j} \text{ N}$$

$$F_2 = -100 \cos 30°\mathbf{i} + 100 \sin 30°\mathbf{j}$$

$$= -86.6\mathbf{i} + 50\mathbf{j} \text{ N}$$

$$F_3 = 173 \cos 60°\mathbf{i} + 173 \sin 60°\mathbf{j}$$

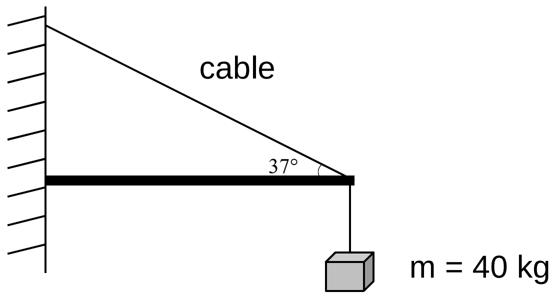
$$= 86.5\mathbf{i} + 150\mathbf{j} \text{ N}$$

In order for the system to be in equilibrium, $F_3 = 173$ N at an angle of $\theta = 60°$ to the horizontal.

Rigid Bodies

Sample Problem:

Consider a body supported by a diagonal cable (at an angle of 37°) and a horizontal rigid bar. The body weighs 40 kg. The force (due to gravity) acting on the body is F = ma. In this case a = g = 9.81m/s^2 (due to the acceleration of gravity). So F = ma = 40(9.81) = 392 N. Since the force is straight down and the system is in equilibrium, the upward force is also 392 N. Let's find the tension in the cable (F) and the resultant force acting on the rigid bar (F_x).



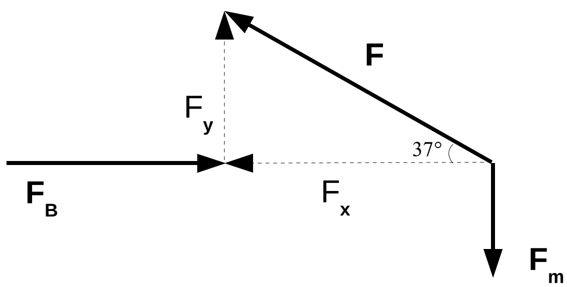


Figure 10 - Body supported by a cable

$$F_y = ma = mg$$
$$= 40(9.81) = 392 \text{ N}$$
$$F_y = F \sin \theta$$

or

$$F = F_y / \sin \theta$$
$$= 392 / \sin 37°$$
$$F = 652 \text{ N (tension in the cable)}$$

$$F_x = F \cos \theta$$
$$= 652 \cos 37°$$
$$F_x = 521 \text{ N}$$

or, in vector form:

$$\mathbf{F} = -521\mathbf{i} + 392\mathbf{j} \text{ N}$$

And, the resultant force acting on the rigid bar:

$$\mathbf{F_B} = 521\mathbf{i} \text{ N}$$

Sample Problem:

Determine the horizontal force (F) that must be applied to the weight (having a mass of 75 lbm) to maintain an angle of 20° with the vertical as shown in Figure 11.

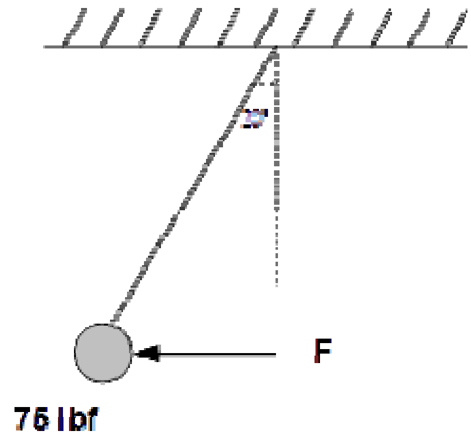
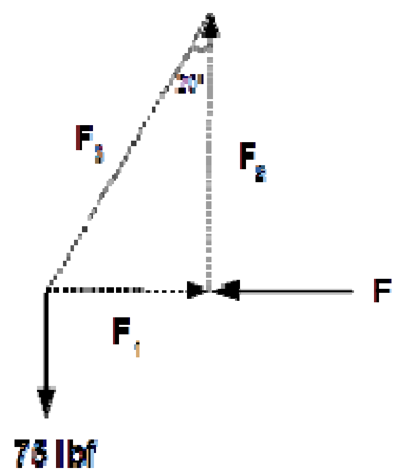


Figure 11 - Weight supported by a cable

First draw the free body diagram:



$$F = ma / g_c$$

where $g_c = 32.174\ (\text{lbm} * \text{ft}) / (\text{lbf} * \text{s}^2)$

$$F_2 = F_3 \cos 20° = 75$$

$$F_1 = F_3 \sin 20° = F$$

$$F_3 = 75 / \cos 20°$$

$$F = (75 / \cos 20°)(\sin 20°)$$

$$F = 27.3 \text{ lbf}$$

Sample Problem:

Consider a weight supported by a cable. Determine the force in cable AB.

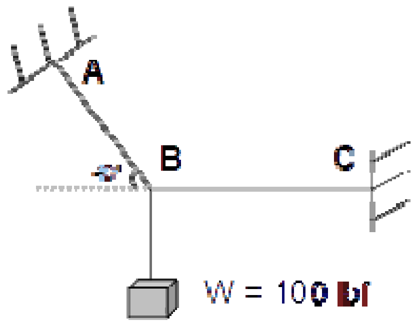
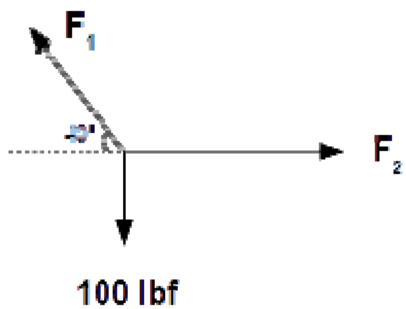


Figure 12 - Weight supported by a cable

First draw the free body diagram.



Next, write the equations for equilibrium.

$$F_1 \cos 40° = F_2$$

$$F_1 \sin 40° = 100$$

$$F_1 = 156 \text{ lbf}$$

$$F_2 = 119 \text{ lbf}$$

Sample Problem:

Now consider a body supported by two cables attached at two points. The mass is 150 kg. The cable of the left makes an angle of 50° to the horizontal and the cable on the right makes an angle of 30° to the horizontal. Assume gravity is not acting on the cables, then the resultant force acts in the upward direction. Let's find the tension in both cables, F_1 and F_2.

