# Chapter 2
# Occurrence and Movement of Groundwater

## 2-1. General

The occurrence and movement of groundwater are related to physical forces acting in the subsurface and the geologic environment in which they occur. This chapter presents a general overview of basic concepts which explain and quantify these forces and environments as related to groundwater. For Corps-specific applications, a section on estimating capture zones of pumping wells is included. Additionally, a discussion on saltwater intrusion is included. For a more detailed understanding of general groundwater concepts, the reader is referred to Fetter (1994).

## 2-2. Hydrologic Cycle

*a.* The Earth's hydrologic cycle consists of many varied and interacting processes involving all three phases of water. A schematic diagram of the flow of water from the atmosphere, to the surface and subsurface, and eventually back to the atmosphere is shown in Figure 2-1.

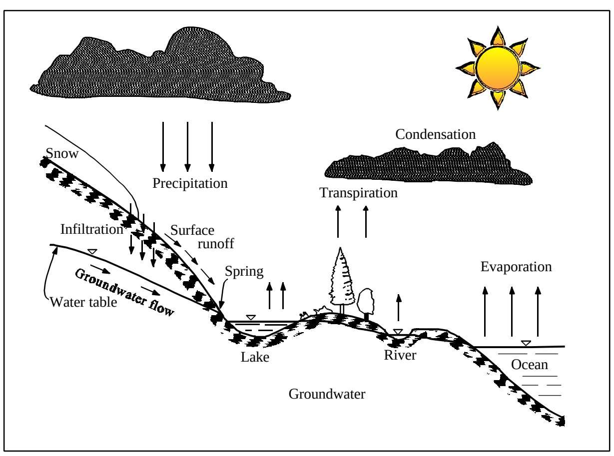


**Figure 2-1. Hydrologic cycle**

*b.* Groundwater flow is but one part of this complex dynamic hydrologic cycle. Saturated formations below the surface act as mediums for the transmission of groundwater, and as reservoirs for the storage of water. Water infiltrates to these formations from the surface and is transmitted slowly for varying distances until it returns to the surface by action of natural flow, vegetation, or man (Todd 1964). Groundwater is the largest source of available water within the United States, accounting for 97 percent of the available fresh water in the United States, and 23 percent of fresh-water usage (Solley and Pierce 1992).

## 2-3. Subsurface Distribution

*a. General.* Groundwater occurs in the subsurface in two broad zones: the unsaturated zone and the saturated zone. The unsaturated zone, also known as the vadose zone, consists of soil pores that are filled to a varying degree with air and water. The zone of saturation consists of water-filled pores that are assumed to be at hydrostatic pressure. For an unconfined aquifer, the zone of saturation is overlain by an unsaturated zone that extends from the water table to the ground surface (Figure 2-2).

*b. Unsaturated zone.* The unsaturated zone (or vadose zone) serves as a vast reservoir which, when recharged, typically discharges water to the saturated zone for a relatively long period after cessation of surface input. The unsaturated zone commonly consists of three sub-zones: the root zone, an intermediate zone, and the capillary fringe. The root zone varies in thickness depending upon growing season and type of vegetation. The water content in the root zone is usually less than that of saturation, except when surface fluxes are of great enough intensity to saturate the surface. This region is subject to large fluctuations in moisture content due to evaporation, plant transpiration, and precipitation. Water below the root zone is either percolating near vertically downward under the influence of gravity, or is suspended due to surface tension after gravity drainage is completed. This intermediate zone does not exist where the capillary fringe or the water table intercepts the root zone. The capillary fringe extends from the water table up to the limit of capillary rise. Water molecules at the water surface are subject to an upward attraction due to surface tension of the air-water interface and the molecular attraction of the liquid and solid phases. The thickness of this zone depends upon the pore size of the soil medium, varying directly with decrease in pore size. Water content can range from very low to saturated, with the

