PDHonline Course G523 (2 PDH)

Forensic Analysis of a Trampoline

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Forensic Analysis of a Trampoline

Peter Chen, P.E.

1. Introduction

Trampolines in and of themselves are a recreation and exercise device. People enjoy using trampolines because they get a feeling of euphoria from jumping higher than normal (without hurting themselves), from a temporary feeling of weightlessness, and from the exercise.

The Consumer Product Safety Commission tracks injuries by products via the National Electronic Injury Surveillance System (NEISS). Based on the number of reporting hospitals, the NEISS produces a historical estimate of the number of injuries by product types. Although not as common as a pedal powered bicycle accident, trampolines are involved in a number of accidents (see Table 1).

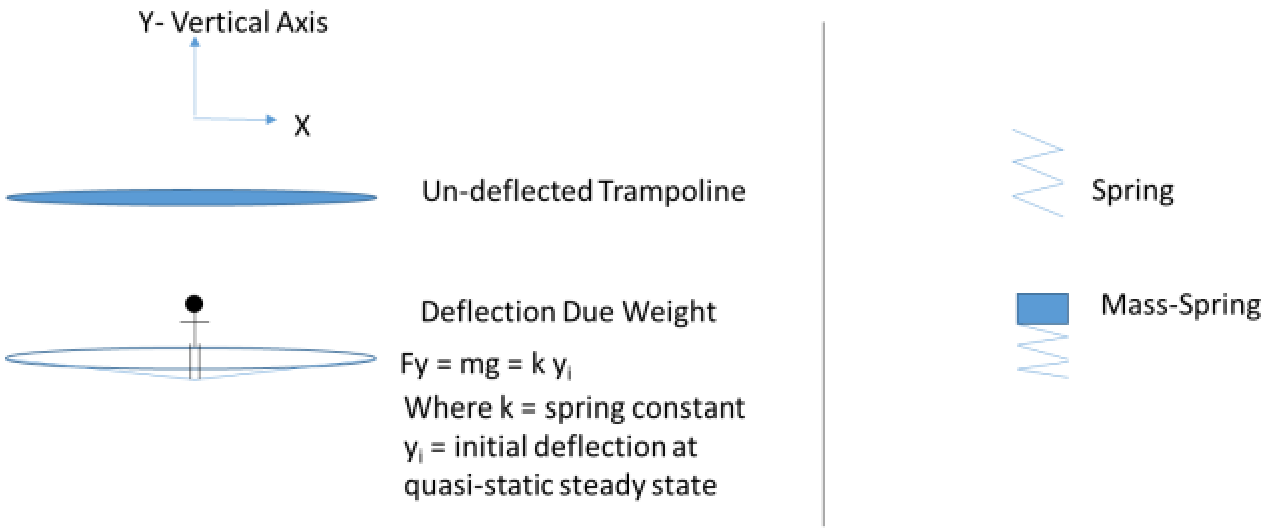
	NEISS Historical Estimate	
Year	Trampoline	Bicycle
2014	104691	664280
2013	83665	690243
2012	94945	730117

Table 1: Summary of Yearly Accidents
(Source: NEISS)

For Forensic Analysis of trampoline accidents, a basic understanding of the physics and engineering of trampolines is required. This training will cover a basic analysis and model of the trampoline, an analysis of the double bounce, and a more in depth analysis of the trampoline, and finally the basics of the Forensic Analysis of a trampoline accident.

2. Basic Analysis and Model of a Trampoline

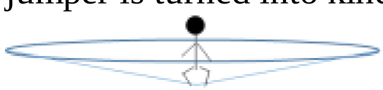
For forensic analysis, a simple and basic analysis of a trampoline can be accomplished by modeling the trampoline as a basic one-dimensional mass-spring system (Hooke's Law). For a single jumper standing on the trampoline, the sum of forces in the vertical direction equals zero and can be modelled as:



The cycle of jumping on the trampoline: the start, airborne, and landing, can be modelled or analyzed from the steady state equilibrium above.

The Start

In the beginning, a jumper initiates the cycle by using the muscles of the leg, back, and/or arms to start a jump. The trampoline has an initial deflection from the weight of the jumper. The initial jump additionally deflects the trampoline. The potential energy of the initial jump deflection, plus the potential energy of the jumper is turned into kinetic energy as the jumper leaves the trampoline surface.