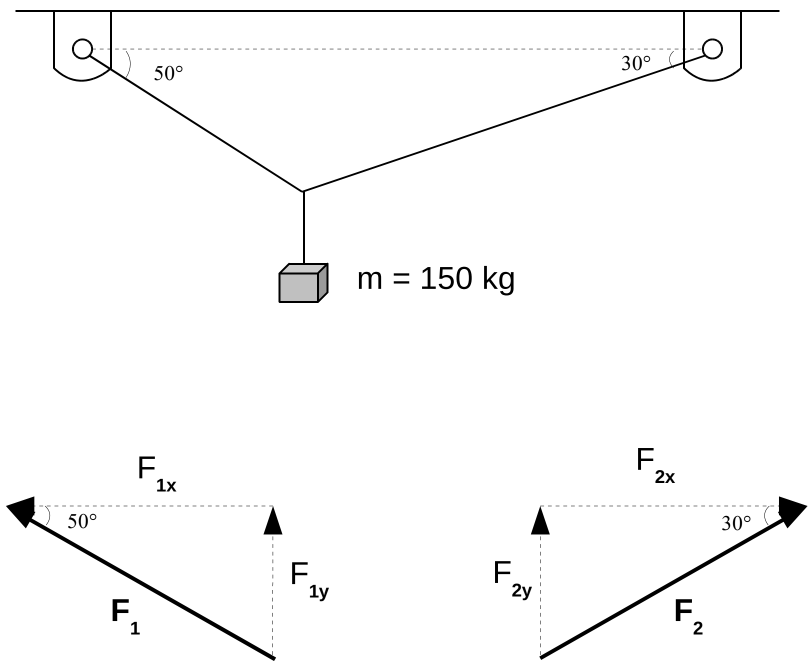


Figure 13 - Body supported by a cable

Here we must also use the concept that the sum of the forces in both the x-direction and the y-direction is zero.

$$\Sigma F_x = 0$$

$$F_{2x} - F_{1x} = 0$$

$$F_{1x} = F_{2x}$$

we know that

$$F_{1x} = F_1 \cos 50°$$

and

$$F_{2x} = F_2 \cos 30°$$

so,

$$F_1 \cos 50° = F_2 \cos 30°$$

therefore,

$$F_1 = F_2 (\cos 30° / \cos 50°)$$

$$\Sigma F_y = 0$$

$$F_{1y} + F_{2y} - mg = 0$$

we know that

$$F_{1y} = F_1 \sin 50°$$

and

$$F_{2y} = F_2 \sin 30°$$

and

$$mg = 150(9.81)$$

so

$$F_1 \sin 50° + F_2 \sin 30° - 150(9.81) = 0$$

$$F_2 \sin 30° = 150(9.81) - F_1 \sin 50°$$

$$F_2 \sin 30° = 150(9.81) - F_2(\cos 30° / \cos 50°) \sin 50°$$

$$F_2 \sin 30° + F_2(\cos 30° / \cos 50°) \sin 50° = 150(9.81)$$

$$F_2[\sin 30° + (\cos 30° / \cos 50°) \sin 50°] = 150(9.81)$$

$$F_2 = 960 \text{ N}$$

and taking F_2 and substituting back into the F_1 equation gives us F_1:

$$F_1 = F_2 (\cos 30° / \cos 50°)$$

$$= 960(\cos 30° / \cos 50°)$$

$$F_1 = 1290 \text{ N}$$

Pulleys

Pulleys are simple machines that reduce the amount of force required to lift an object. In statics pulleys are modeled as being frictionless. Therefore, the tension in the cable on either side of a pulley is equal. The concept of mechanical advantage is used to describe how a pulley or combination of pulleys reduces the amount of force required to lift an object. The mechanical advantage (MA) is simply the force output divided by the force input:

$$MA = (\text{output force}) / (\text{input force})$$

A mechanical advantage of one (MA = 1) means there is no advantage: the input force equals the output force (no help). A mechanical advantage of two (MA = 2) means that is takes half the force to lift an object. Mechanical advantage is equal to the number of ropes coming to and going from the load-carrying pulley. The diameters of the pulleys are not a factor in calculating the mechanical advantage.

Figure 14 shows a pulley with no mechanical advantage, MA = 1. The amount of rope pulled is the same length as the height that the weight is lifted.

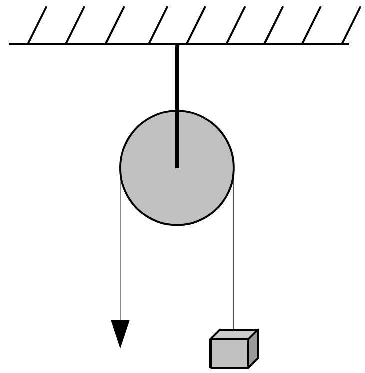


Figure 14 - Single pulley, MA = 1

$$T = mg$$

Figure 15 shows a pulley with a mechanical advantage of MA = 2. The amount of rope pulled is equal to twice the height that the weight is lifted.

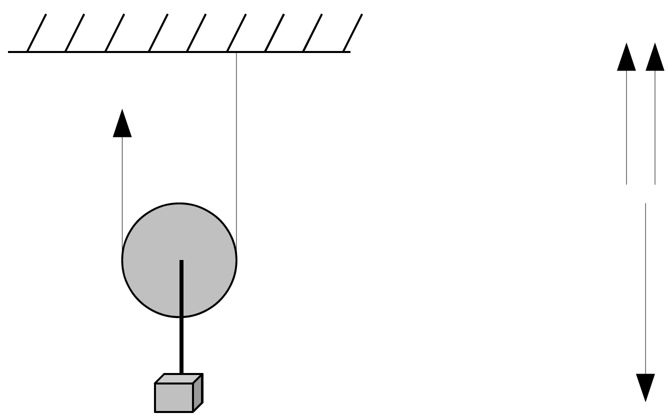


Figure 15 - Single pulley, MA = 2

$$T + T = mg$$

$$2T = mg$$

$$T = 0.5mg$$

Figure 16 shows a pulley system with a mechanical advantage of MA = 2. The amount of rope pulled is equal to twice the height that the weight is lifted. So with a mechanical advantage of MA = 2, the rope will have to be pulled 10 ft to raise the load by 5 ft.

All of the forces sum to zero. For equilibrium, all of the upward forces equal the downward force of the weight.

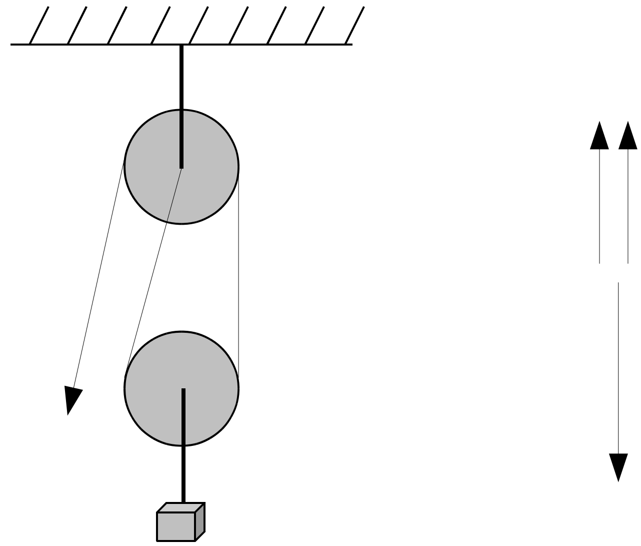


Figure 16 - Two pulleys, MA = 2

$$T + T = mg$$

$$2T = mg$$

$$T = 0.5mg$$

Figure 17 shows a pulley system with a mechanical advantage of MA = 3. There are 3 ropes either coming to or going from the load-carrying pulley.