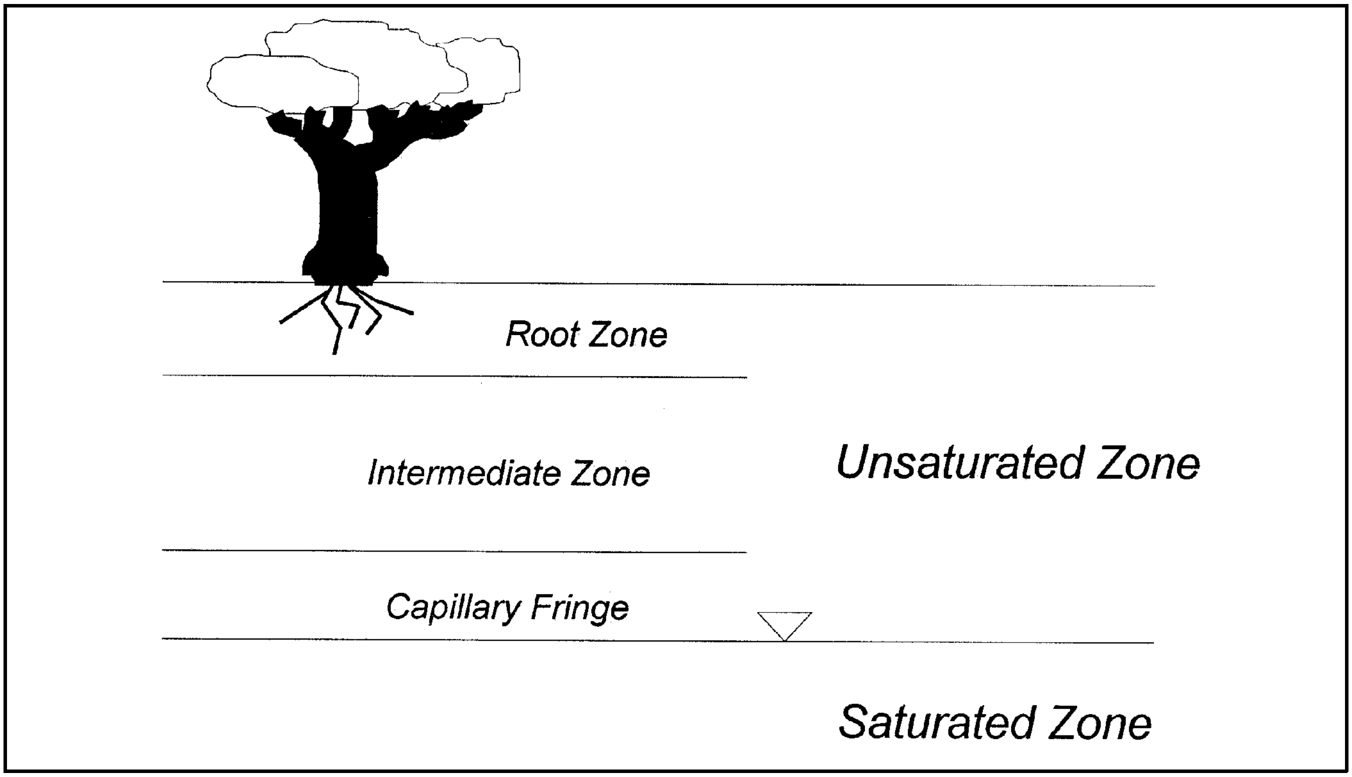


**Figure 2-2. Subsurface distribution of water**

lower part of the capillary fringe often being saturated. Infiltration and flow in the unsaturated zone are discussed in Section 6-3.

*c. Saturated zone.* In the zone of saturation, all communicating voids are filled with water under hydrostatic pressure. Water in the saturated zone is known as groundwater or phreatic water.

## 2-4. Forces Acting on Groundwater

External forces which act on water in the subsurface include gravity, pressure from the atmosphere and overlying water, and molecular attraction between solids and water. In the subsurface, water can occur in the following: as water vapor which moves from regions of higher pressure to lower pressure, as condensed water which is absorbed by dry soil particles, as water which is retained on particles under the molecular force of adhesion, and as water which is not subject to attractive forces towards the surface of solid particles and is under the influence of gravitational forces. In the saturated zone, groundwater flows through interconnected voids in response to the difference in fluid pressure and elevation. The driving force is measured in terms of hydraulic head. Hydraulic head (or potentiometric head) is defined by Bernoulli's equation:

$$h = z + \frac{p}{\rho g} + \frac{v^2}{2g} \tag{2-1}$$

where

$h$ = hydraulic head

$z$ = elevation above datum

$p$ = fluid pressure with constant density $\rho$

$g$ = acceleration due to gravity

$v$ = fluid velocity

Pressure head (or fluid pressure) $h_p$ is defined as:

$$h_p = \frac{p}{\rho g} \tag{2-2}$$

By convention, pressure head is expressed in units above atmospheric pressure. In the unsaturated zone, water is held in tension and pressure head is less than atmospheric pressure ($h_p < 0$). Below the water table, in the saturated zone, pressure head is greater than atmospheric pressure ($h_p > 0$). Because groundwater velocities are usually very low, the velocity component of hydraulic head can be neglected. Thus, hydraulic head can usually be expressed as:

$$h = z + h_p \tag{2-3}$$

Figure 2-3 depicts Equation 2-3 within a well.

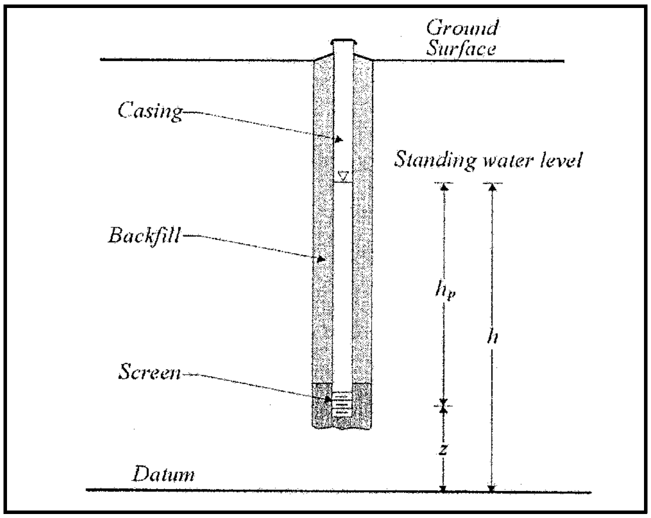


**Figure 2-3. Relationship between hydraulic head, pressure head, and elevation head within a well**

## 2-5. Water Table

As illustrated by Figure 2-3, the height of water measured in wells is the sum of elevation head and pressure head, where the pressure head is equal to the height of the water column above the screened interval within the well. Freeze and Cherry (1979) define the water table as located at the level at which water stands within a shallow well which penetrates the surficial deposits just deeply enough to encounter standing water. Thus, the hydraulic head at the water table is equal to the elevation head; and the pore water pressure at the water table is equivalent to atmospheric pressure.