Deflection Due Initial Jump

$Fy = k\,\Delta y$

Where k = spring constant

Δy = deflection due to jump

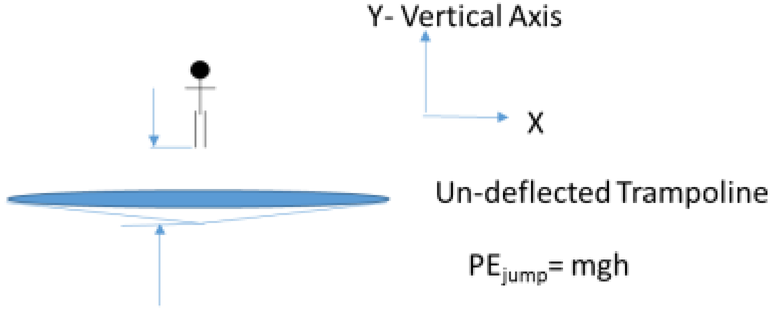
Because force increases linearly with deflection

$PE_{trampoline} = ½\,Fy\,d = ½\,k\,\Delta y^2$

Note that Δy for the trampoline will be the total deflection from the lowest point of deflection until the trampoline returns to the undeformed shape.

Airborne

Once the jumper leaves the trampoline, the jumper reaches a maximum height above the steady state equilibrium surface and again achieves the maximum potential energy of gravity.



If we use a height from the un-deflected trampoline we would be under-estimating the potential energy of the jumper.

After the initial jump, we can consider the height, h, to be the height above the un-deflected trampoline plus the total height the trampoline deflects when the jumper lands.

Landing

When landing, the jumper transfers all the potential energy from the airborne state back into potential energy of the spring force from the trampoline. If there were no damping losses due to aero drag on the jumper and on the trampoline surface, shock absorption in the body, and friction and inefficiencies in the springs, theoretically, the jumper could sustain the same height bounce indefinitely without jumping. Because of energy loss, the jumper typically times an additional jumping motion to maintain bounce height.

Upon reaching the maximum trampoline deflection, then the potential energy of the spring force from the trampoline becomes:

$PE_{trampoline} = ½\ k\ \Delta y^2$

Where Δy is now the total deflection from the undeformed trampoline shape.

Implications

Some implications can be drawn from this analysis and model. Jump height may be difficult to be assessed by the jumper. The jumper may think that they are jumping higher relative to the un-deflected surface of an empty trampoline than they actually are. The jumper is starting in equilibrium from a surface already deflected downward.

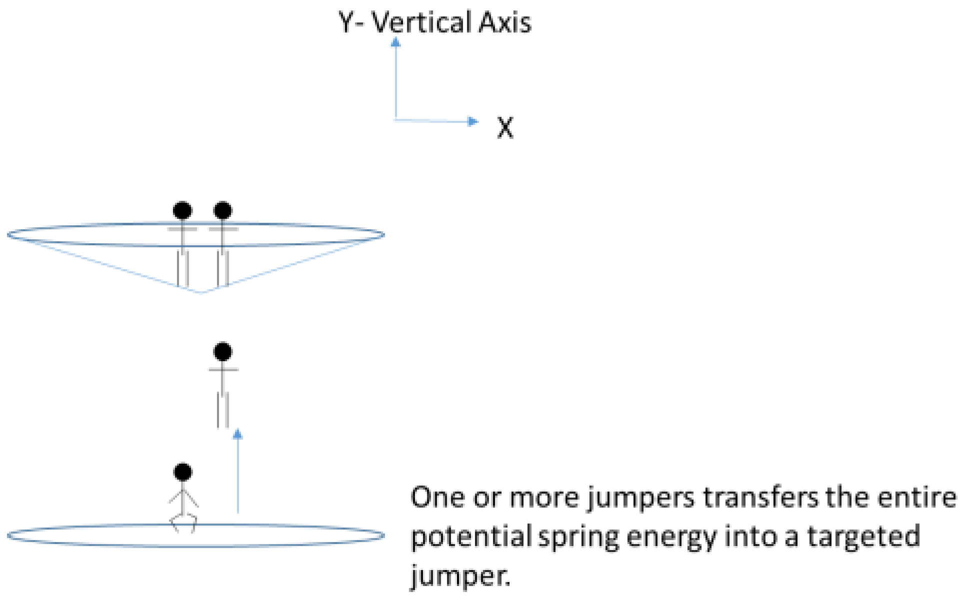
Also from this model, Kinetic Energy would be highest just as the person leaves the trampoline and just as the person lands on the trampoline. Potential Energy would be highest at the maximum jump height and at the maximum trampoline deflection.