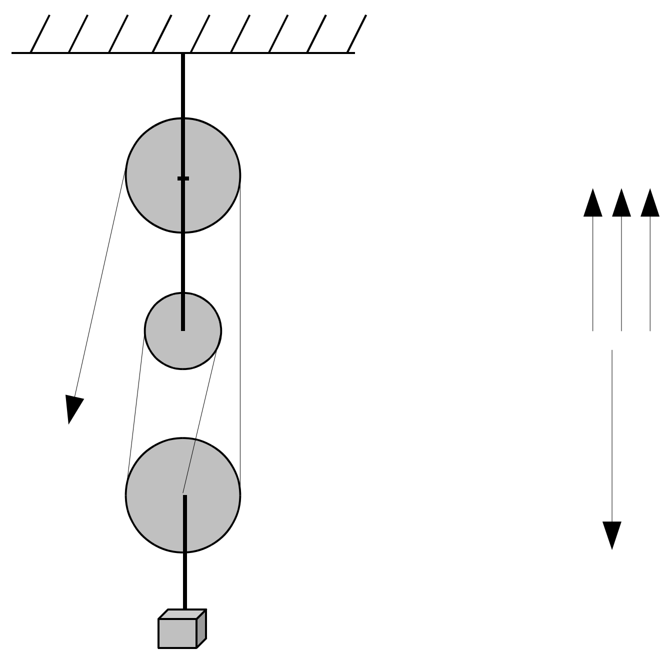


Figure 17 - Three pulleys, MA = 3

$$T + T + T = mg$$
$$3T = mg$$
$$T = 0.333mg$$

Figure 18 shows a pulley system with a mechanical advantage of MA = 4. There are 4 ropes either coming to or going from the load-carrying pulley.

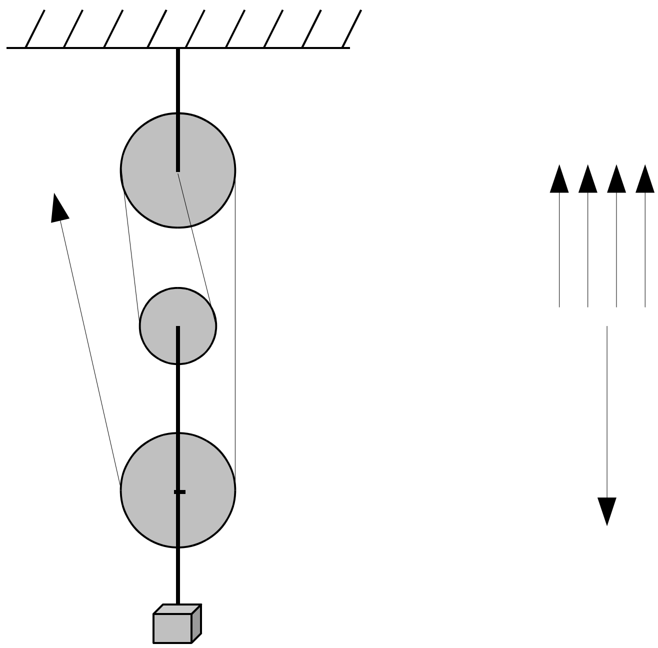


Figure 18 - Three pulleys, MA = 4

$$T + T + T + T = mg$$
$$4T = mg$$
$$T = 0.25mg$$

Moments

As well as displacing an object, a force can also tend to rotate a body about an axis. This rotational tendency is known as the moment (M) of the force. Moment is also referred to as torque.

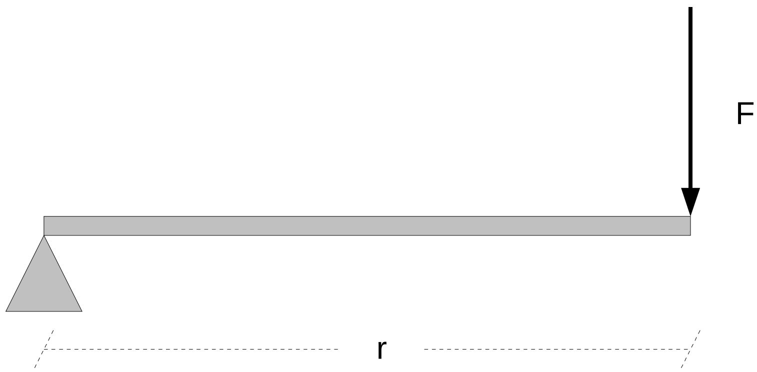


Figure 19 – Moment

The magnitude is simply the force times distance. The distance (r) is also called the moment arm. The moment **M** is a vector which means it has a magnitude and a direction. Its direction is given by the right hand rule. In Figure 19 the moment is directed perpendicular to the line of force into the page. A clockwise rotation gives a direction into the page, and a counterclockwise rotation gives a direction out of the page.

Moment of a Force About a Point

The moment, vector, **M**, for a force about a point, O, is the cross product of the force, **F**, and the vector from the point O to the point of application of the force given by the position vector, **r**:

In vector format, the moment is given by a vector cross product:

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Recall that the cross product or vector product of two vectors (**r** and **F**) may be expressed as the following determinant:

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

$$\mathbf{r} \times \mathbf{F} = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The scalar product $|\mathbf{r}| \sin \varphi$ is known as the moment arm, d.

Sample Problem:

A boom AB is fixed at point A. A cable is attached to point B on the boom and to a wall at point C. The boom is 5m in length and the tension in the cable is known to be 1500 N. Determine the moment about point A of the force exerted by the cable at point B.

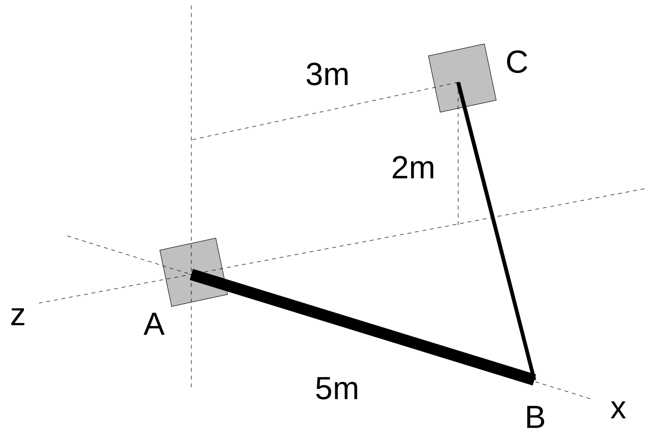


Figure 20 - Moment about a point

Recall that the moment of a force about a point is given by the vector cross product **M** = **r** × **F** where r is the position vector or moment arm that extends from the point at which the boom is attached to (A) to the point at which the force is applied (B). In this case the position vector is given by

$$\mathbf{r} = 5\mathbf{i} \text{ meters}$$

The force may be computed first by determining the unit vector of the force:

$$\mathbf{u_F} = -5\mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \,/\, \sqrt{5^2 + 2^2 + 3^2}$$

$$= -0.811\mathbf{i} + 0.324\mathbf{j} - 0.487\mathbf{k}$$

The force is therefore given by

$$\mathbf{F} = 1500\ \mathbf{u_F}$$

$$= -1217\mathbf{i} + 486\mathbf{j} - 730\mathbf{k} \text{ Newtons}$$

So, the moment is

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

$$= 3650\mathbf{j} + 2430\mathbf{k} \text{ Nm}$$

Moment of a Force About a Line

Most rotating machines revolve around an axis. Machines tend to turn around a line, not a point. The moment of a force about a line is not the same as the moment of a force about a point. The moment about a line is a scalar. The moment, M_{OL}, of a force, F, about a line OL is the projection of the moment, M_O onto the line.