## 2-6. Potentiometric Surface

The water table is defined as the surface in a groundwater body at which the pressure is atmospheric, and is measured by the level at which water stands in wells that penetrate the water body just far enough to hold standing water. The potentiometric surface approximates the level to which water will rise in a tightly cased well which can be screened at the water table or at greater depth. In wells that penetrate to greater depths within the aquifer, the potentiometric surface may be above or below the water table depending on whether an upward or downward component of flow exists. The potentiometric surface can vary with the depth of a well. In confined aquifers (Section 2-6), the potentiometric surface will rise above the aquifer surface. The water table is the potentiometric surface for an unconfined aquifer (Section 2-6). Where the head varies appreciably with depth in an aquifer, a potentiometric surface is meaningful only if it describes the static head along a particular specified stratum in that aquifer. The concept of potentiometric surface is only rigorously valid for defining horizontal flow directions from horizontal aquifers.

## 2-7. Aquifer Formations

*a. General.* An aquifer is a geologic unit that can store and transmit water. Aquifers are generally categorized into four basic formation types depending on the geologic environment in which they occur: unconfined, confined, semi-confined, and perched. Figure 2-4 describes basic aquifer formations.

*b. Unconfined aquifers.* Unconfined aquifers contain a phreatic surface (water table) as an upper boundary that fluctuates in response to recharge and discharge (such as from a pumping well). Unconfined aquifers are generally close to the land surface, with continuous layers of materials of high intrinsic permeability (Section 2-11) extending from the land surface to the base of the aquifer.

*c. Confined aquifers.* Confined, or artesian, aquifers are created when groundwater is trapped between

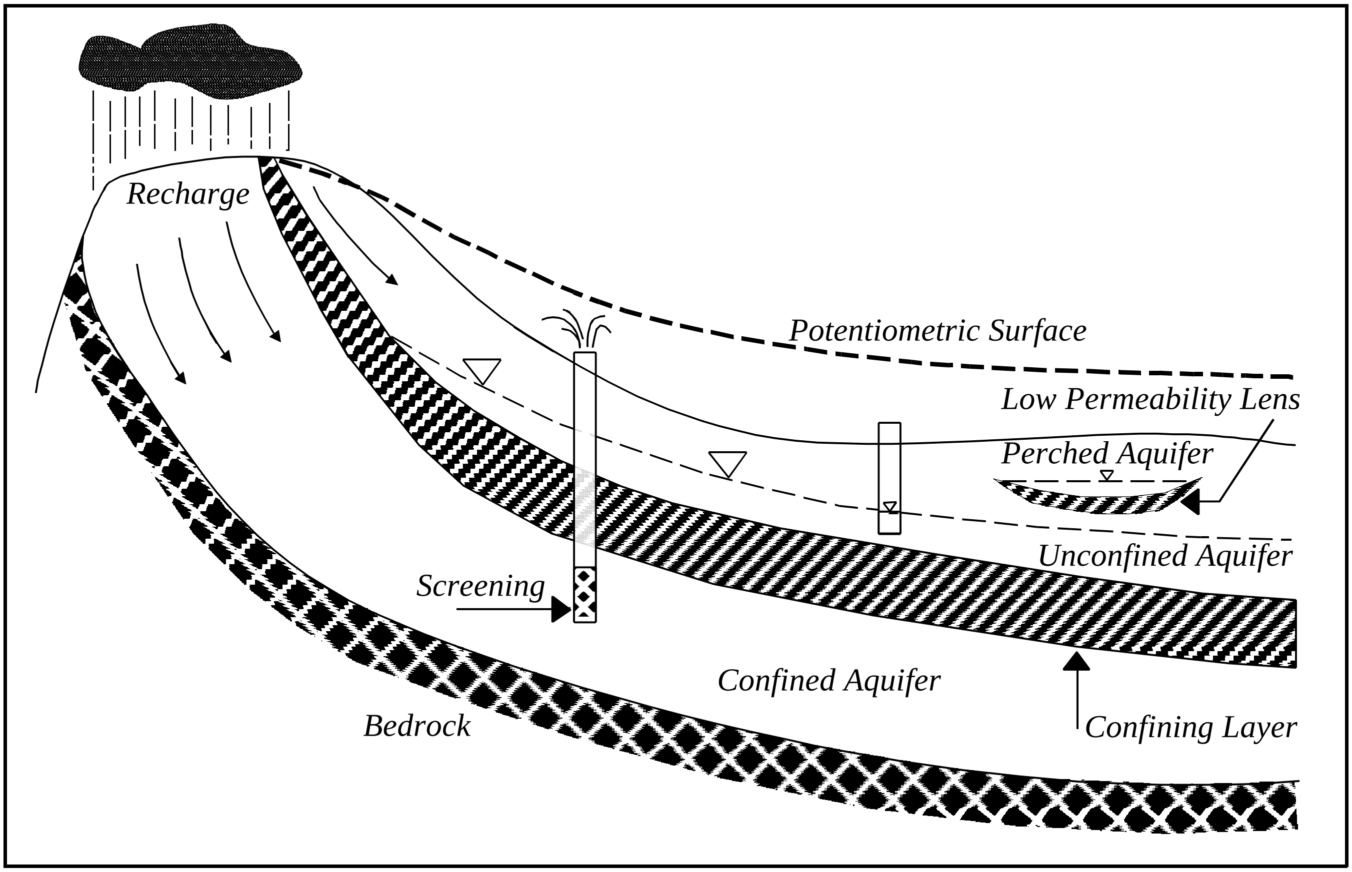


**Figure 2-4. Aquifer formations**

two layers of low permeability known as aquitards. In a confined aquifer, the groundwater is under pressure and the water level in a well rises above the upper boundary of the aquifer. Flowing artesian conditions exist when the water level in a well rises above land surface. Recharge to confined aquifers is predominantly from areas where the confining bed is breached, either by erosional unconformity, fracturing, or depositional absence.