3. The Double Bounce

If two (or more) people are jumping on a trampoline, theoretically from the basic model, there should be no change in jump height provided all jumpers continue to jump in a normal manner. If the multiple people are jumping off phase, then the trampoline model would be the same as a single jumper. If multiple people are jumping in phase (landing at the same time), then the trampoline model would be the same as a single jumper just with a much larger mass.

(Note: this training is not condoning multiple jumpers on a trampoline. Trampoline use should follow the manufacturer's warnings and instructions, or facility rules and instructions, or direction of a coach, trainer, or other skilled fitness or competition instructor).

A double bounce (sometimes called double jump) occurs when one or more people are jumping on the trampoline and the jumpers decide to transfer their potential energies into one jumper. The result is a higher jump by the targeted individual. This is accomplished by the other jumpers purposely bending their knees and/or hips thereby reducing the amount of potential spring energy going into their body.



The following are examples taken from social media demonstrating the double bounce:

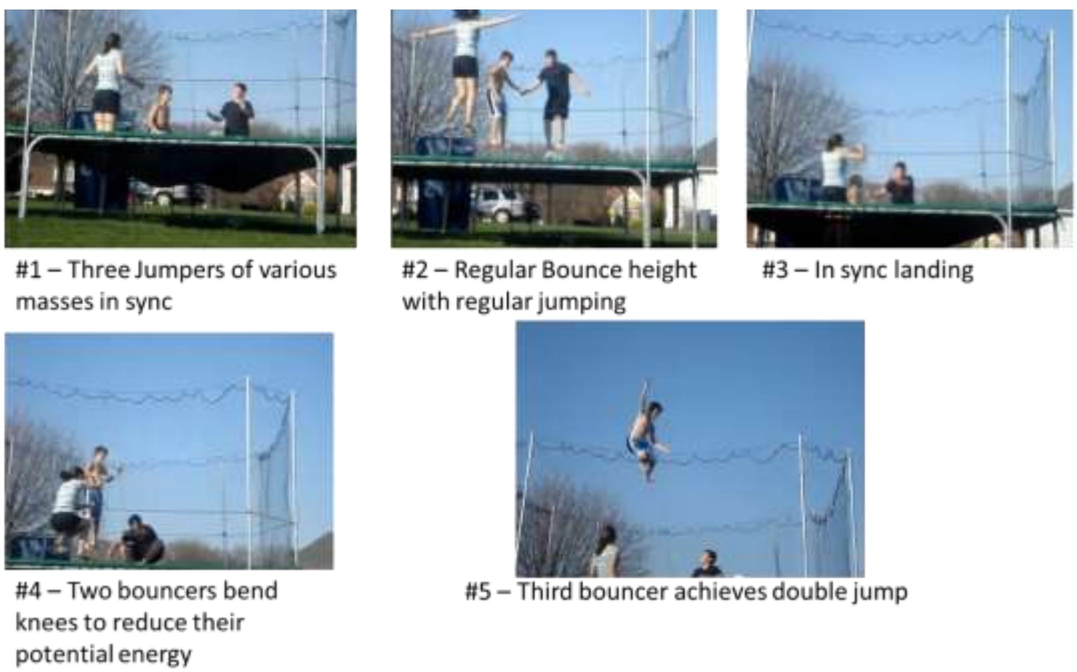


Figure 1: First Example of a Double Bounce
(Source: Link)

Figure 2: Second Example of a Double Bounce
(Source: Link 2)

With an increased number people or mass, the Double Bounce can achieve impressive heights:

#1 – One person bouncing, two waiting, and a third running in.