$$\mathbf{M_{OL}} = \mathbf{a} \cdot \boldsymbol{M_O}$$

$$= \mathbf{a} \cdot \mathbf{r} \times \mathbf{F}$$

where **a** is the unit vector directed along the line.

In statics problems the sum of forces acting on a body equals zero. When there is a rotational component, the sum of all moments acting on the body also equals zero.

Reduction of a System of Forces

A couple is a pair of forces, equal in magnitude, oppositely directed, and displaced by a perpendicular distance. Since the forces are equal, the resultant force is zero, but the displacement of the force-couple causes a moment.

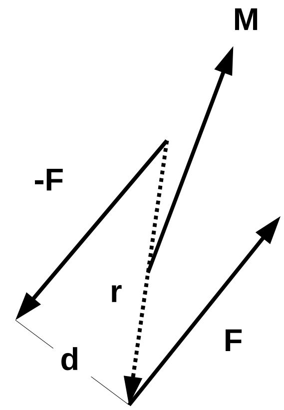


Figure 21 – Force-couple

A force-couple is defined as follows:

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

$$M = (r \sin \theta)\, F$$

$$M = Fd$$

Any system of forces can be reduced to a single force and a single couple.

The equivalent force-couple system is defined as the summation of all forces and the summation of all moments of the couples about point O:

$$\mathbf{R} = \Sigma\mathbf{F}$$

$$\mathbf{M_R} = \Sigma\mathbf{M_O}$$

Sample Problem:

As an example, a 5m beam is subjected to the forces shown. Reduce the system of forces to

(a) an equivalent force-couple system at A
(b) an equivalent force-couple system at B
(c) a single force or resultant

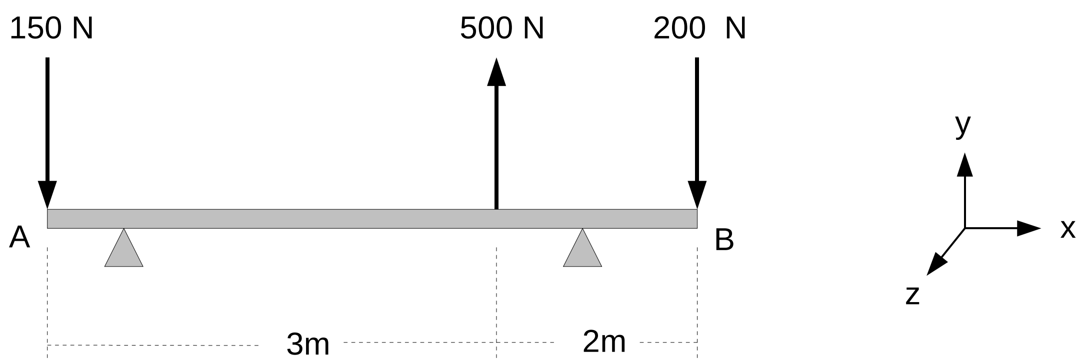


Figure 22 - Force-couple system example

(a)

$$\mathbf{R} = \Sigma\mathbf{F}$$
$$= -150\mathbf{j} + 500\mathbf{j} - 200\mathbf{j}$$
$$= 150\mathbf{j}\text{ N}$$

$$\mathbf{M_A} = 3\mathbf{i} \times 500\mathbf{j} + 5\mathbf{i} \times (-200\mathbf{j})$$
$$= 1500\mathbf{k} + (-1000\mathbf{k})$$
$$= 500\mathbf{k}\text{ Nm}$$

(b)

$$\mathbf{R} = 150\mathbf{j}\text{ N} \quad \text{same as (a)}$$

$$\mathbf{M_B} = -5\mathbf{i} \times (-150\mathbf{j}) + (-2\mathbf{i}) \times (500\mathbf{j})$$
$$= 750\mathbf{k} - 1000\mathbf{k}$$
$$= -250\mathbf{k}\text{ Nm}$$

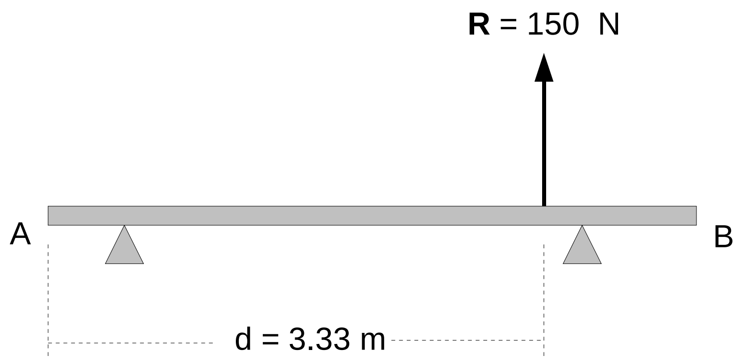
(c) The resultant force is equivalent to the resultant force **R** acting at a point (d) which will be calculated such that Rd is equal to $\mathbf{M_A}$.

$$\mathbf{M_A} = d\mathbf{i} \times \mathbf{R}$$
$$= d\mathbf{i} \times 150\mathbf{j}$$
$$= 500\mathbf{k}\text{ Nm}$$

Therefore,

$$\mathbf{R} = 150\mathbf{j}\text{ N}$$
$$d = 3.33\text{ m}$$

or,



Sample Problem:

As another example, determine the loading of the beam at point A:

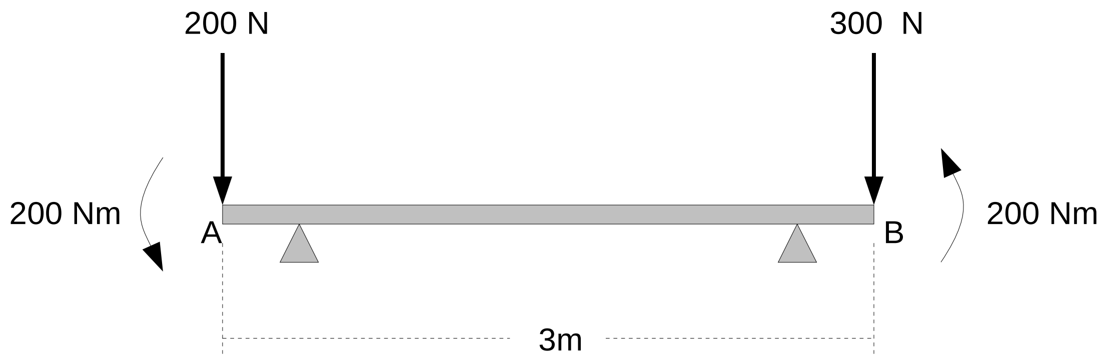


Figure 23 - Force-couple system example

$$\mathbf{R} = \Sigma\mathbf{F}$$
$$= -200\mathbf{j} - 300\mathbf{j}$$
$$= -500\mathbf{j}\text{ N}$$

$$\mathbf{M_A} = 3\mathbf{i} \times (-300\mathbf{j}) + 200\mathbf{k} + 200\mathbf{k}$$
$$= -900\mathbf{k} + 200\mathbf{k} + 200\mathbf{k}$$
$$= -500\mathbf{k}\text{ Nm}$$

Trusses

A truss is one of the major types of engineering structures. It provides a practical and economical solution to many engineering and construction challenges, especially in the design of bridges and buildings.

A truss is a structure comprising triangular units constructed with straight beams called members whose ends are connected at points called joints. External forces and reactions are considered to act only at the nodes and result in internal forces which are either tensile (pulled apart) or compressive forces. A joint in compression will be shown with the force arrows pointing toward the joint. A joint in tension will be shown with the force arrows pointing away from the joint. The weights of the members of the truss are assumed to be applied to the joints, half of the weight of each member being applied to each of the two joints the member connects. A truss is analyzed as a two-dimensional structure. Truss loads are considered to act only in the plane of a truss.

The following three basic concepts apply to all trusses:

- all joints are pinned
- all members are two-force members
- all loading is applied at the joints

A structural cell consists of all members in a closed loop of members. For the truss to be stable (that is rigid), all of the structural cells must be triangles. A truss will be statically determinate if the following equation holds true:

$$m = 2n - 3$$

where m = number of members, n = number of joints

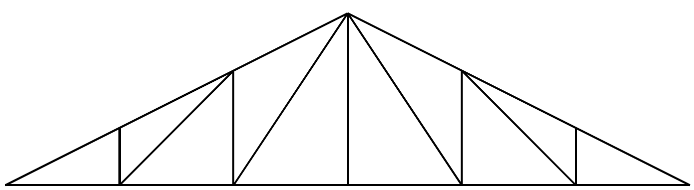
If following is true, then there are redundant members in the truss:

$$m > 2n - 3$$

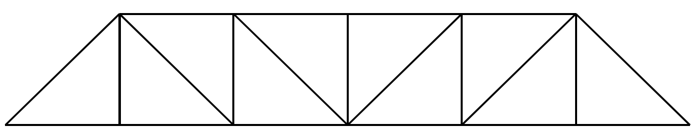
If the following is true, then the truss in unstable and will collapse under certain loading conditions:

$$m < 2n - 3$$

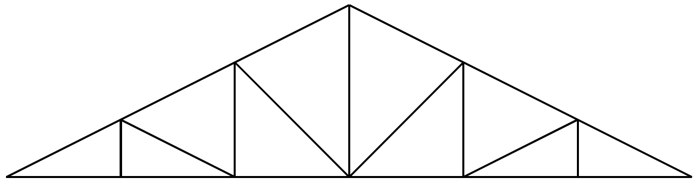
Figure 24 shows some typical trusses.



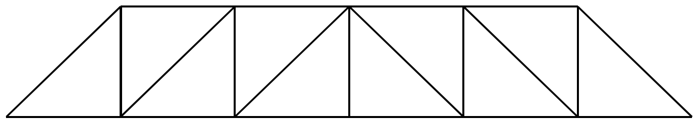
Pratt Roof Truss



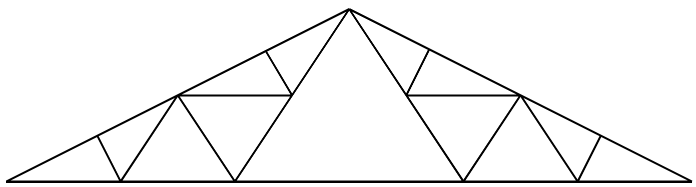
Pratt Bridge Truss



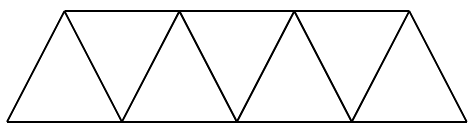
Howe Roof Truss



Howe Bridge Truss



Fink Roof Truss



Fink Bridge Truss

Figure 24 - Typical Trusses

There are three methods that can be used to determine the internal forces in each truss members: the method of sections, the method of joints, and the cut-and-sum method. The first two methods will be discussed here.

Method of Joints

The method of joints is one method which can be used to find the internal forces in each truss member. This method is useful when most or all of the truss member forces are to be calculated; it is inconvenient when a single isolated member force is to be calculated.

If a truss is in equilibrium then each of its joints must be in equilibrium. Therefore, with the method joints the following equilibrium conditions must be satisfied at each joint:

$$\Sigma \mathbf{M} = 0 \qquad \Sigma \mathbf{F_x} = 0 \qquad \Sigma \mathbf{F_y} = 0$$

Sample Problem:

The following example calculates the axial forces in each member.

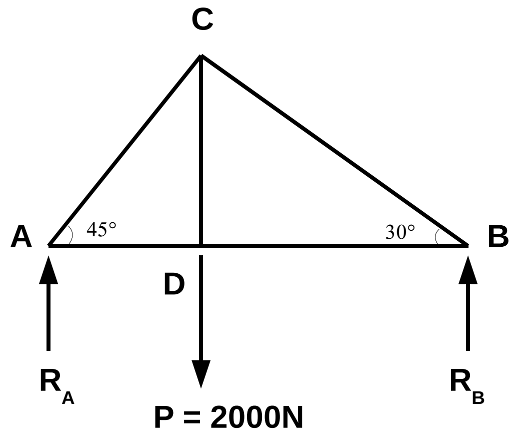


Figure 25 - Method of Joints Example

First consider the moments.

$$\Sigma M_A = 0$$

$$-2000r_{AD} + R_B(r_{AD} + r_{BD}) = 0$$

This equation introduces two extra unknowns since the distances between AD and BD are unknown. So, this equation will not be used for this problem.

At each joint, select the direction of the member forces then show the reaction forces on each node (or joint). If the wrong direction is chosen for any given force, the decision is still valid, but the force will be negative.