*d. Semi-confined aquifers.* Semi-confined, or leaky, aquifers occur when water-bearing strata are confined, either above or below, by a semipermeable layer. When water is pumped from a leaky aquifer, water moves both horizontally within the aquifer and vertically through the semipermeable layer.

*e. Perched aquifers.* A perched aquifer is a special type of unconfined aquifer where a groundwater body is separated above the water table by a layer of unsaturated material. A perched aquifer occurs when water moving down through the unsaturated zone is intercepted by an impermeable formation. Clay lenses in sedimentary deposits often have shallow perched water bodies overlying them. Wells tapping perched aquifers generally yield temporary or small quantities of water.

## 2-8. Principal Types of Aquifer Materials

*a. General.* Earth materials which can have the potential to transmit water can be classified into four broad groups: unconsolidated materials, porous sedimentary rock, porous volcanics, and fractured rock. In unconsolidated material, water is transported through the primary openings in the rock/soil matrix. Consolidation is the process where loose materials become firm and coherent. Sandstone and conglomerate are common consolidated sedimentary rocks formed by compaction and cementation. Carbonate rocks (such as limestone and dolomite) are sedimentary rocks which can be formed by chemical precipitation. Water is usually transported through secondary openings in carbonate rocks enlarged by the dissolution of rock by water. The movement of water through volcanics and fractured rock is dependent upon the interconnectedness and frequency of flow pathways.

*b. Gravel and sand.* Gravel and sand aquifers are the source of most water pumped in the United States. Gravels and sands originate from alluvial, lacustrine, marine, or eolian glacial deposition.

(1) Alluvial deposits. Alluvial deposits of perennial streams are usually fairly well sorted and therefore permeable. Ephemeral streams typically deposit sand and gravel with much less sorting. Stream channels are sensitive to changes in sediment load, gradient, and velocity. This can result in lateral distribution of alluvial deposits over large areas. Areas with greater streambed slope typically contain coarser deposits. Alluvial fans occur in arid or semiarid regions where a stream issues from a narrow canyon onto a plain or valley floor. Viewed from above, they have the shape of an open fan, the apex being at the valley mouth. These alluvial deposits are coarsest at the point where the stream exits the canyon mouth, and become finer with increasing distance from the point of initial deposition.

(2) Lacustrine deposits. The central and lower portions of alluvium-filled valleys may consist of fine-grained lacustrine (or lakebed) deposits. When a stream flows into a lake, the current is abruptly checked. The coarser sediment settles rapidly to the bottom, while finer materials are transported further into the body of relatively still water. Thus, the central areas of valleys which have received lacustrine deposits often consist of finer-grained, lower-permeability materials. Lacustrine deposits usually consist of fine-grained materials that are not normally considered aquifers.

(3) Marine deposits. Marine deposits originate from sediment transported to the ocean by rivers and erosion of the ocean floor. As a sea moves inland, deposits at a point in the ocean bottom near the shore become gradually finer due to uniform wave energy. Conversely, as the sea regresses, deposition progresses gradually from finer to coarser deposits. This is a common sequence in the southern United States. Additionally, coral reefs, shells, and other calcite-rich deposits commonly occur in areas with temperate climatic conditions.

(4) Eolian deposits. Materials which are transported by the wind are known as eolian deposits. The sorting action of the wind tends to produce deposits that are uniform on a local scale, and in some cases quite uniform over large areas. Eolian deposits consist of silt or sand. Eolian sands occur wherever surface sediments are available for transport. In comparison with alluvial deposits, eolian sands are quite homogeneous and are as isotropic (Section 2-13) as any deposits occurring in nature. Eolian deposits of silt, called loess, are associated with the abnormally high wind velocities associated with glacial ice fronts. Loess occurs in the shallow subsurface in large areas of the Midwest and Great Plains regions of North America.

(5) Glacial deposits. Unlike water and wind, glacial ice can entrain unconsolidated deposits of all sizes from sediments to boulders. Glacial till is non-sorted, non-stratified sediment deposited beneath, from within, or from the top of glacial ice. Glacial outwash (or glaciofluvial) deposits consist of coarse-grained sediments deposited by meltwater in front of a glacier. The sorting and homogeneity of glaciofluvial deposits depend upon environmental conditions and distance from the glacial front.

*c. Sandstone and conglomerate.* Sandstone and conglomerates are the consolidated equivalents of sand and gravel. Consolidation results from compaction and cementation. The highest yielding sandstone aquifers occur where partial consolidation takes place. These yield water from the pores between grains, although secondary openings such as fractures and joints can also serve as channels of flow.

*d. Carbonate rocks.* Carbonate rocks, formed from calcium, magnesium, or iron, are widespread throughout the United States. Limestone and dolomite, which originate from calcium-rich deposits, are the most common carbonate rocks. Carbonates are typically brittle and susceptible to fracturing. Fractures and joints in limestone yield water in small to moderate amounts. However, because water acts as a weak acid to carbonates, dissolution of rock by water enlarges openings. The limestones that yield the highest amount of water are those in which a sizable portion of the original rock has been dissolved or removed. These areas are commonly referred to as karst. Thus, large amounts of flow can potentially be transmitted in carbonate rocks.

*e. Volcanics.* Basalt is an important aquifer material in parts of the western United States, most notably central Idaho, where enormous flows of lava have spread out over large areas in successive sheets of varying thickness. The ability of basalt formations to transmit water is dependent on the presence of fractures, cracks, and tubes or caverns, and can be significant. Near the surface, rapid cooling produces jointing. Fracturing below the surface occurs as the crust cools, causing differential flow velocities with depth. Other volcanic rocks, including rhyolite and other more siliceous rocks, do not usually yield water in quantities comparable to those secured from basalt. Another major source of groundwater in some parts of the western United States is found in sedimentary "interbed" materials which occur between basalt flows. This interbedded material is generally alluvial or colluvial in nature, consisting of sands, gravels, and residuum (particularly granite). When the interbedded materials tend to be finer-grained, the interbed acts as a confining layer.