#2 – Three people jump while jumper lands on his back.

#3 – Jumper is bounced high.

Figure 3: Third Example of Double Bounce
(Source: Link 3)

4. Other Analysis Models of Trampoline

Damping Factor

The 1-D model of the trampoline could be further supplemented with a damping factor. For a spring mass system we know that the damping factor, c, is related to the critical damping factor, c_{cr} , by the damping ratio:

$\varsigma = c/ c_{cr}$

Where, c_{cr}=

The critical damping factor is related to the logarithmic decrement, δ, by the equation:

ς =

The logarithmic decrement can be determined by videotaping or measuring via an accelerometer the dropping of a mass from a known height onto the trampoline and measuring the successive heights of the return bounce.

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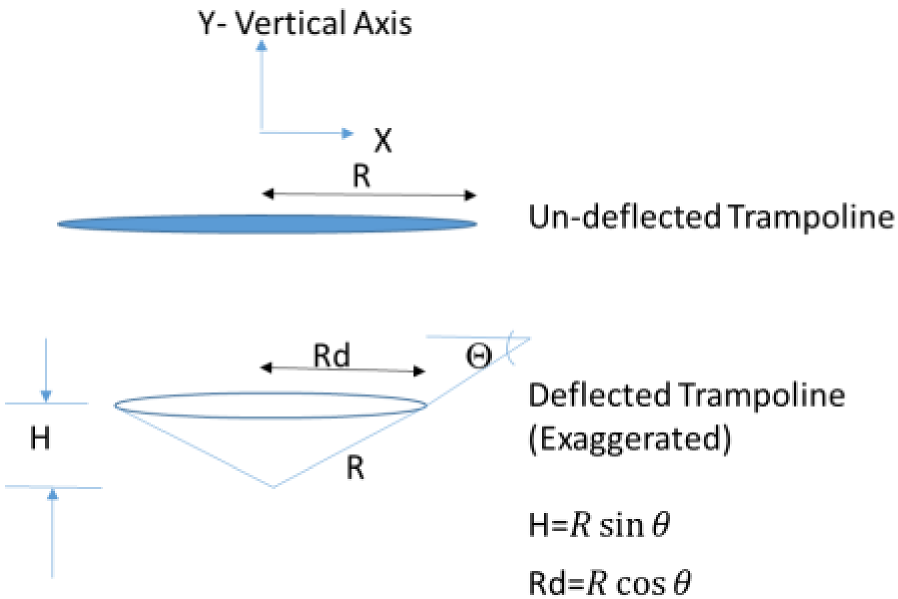
Two-Dimensional Model of Trampoline
A round trampoline could be modelled in two dimensions assuming a vertical direction and a radial direction.

Assuming each spring on the trampoline is behaving in an identical manner, a spring constant, ks, can be measured for the springs by testing a single spring for load and deflection.

Now assuming that the deflection of the trampoline is a perfect cone, we know by geometry that:



We know that the springs are already stretched and the surface of the trampoline is pre-tensioned. The pre-tensioned radial force can be calculated by:

Fo = N ks Δx

Where, N is the number of springs and Δx is the initial spring deflection required for installation. The initial spring deflection can be measured from the frame to the trampoline surface fastener after installation.

The force from the springs in the radial direction, Fr, would have to be:
Fr=Fo+N ks (R-Rd),

Newton's Law on a mass in the Y direction for steady state deflection results in:

$\Sigma Fy = ma_y = 0 = Fr \sin(\Theta) - W$
Where W is the weight of the jumper.