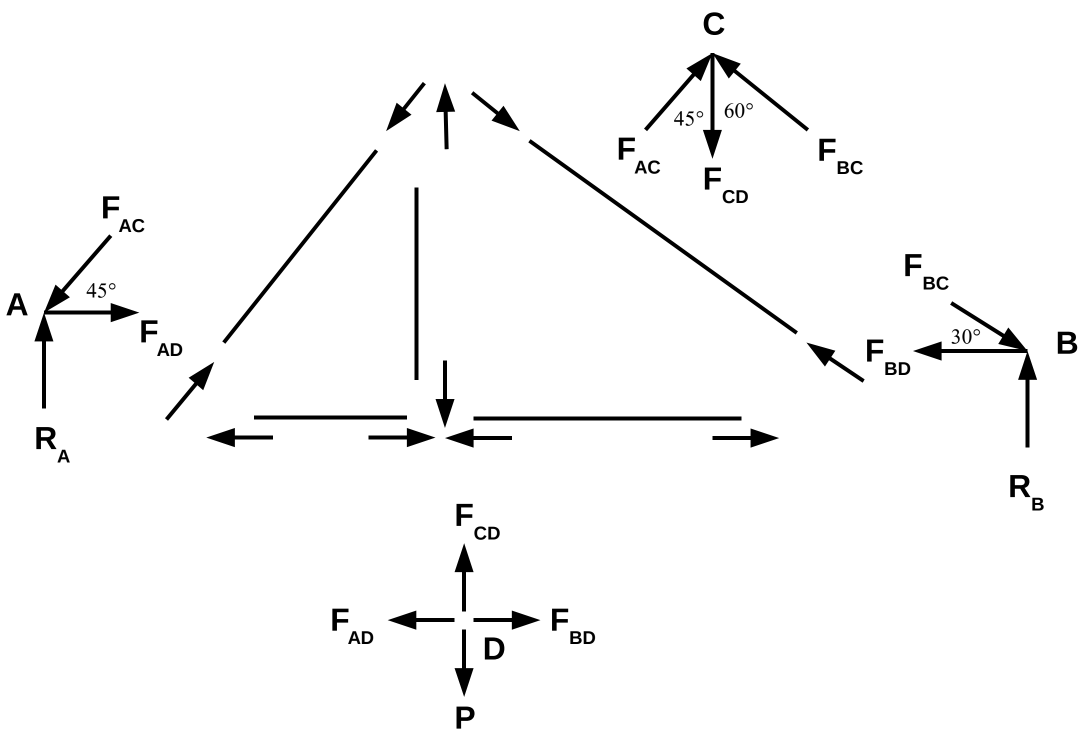
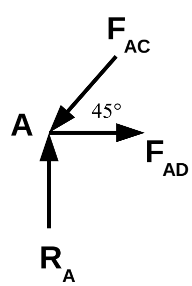


Figure 26 - Trusses - method of joints

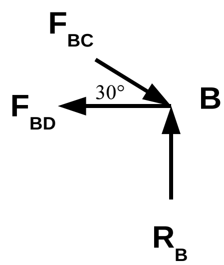
Write the equations of equilibrium for each joint.

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$



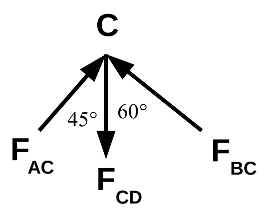
$$F_{AD} - F_{AC}\cos 45° = 0$$

$$R_A - F_{AC}\sin 45° = 0$$



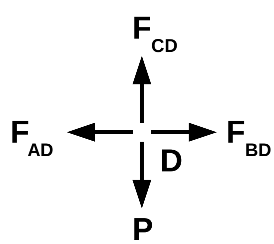
$$F_{BC}\cos 30° - F_{BD} = 0$$

$$R_B - F_{BC}\sin 30° = 0$$



$$F_{AC}\sin 45° - F_{BC}\sin 60° = 0$$

$$-F_{CD} + F_{AC}\cos 45° + F_{BC}\cos 60° = 0$$



$$F_{BD} - F_{AD} = 0$$

$$F_{BD} = F_{AD}$$

$$F_{CD} - P = 0$$

$$F_{CD} = P$$

Here, there are 8 equations and 7 unknowns.

$$F_{CD} = P;\ \ P = 2000$$

$$F_{CD} = P$$

$$F_{CD} = 2000\text{ N}$$

$$F_{AC} = F_{BC} \sin 60° / \sin 45°$$

$$-F_{CD} + F_{AC} \cos 45° + F_{BC} \cos 60° = 0$$

$$-2000 + F_{BC} (\sin 60° / \sin 45°)(\cos 45°) + F_{BC} \cos 60° = 0$$

$$F_{BC}[(\sin 60° / \sin 45°)\cos 45° + \cos 60°] = 2000$$

$$F_{BC} = 1464 \text{ N}$$

$$F_{AC} = F_{BC} \sin 60° / \sin 45°$$

$$F_{AC} = 1793 \text{ N}$$

$$F_{BC} \cos 30° - F_{BD} = 0$$

$$F_{BD} = F_{BC} \cos 30°$$

$$F_{BD} = 1268 \text{ N}$$

$$R_B - F_{BC} \sin 30° = 0$$

$$R_B = F_{BC} \sin 30°$$

$$R_B = 732 \text{ N}$$

$$F_{AD} - F_{AC} \cos 45° = 0$$

$$F_{AD} = F_{AC} \cos 45°$$

$$F_{AD} = 1268 \text{ N}$$

$$R_A - F_{AC} \sin 45° = 0$$

$$R_A = F_{AC} \sin 45°$$

$$R_A = 1268 \text{ N}$$

We already have a solution for all of the force members, but for verification, we can check the moment equations, $\Sigma M_A = 0$ and $\Sigma M_B = 0$. The values r_{AD} and r_{BD} are the distances between AD and BD respectively.

$$\Sigma M_A = 0$$

$$-2000r_{AD} + R_B(r_{AD} + r_{BD}) = 0$$

$$r_{AD}(732 - 2000) + 732\, r_{BD} = 0$$

$$r_{AD} = 0.577r_{BD}$$

$$\Sigma M_B = 0$$

$$-R_A(r_{AD} + r_{BD}) + 2000r_{BD} = 0$$

$$(-1268 + 2000)r_{BD} - 1268r_{AD} = 0$$

$$r_{AD} = 0.577r_{BD}$$

Both of these equations are equivalent, so the solution is correct.

Method of Sections

The method of sections is useful if the forces in only a few members of a truss are to be determined or if only a few truss member forces are unknown. It is a direct approach to finding forces in any truss member.

Just as with the method of joints method, the first step is to find the support reactions. Then decide how the truss should be cut into sections and draw the free-body diagrams. The cut should pass through the unknown member. Then write the equations of equilibrium for each section. When determining how to cut the truss, keep in mind that a cut cannot pass through more than three members in which the forces are unknown since there are three equations of equilibrium:

$$\Sigma \mathbf{M} = 0 \qquad \Sigma \mathbf{F_x} = 0 \qquad \Sigma \mathbf{F_y} = 0$$

Sample Problem:

The following example calculates the axial forces in members CD and DE.