*f.* Fractured rock. Crystalline and metamorphic rocks, including granite, basic igneous rocks, gneiss, schist, quartzite, and slate are relatively impermeable. Water in these areas is supplied as a result of jointing and fracturing. The yield of water from fractured rock is dependent upon the frequency and interconnectedness of flow pathways.

## 2-9. Movement of Groundwater

Groundwater moves through the sub-surface from areas of greater hydraulic head to areas of lower hydraulic head (Equation 2-3). The rate of groundwater movement depends upon the slope of the hydraulic head (hydraulic gradient), and intrinsic aquifer and fluid properties.

## 2-10. Porosity and Specific Yield

*a. Porosity. Porosity n* is defined as the ratio of void space to the total volume of media:

$$n = \frac{V_v}{V_T} \tag{2-4}$$

where

$V_v$ = volume of void space [$L^3$]

$V_T$ = total volume (volume of solids plus volume of voids) [$L^3$]

In unconsolidated materials, porosity is principally governed by three properties of the media: grain packing, grain shape, and grain size distribution. The effect of packing may be observed in two-dimensional models comprised of spherical, uniform-sized balls. Arranging the balls in a cubic configuration (each ball touching four other balls) yields a porosity of 0.476 whereas rhombohedral packing of the balls (each ball touching eight other balls) results in a porosity of 0.260. Porosity is not a function of grain size, but rather grain size distribution. Spherical models comprised of different sized balls will always yield a lower porosity than the uniform model arranged in a similar packing arrangement. *Primary porosity* in a material is due to the properties of the soil or rock matrix, while *secondary porosity* is developed in the material after its emplacement through such processes as solution and fracturing. Representative porosity ranges for sedimentary materials are given in Table 2-1.

**Table 2-1**
**Porosity Ranges for Sedimentary Materials**

| Material | Porosity |
|---|---|
| Clay | .45 - .55 |
| Silt | .40 - .50 |
| Medium to coarse mixed sand | .35 - .40 |
| Uniform sand | .30 - .40 |
| Fine to medium mixed sand | .30 - .35 |
| Gravel | .30 - .40 |
| Gravel and sand | .20 - .35 |

*b. Effective porosity. Effective porosity $n_e$* is the porosity available for fluid flow. The effective porosity of a unit of media is equal to the ratio of the volume of interconnected pores that are large enough to contain water molecules to the total volume of the rock or soil.

*c. Specific yield. Specific yield $S_y$* is the ratio of the water that will drain from a saturated rock owing to the force of gravity to the total volume of the media. *Specific retention Sr* is defined as the ratio of the volume of water that a unit of media can retain against the attraction of gravity to the total volume of the media. The porosity of a rock is equal to the sum of the specific yield and specific retention of the media. For most practical applications in sands and gravels, the value of effective porosity can be considered equivalent to specific yield. In clays, there is a much greater surface area and corresponding adhesion of water molecules. Figure 2-5 illustrates a typical relationship of specific yield and specific retention to total porosity for different soil types.

## 2-11. Darcy's Law and Hydraulic Conductivity

*a. Darcy's Law.* Henry Darcy, a French hydraulic engineer, observed that the rate of laminar flow of a fluid (of constant density and temperature) between two points in a porous medium is proportional to the hydraulic gradient (dh/dl) between the two points (Darcy 1856). The equation describing the rate of flow through a porous medium is known as Darcy's Law and is given as:

$$Q = -KA\frac{dh}{dl} \tag{2-5}$$

where

$Q$ = volumetric flow rate [$L^3T^{-1}$]

$K$ = hydraulic conductivity [$LT^{-1}$]

$A$ = cross-sectional area of flow [$L^2$]