Therefore, $W = Fr \sin(\Theta)$

Including Rotation
So far with all the trampoline models, we are assuming that the jumpers are jumping up and down without flipping. A flip involves expending some of the potential energy into rotational motion. From engineering, we know that the energy for rotation, Er, is:

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Where,
I = moment of Inertia
ω = angular velocity

Angular velocity is fairly easy to determine. With a given jump height (either estimated or determined from testing), we can get the time between maximum jump height and landing on the trampoline. From Newton's Law, and equations of motion we know that:

$h=1/2\ a\ t^2$
or
$h=1/2g\ t^2$
Where,
h=jump height
g=gravitational acceleration

Furthermore, if we know how many rotations or partial rotations were completed during the jump, then we know the angular velocity. For example, if someone jumped 4 feet high and completed 1 flip, then:
h=4 ft
g=32.2 ft/sec^2
Therefore, t = 0.5 sec

1 flip= 1 rotation= 2Π radians = 6.28 radians

ω = 12.6 radians/sec

The difficulty in including rotation into the trampoline model and analysis would be determining the moment of Inertia for the jumper. There are some simple published models from gymnastics that could be scaled for the mass and height of a particular jumper (see Swinging Around the High Bar). For a 60 kg mass person, the paper uses a rotational moment of Inertia of 10 kg m^2, and a radius of 1 m for the person's center of gravity. For a ratio of the moment of inertia, the ratio would be relative to the mass and the radius squared ($I = mr^2$), so the ratio isn't linear to mass or the radius.

Or if more detailed biomechanical measurements or information about the jumper is available, a more complex analysis can be performed with the basics of mechanical engineering (see Geometry and inertia of the human body). In this paper, the human body is separated into components and the total moment of inertia is calculated based on each component part and the position of those parts (see Figure 4). This method allows for greater capability in assessing the moment of inertia for difficult or different body positions at the time of the flip.

The following paper published for the Airforce and Department of Transportation has empirical measurements of different parts of the human body, which again could be scaled for the purposes of trampoline analysis (see Anthropometric Relationships of Body and Body Segment Moments of Inertia).

Please take the time to skim through each of the papers.

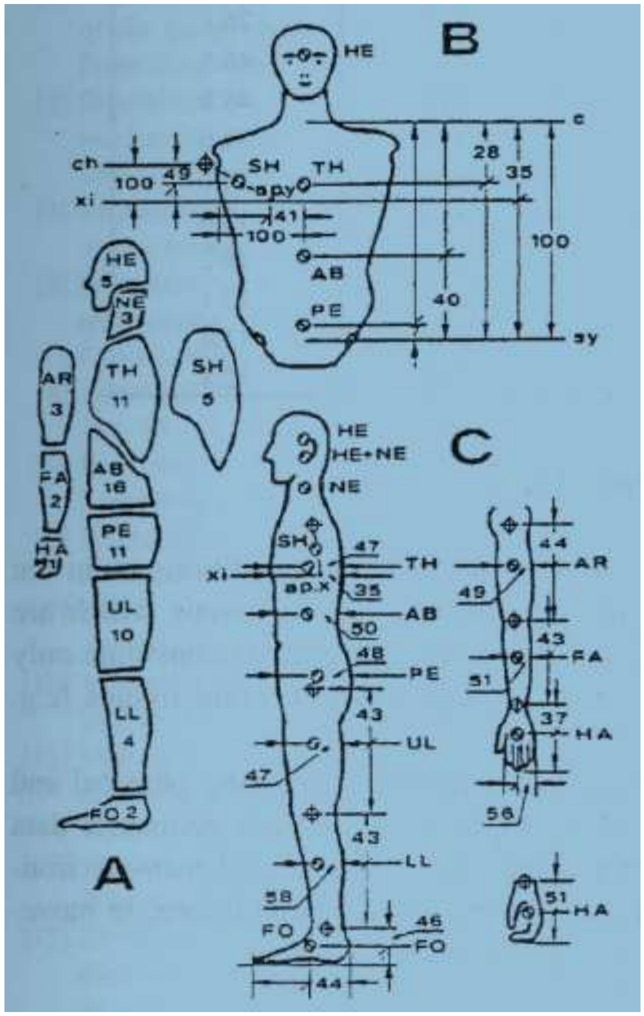

Figure 4: Diagram from “Geometry and inertia of the human body”

Bungee Trampoline

The basic model of trampoline jumping can be applied with the situation where we have bungee trampoline jumping. Bungee trampolines typically have two or more bungee cords attached to a harness for the jumper over a trampoline (see Figure 5).

In this case we can get a spring rate or each of the bungee cords, k_b, by measuring the free length of the bungee cord, L, and the Force, F, required to produce a deflection. We can measure the starting height of the harness empty. Note that for many bungee trampolines, the elastic cord is attached to a cable winch in which the height can be adjusted for different jumpers.