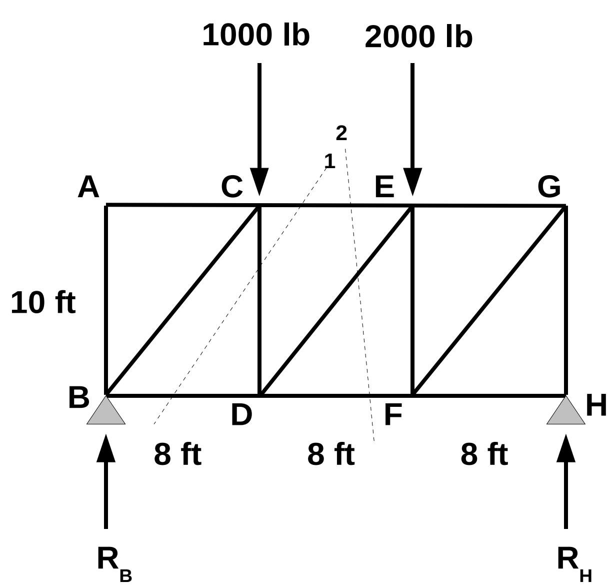


Figure 27 - Trusses - method of sections

First, find the equilibrium of the entire truss:

$\Sigma \mathbf{M_B} = 0$

$$-1000(8) - 2000(16) + R_H(24) = 0$$

$$R_H = 1667 \text{ lb}$$

$\Sigma \mathbf{M_H} = 0$

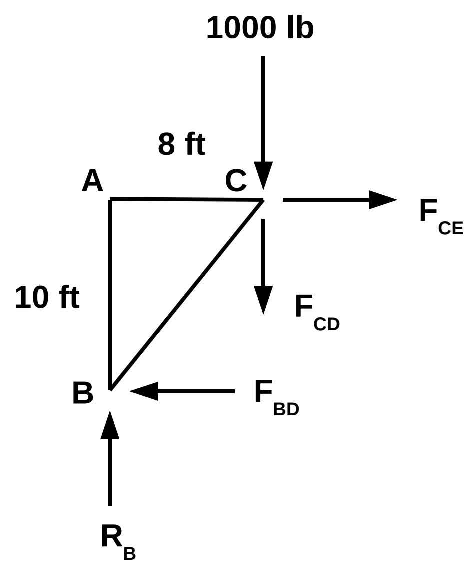
$$1000(16) + 2000(8) - R_B(24) = 0$$

$$R_B = 1333 \text{ lb}$$

$\Sigma \mathbf{F_y} = 0$

$$R_B + R_H - 3000 = 0$$

To find F_{CD} the truss is cut at section 1:



$\Sigma \mathbf{M_B} = 0$

$$-1000(8) - F_{CD}(8) - F_{CE}(10) = 0$$

$\Sigma \mathbf{F_x} = 0$

$$F_{CE} - F_{BD} = 0$$

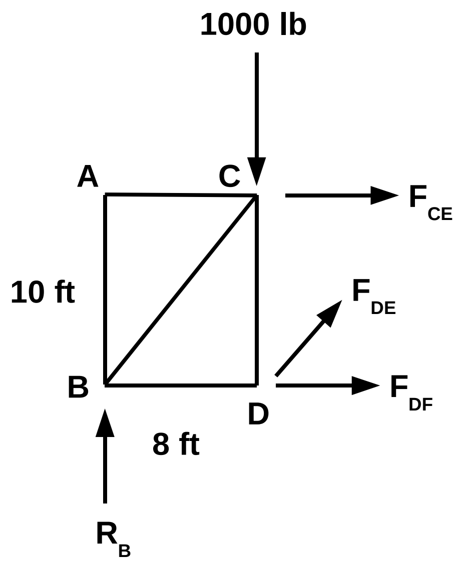
$\Sigma \mathbf{F_y} = 0$

$$R_B - 1000 - F_{CD} = 0$$

$$1333 - 1000 - F_{CD} = 0$$

$$F_{CD} = 333 \text{ lb}$$

To find F_{DE} the truss is cut at section 2:



From the first section:

$$-1000(8) - F_{CD}(8) - F_{CE}(10) = 0$$

$$F_{CE} = -1066 \text{ lb}$$

The force is negative because it is opposite in direction than as shown.

$\Sigma \mathbf{M_B} = 0$

$$-1000(8) - F_{CE}(10) + F_{DE} \sin\theta \,(8) = 0$$

Where

$$\theta = \tan^{-1}(10/8)$$

$$\theta = 51.3°$$

$$-1000(8) - (-1066)(10) + F_{DE} \sin 51.3° \,(8) = 0$$

$$F_{DE} = -426 \text{ lb}$$

The force is negative because it is opposite in direction than as shown.

Friction

Friction is a force that always resists motion and impending motion. It acts in parallel to the surface with which an object is in contact. Frictional force exerted on a stationary body is known as static friction. If the body is moving, the friction is known as dynamic (or kinetic) friction. Dynamic friction for a given body is less than static friction on the same body.

The magnitude of the frictional force depends on the normal force (N) and the coefficient of friction (μ) between the object and the surface that it is touching. For a body resting on a horizontal surface the normal force is the object's weight:

$$N = W = mg$$

A block with weight W is placed on a horizontal surface. The forces acting on the block are its weight W and the reaction force of the surface as in Figure 28.

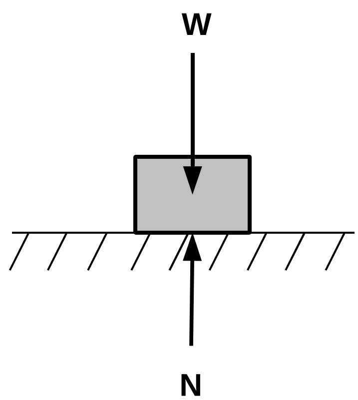


Figure 28 - Weight and normal forces

Since the weight has no horizontal force component, the reaction of the surface also has no horizontal force component. Suppose now that a horizontal force P is applied to the block as in Figure 29.

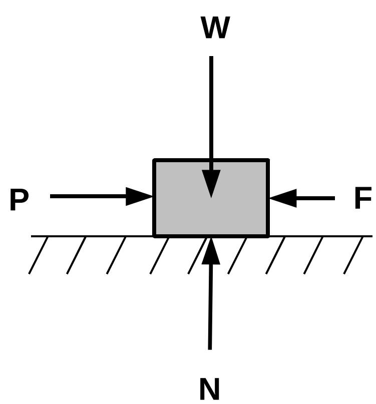


Figure 29 - Weight and horizontal forces and their reactions

If P is small, the block will not move. There is a reactionary force F that pushes in the opposite direction to counteract the force P. The force F depends on the normal force N and the coefficient of static friction:

$$F = \mu_s N$$

If the body is on an inclined surface then the normal force becomes

$$N = W \cos \varphi$$
$$N = mg \cos \varphi$$

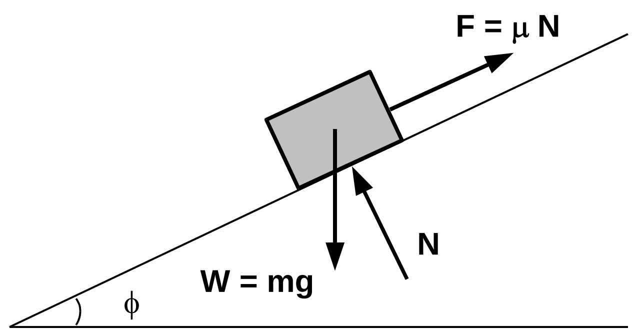


Figure 30 - Friction

The frictional force is given by the following equation:

$$F = \mu N$$
$$= \mu mg \cos \varphi$$

Sample Problem:

Suppose, for example, a 100kg block is on a plane inclined at 30°. The coefficient of static friction is $\mu_s = 0.3$.