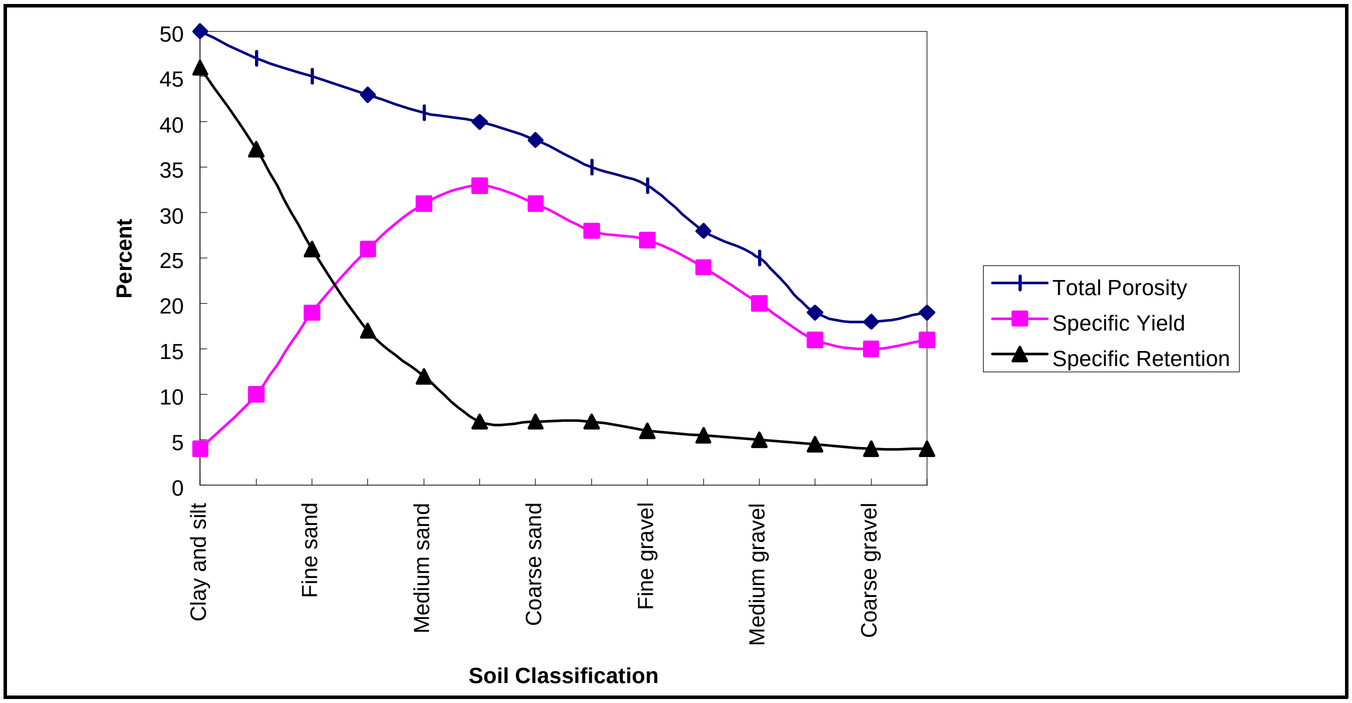


**Figure 2-5. Typical relationship between specific yield, specific retention, and total porosity for different soil types**

$h$ = hydraulic head [L]

$l$ = distance between two points [L]

The negative sign on the right-hand side of Equation 2-5 (Darcy's Law) is used by convention to indicate a downward trending flow gradient.

*b. Hydraulic conductivity.* The hydraulic conductivity of a given medium is a function of the properties of the medium and the properties of the fluid. Using empirically derived proportionality relationships and dimensional analysis, the hydraulic conductivity of a given medium transmitting a given fluid is given as:

$$K = \frac{k \rho g}{\mu} \tag{2-6}$$

where

$k$ = intrinsic permeability of porous medium [$L^2$]

$\rho$ = fluid density [$ML^{-3}$]

$\mu$ = dynamic viscosity of fluid [$ML^{-1}T^{-1}$]

$g$ = acceleration of gravity [$LT^{-2}$]

The intrinsic permeability of a medium is a function of the shape and diameter of the pore spaces. Several empirical relationships describing intrinsic permeability have been presented. Fair and Hatch (1933) used a packing factor, shape factor, and the geometric mean of the grain size to estimate intrinsic permeability. Krumbein (1943) uses the square of the average grain diameter to approximate the intrinsic permeability of a porous medium. Values of fluid density and dynamic viscosity are dependent upon water temperature. Fluid density is additionally dependent upon total dissolved solids (TDS). Ranges of intrinsic permeability and hydraulic conductivity values for unconsolidated sediments are presented in Table 2-2.

*c. Specific discharge.* The volumetric flow velocity $v$ can be determined by dividing the volumetric flow rate by the cross-sectional area of flow as:

**Table 2-2**
**Ranges of Intrinsic Permeability and Hydraulic Conductivity for Unconsolidated Sediments**

| Material | Intrinsic Permeability ($cm^2$) | Hydraulic Conductivity (cm/s) |
|---|---|---|
| Clay | $10^{-6}$ - $10^{-3}$ | $10^{-9}$ - $10^{-6}$ |
| Silt, sandy silts, clayey sands, till | $10^{-3}$ - $10^{-1}$ | $10^{-6}$ - $10^{-4}$ |
| Silty sands, fine sands | $10^{-2}$ - 1 | $10^{-5}$ - $10^{-3}$ |
| Well-sorted sands, glacial outwash | 1 - $10^{2}$ | $10^{-3}$ - $10^{-1}$ |
| Well-sorted gravels | 10 - $10^{3}$ | $10^{-2}$ - 1 |

$$v = \frac{Q}{A} = -K\frac{dh}{dl} \tag{2-7}$$

The velocity given by Equation 2-7 is termed the *specific discharge*, or *Darcy flux*. The specific discharge is actually an apparent velocity, representing the velocity at which water would move through an aquifer if the aquifer were an open conduit. The cross-sectional area is not entirely available for flow due to the presence of the porous matrix.

*d. Pore water velocity.* The average linear velocity of water in a porous medium is derived by dividing specific discharge by effective porosity ($n_e$) to account for the actual open space available for the flow. The resulting velocity is termed the *pore water velocity*, or the *seepage velocity*. The pore water velocity $V_x$ represents the average rate at which the water moves between two points and is given by

$$V_x = \frac{Q}{n_e A} = -\frac{K dh}{n_e dl} \tag{2-8}$$

## 2-12. Flow and Transmissivity

*Transmissivity T* is a measure of the amount of water that can be transmitted horizontally through a unit width by the fully saturated thickness of an aquifer under a hydraulic gradient equal to 1. Transmissivity is equal to the hydraulic conductivity multiplied by the saturated thickness of the aquifer and is given by:

$$T = Kb \tag{2-9}$$

where

$K$ = hydraulic conductivity [$LT^{-1}$]

$b$ = saturated thickness of the aquifer [L]

Since transmissivity depends on hydraulic conductivity and saturated thickness, its value will differ at different locations within aquifers comprised of heterogeneous material, bounded by sloping confining beds, or under unconfined conditions where the saturated thickness will vary with the water table.