Figure 5: Bungee Trampoline

At maximum trampoline deflection, the total spring potential energy becomes:

$PE_{total} = PE_{trampoline} + PE_{bungee}$

$PE_{trampoline} = ½\ k\ \Delta y^2$

Where Δy is now the total deflection from the undeformed trampoline shape.

$PE_{bungee} = ½\ N\ k_b\ \delta^2$

Where δ is now the total stretch of the bungee cord at maximum trampoline deflection, and N = number of bungee cords.

$PE_{gravity} = mgh$
Where h is the maximum jump height above the deflected trampoline surface.

The Kinetic energy story becomes complicated because the kinetic energy may no longer be maximum upon the jumper leaving the trampoline surface. If the bungee cord stiffness is much less than the trampoline stiffness and the jumper is touching or in contact with the trampoline at the start of the jumping, then the maximum KE may be upon the jumper leaving the trampoline surface. If the bungee cord stiffness is higher than the trampoline stiffness, then the maximum KE is going to occur above the trampoline surface.

5. Forensic Analysis of Trampoline Accidents

Analysis Modeling of the Trampoline and Forensic Analysis
Analysis modeling of a trampoline isn't always necessary for forensic analysis of trampoline accidents. Certainly if the trampoline no longer exists, the identity (make and model) of the trampoline can't be ascertained, and/or the trampoline has deteriorated; getting the engineering/physics properties in order to perform analysis modeling of the condition at the time of the accident may not be possible.

Analysis modeling may be useful in attempting to quantify forces or energy, or principal directions of forces. This information can then be sent to a biomechanical engineer or medical person for injury evaluation.

Analysis modeling is useful in evaluating whether or not an accident occurred the way a person, claimant, or witness said it occurred. In other words, does the story violate the laws of Physics or conservation of energy?

If a trampoline involved in an incident has been preserved or retained for inspection, or an exemplar (same make, model, and year) can be acquired, the forensic engineer may have the ability to test and measure the various components to come up with a model of the trampoline and jumper.

Inspection of Trampoline

Typically during the trampoline inspection, a forensic engineer is looking to see if the trampoline was assembled correctly, put on a suitably prepared or flat surface, or if any components failed. Additionally, a forensic engineer is looking for missing components, safety devices (pads, mats, protective nets and walls), warnings and instructions, and if applicable facility warnings and instructions, and coaching, instruction, or monitoring.

Human Factors

There are inherent risks to users that are typically foreseeable. From a human factors standpoint, once a person enters the airborne phase, the ability of a person to turn or control their path is limited. Typically the desired trajectory would be straight up and down. However, problems may arise as a result of an intentional or unintentional trajectory. As a result, incidents that occur on a trampoline can involve collision with stationary parts of the trampoline (springs, frame, protective posts), collision with other jumpers, and collision with adjacent structures. In addition to collisions, a jumper can fall onto another jumper or bystanders or other stationary objects or onto the ground.

In modeling analysis without an attempt at quantify rotation or spinning, we simplify the person to either a point mass, or a mass/spring, or a mass that can create it's own energy to overcome the damping factor or energy losses. In reality the human body is very complex. The human body has a mass, spring rate, and damping factor, but with four limbs and a head with varying degrees of freedom, the physics model for a human jumping on a trampoline can become daunting. However, the point mass model provides and upper bound of the conservation of energy analysis. Any flipping, twisting, flapping of limbs, or etc. will tend to reduce the amount of energy available for jump height as energy is being converted into 3 dimensional motion.

Causation

There are a variety of trampolines used in a variety of manner. A consumer product trampoline would have different warnings, instructions, and requirements, than would an in-ground trampoline used at a closed gymnastics training center, or a bungee trampoline at an amusement park. A manner of use in one situation, may not be appropriate in another.

An experienced jumper in a specific situation (i.e. stunt basketball dunker with a mini-trampoline) may have no issues performing a stunt, whereas a novice or minor may have an incident attempting to perform the same stunt.

Furthermore, we have to keep in mind that a misuse of a trampoline may or may not be the cause of an accident or injury.