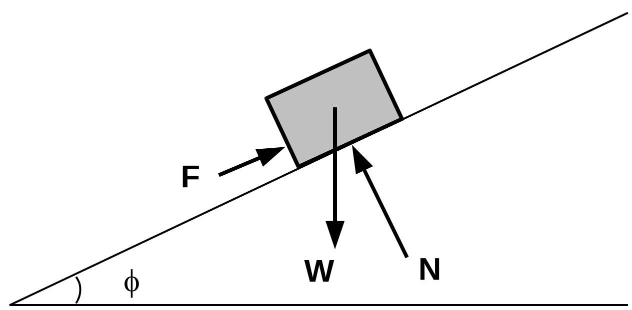


Figure 31 - Friction force caused by weight on inclined plane

Assuming the block is not moving, the force of friction may be calculated to keep the block from sliding.

The force of friction is given by

$$F = \mu_s N$$

The weight of the object is given by

$$W = mg$$

Therefore,

$$N = W \cos 30°$$

$$N = mg \cos 30°$$

So, in the following equation, substitute in for N:

$$F = \mu_s N$$

$$= \mu_s mg \cos 30°$$

$$= 0.3(100)(9.81)\cos 30°$$

$$F = 255 \text{ N}$$

Where μ is the coefficient of either static friction, μ_s, or the coefficient of kinetic friction, μ_k. The coefficient of static friction is always greater than the coefficient of kinetic friction:

$$\mu_s > \mu_k$$

Sample Problem:

As an example, a 200N force acts on a 100kg block on a plane inclined at 30°. The coefficients of static and kinetic friction between the block and the plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Find the value of the friction force and determine if the block is in equilibrium.

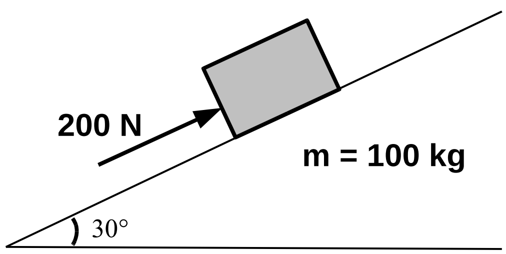
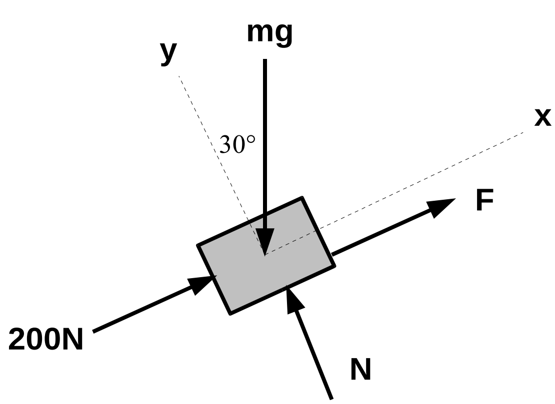


Figure 32 - Friction example

The first step is to determine the magnitude of the friction force (F) required to maintain equilibrium.



$\Sigma F_x = 0$

$$200 + F - mg\sin 30° = 0$$

$$200 + F - 100(9.81)\sin 30° = 0$$

$$F = 290\text{ N}$$

$\Sigma F_y = 0$

$$N - mg\cos 30° = 0$$

$$N = 100(9.81)\cos 30°$$

$$N = 850\text{ N}$$

The frictional force required to maintain equilibrium is F = 290 N up the plane. The maximum friction force which may be developed is computed using μ_s:

$$F = \mu_s N$$

$$= 0.25(850)$$

$$= 213\text{ N}$$

which is insufficient to hold the block in place, so the block is in motion down the inclined plane. The actual frictional force given that the block is in motion is

$$F = \mu_k N$$

$$= 0.20(850)$$

$$= 170\text{ N}$$

Summary

Mechanics is the branch of science concerned with the behavior of physical bodies when subjected to forces or displacements, and the effects of the bodies on their environment. Mechanics is a physical science incorporating mathematical concepts directly applicable to many fields of engineering such as mechanical, civil, structural and electrical engineering.

Vector analysis is a mathematical tool used in mechanics to explain and predict physical phenomena. Statics (or vector mechanics) is the branch of mechanics that is concerned with the analysis of loads (or forces and moments) on physical systems in static equilibrium. Problems involving statics use trigonometry to find a solution.

Statics is the study of forces and moments on physical systems in static equilibrium. Unlike dynamics, where the components of the system are in motion, components of a system in static equilibrium do not move or vary in position relative to one another over time. Statics is concerned with physical systems in equilibrium and the conditions that require equilibrium given the forces and moments that are acting on the components of the systems. These physical systems can include but are not limited to trusses, beams, pulleys and systems that use support cables.

References

1. “Mechanics.” 14 January 2014 <http://en.wikipedia.org/wiki/Mechanics>

2. “Statics.” 28 December 2013 <http://en.wikipedia.org/wiki/Statics>

3. “Engineering Statics.” Spring 2003 <http://emweb.unl.edu/NEGAHBAN/EM223/Intro.htm>

4. “Vectors: Motion and Forces in Two Dimensions – Lesson 3.” visited 28 December 2013 <http://www.physicsclassroom.com/class/vectors/u3l3c.cfm>

5. “Newton’s Laws of Motion.” 15 January 2014 <http://en.wikipedia.org/wiki/Newton's_laws_of_motion>

6. “Statics.” visited 28 December 2013 <http://www.valpo.edu/student/asme/FE%20Slides/StaticsSlides.pdf>

7. “Statics FE Review.” visited 28 December 2013 <http://engineering.utsa.edu/ce/pdf/FE%20Review%202012/10.4Statics.Montoya.pdf>

8. Lindeburg, Michael R., *P.E., Engineer-In-Training Reference Manual, 7th Edition*, Professional Publications, Inc., Belmont, CA, 1990.

9. “Trusses – Method of Joints.” visited 28 December 2013 <http://www.ce.memphis.edu/3121/notes/notes_03b.pdf>

10. “Trusses – Method of Sections.” visited 28 December 2013 <http://www.ce.memphis.edu/3121/notes/notes_03c.pdf>