## 2-13. Homogeneity and Isotropy

*a. Definition.* If hydraulic conductivity is consistent throughout a formation, regardless of position, the formation is homogeneous. If hydraulic conductivity within a formation is dependent on location, the formation is heterogeneous. When hydraulic conductivity is independent of the direction of measurement at a point within a formation, the formation is isotropic at that point. If the hydraulic conductivity varies with the direction of measurement at a point within a formation, the formation is anisotropic at that point. Figure 2-6 is a graphical representation of homogeneity and isotropy.

*b. Geologic controls.* Geologic material is very rarely homogeneous in all directions. A more probable condition is that the properties, such as hydraulic conductivity, are approximately constant in one direction. This condition results because: a) of effects of the shape of soil particles, and b) different materials incorporate the alluvium at different locations. As geologic strata are formed, individual particles usually rest with their flat sides down in a process called imbrication. Consequently, flow is generally less restricted in the horizontal direction than the vertical and $K_x$ is greater than $K_z$ for most situations. Layered heterogeneity occurs when stratum of homogeneous, isotropic materials are overlain upon each other. Layered conditions commonly occur in alluvial, lacustrine, and marine deposits. At a large scale, there is a relationship between anisotropy and layered heterogeneity. In the field it is not uncommon for sites with layered heterogeneity to have large scale anisotropy values of 100:1 or greater. Discontinuous heterogeneity results from geologic structures such as bedrock outcrop contacts, clay lenses, and buried oxbow stream cutoffs. Trending heterogeneity commonly occurs in sedimentary formations of deltaic, alluvial, and glacial origin.

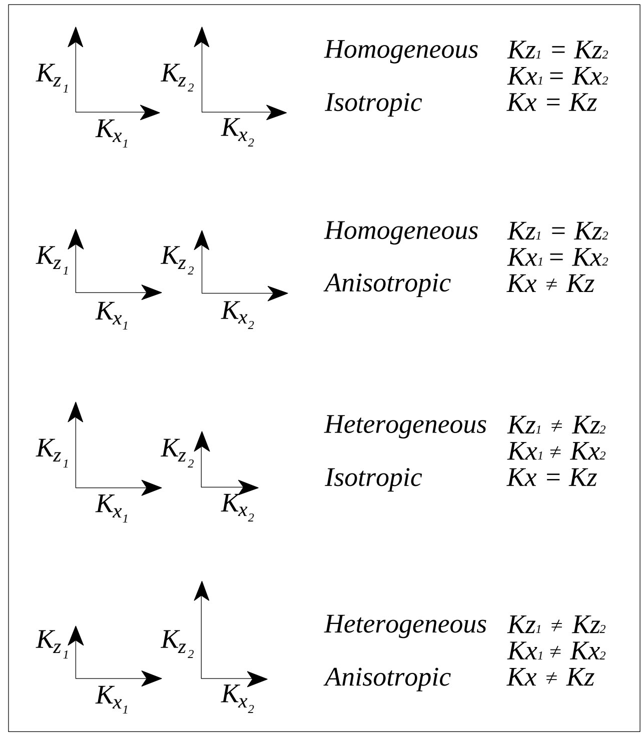


**Figure 2-6. Homogeneity and isotropy**

## 2-14. Flow in Stratified Media

*a. General.* Flow through stratified media can be described through the definition of a hydraulically equivalent conductivity (or effective hydraulic conductivity). Expressions for horizontal and vertical equivalent conductivities can be generalized from expressions developed for flow through porous media comprised of three parallel homogeneous, isotropic strata (Figure 2-7).

*b. Horizontal flow.* Horizontal flow through the media is given by Darcy's Law,

$$Q_x = K_x A_x \frac{\Delta h_T}{x} \tag{2-10}$$

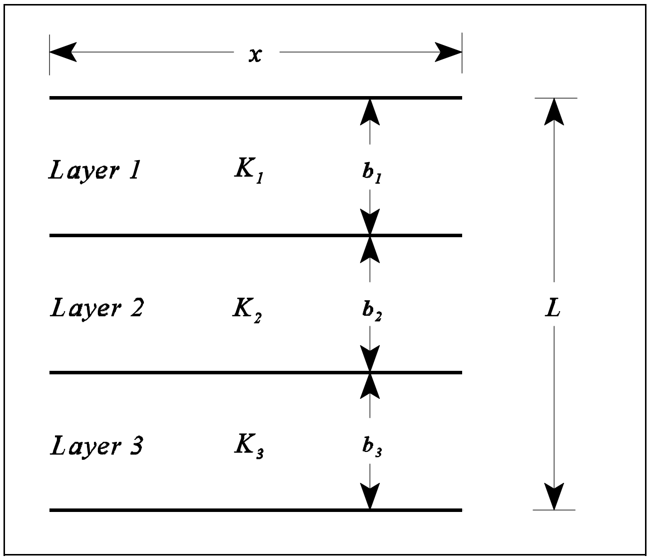


**Figure 2-7. Stratified media**

where

$\Delta h_T$ = total hydraulic head drop across flow distance x

For the case of *i*, the strata method gives the expression for the *horizontal equivalent hydraulic conductivity* as:

$$K_x = \frac{\sum K_i b_i}{L} \quad (2\text{-}11)$$

*c. Vertical flow.* Similarly, vertical flow is given by Darcy's Law as:

$$Q_z = K_z A_z \frac{\Delta h_T}{L} \quad (2\text{-}12)$$

For the case of *i*, the strata method gives the expression for the *vertical equivalent hydraulic conductivity* as:

$$K_z = \frac{L}{\sum \frac{b_i}{K_i}} \quad (2\text{-}13)$$

## 2-15. Aquifer Storage

*a. Storage coefficient.* The *storage coefficient*, or *storativity S* is the volume of water that a permeable unit will absorb or expel from storage per unit surface area per unit change in head. At the water table, water is released from storage by gravity drainage. Below the water table, water is released from storage due to the release of hydrostatic pressure within the pore spaces which accompanies the withdrawal of water from the aquifer. The total load above an aquifer is supported by a combination of the solids skeleton of the aquifer and by the hydraulic pressure exerted by the water in the aquifer. Withdrawal of water from the aquifer results in a decline in the pore water pressure and subsequently more of the load must be supported by the solids skeleton. As a result, the rock particles are distorted and the skeleton is compressed, leading to a reduction in effective porosity. Additionally, the decreased water pressure causes the pore water to expand. Compression of the skeleton and expansion of the pore water both cause water to be expelled from the aquifer.

*b. Specific storage.* The *specific storage* $S_s$ is the amount of water per unit volume of a saturated formation that is stored or expelled from storage owing to compression and expansion of the mineral skeleton and the pore water per unit change in hydraulic head. The specific storage (1/L) is given by:

$$S_s = \rho_w g(\alpha + n\beta) \quad (2\text{-}14)$$

where

$\rho_w$ = density of water [$ML^{-3}T^{-2}$]

$g$ = acceleration of gravity [$LT^{-2}$]

$\alpha$ = compressibility of the aquifer skeleton [$1/(ML^{-1}T^{-2}$]

$n$ = porosity

$\beta$ = compressibility of water [$1/(ML^{-1}T^{-2}$]

From field data, Helm (1975) estimated the specific storage of sands and gravels as $1\times10^{-6}$ ft$^{-1}$ and clays and silts as $3.5\times10^{-6}$ ft$^{-1}$.

*c. Storage coefficient of a confined aquifer.* Within a confined aquifer the full thickness of the aquifer remains saturated when water is released or stored. Therefore, all water is released due to the compaction of the skeleton and expansion of the pore water and the storage coefficient (dimensionless) is given as:

$$S = bS_s \tag{2-15}$$

where

$b$ = thickness of the aquifer [L]

Values of storage coefficient in confined aquifers are generally less than 0.005 (Todd 1980). Values between 0.005 and 0.10 generally indicate a leaky confined aquifer.

*d. Storage coefficient of an unconfined aquifer.* Within an unconfined aquifer the level of saturation varies as water is added to or removed from the aquifer. As the water table falls, water is released by gravity drainage plus compaction of the skeleton and expansion of the pore water. The volume of water released by gravity drainage is given by the specific yield of the aquifer. The storage coefficient of an unconfined aquifer is therefore given by the sum of the specific yield and the volume of water released due to the specific storage as:

$$S = S_y + hS_s \tag{2-16}$$

The value of specific storage is typically very small, generally less than 1 x $10^{-4}$ ft$^{-1}$. As the value of specific yield is usually several orders of magnitude greater than specific storage, the storage coefficient of an unconfined aquifer approximates its specific yield. The storage coefficient of unconfined aquifers typically ranges from 0.10 to 0.30. Estimates of specific yield for various deposits can be found in Johnson (1967).

*e. Volumetric drainage.* The volume of water drained from an aquifer due to a lowering of the hydraulic head can be computed from:

$$V_w = SA\Delta h \tag{2-17}$$

where

$V_w$ = volume of water drained from aquifer [L$^3$]

$S$ = storage coefficient (dimensionless)

$A$ = surface area overlying the drained aquifer [L$^2$]

$\Delta h$ = average decline in hydraulic head [L]

## 2-16. General Flow Equations

*a. Confined aquifer.* The governing flow equation for confined aquifers is developed from application of the law of mass conservation (continuity principle) to the elemental volume shown in Figure 2-8. Continuity is given by:

Rate of mass accumulation = Rate of mass inflow - Rate of mass outflow (2-18)

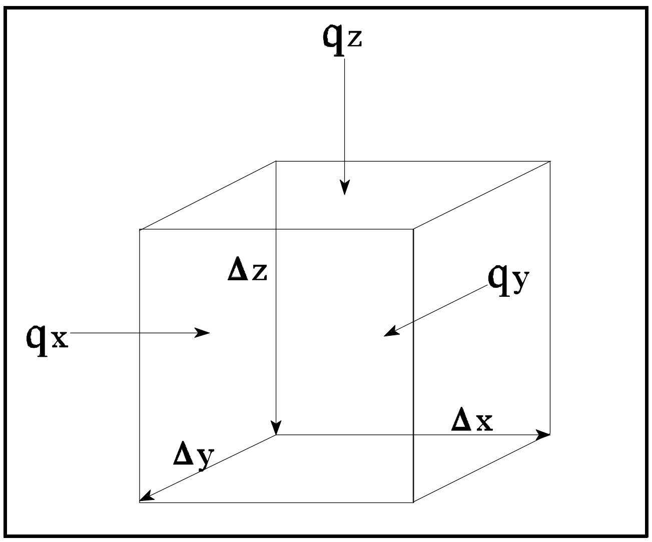


**Figure 2-8. Elemental control volume**

Integrating the conservation of mass (under constant density) with Darcy's Law, the general flow equation in three dimensions for a heterogeneous anisotropic material is derived:

$$S_s \frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) \quad (2\text{-}19)$$

Equation 2-19 is the general flow equation in three dimensions for a heterogeneous anisotropic material. Discharge (from a pumping well, etc.) or recharge to or from the control volume is represented as volumetric flux per unit volume ($L^3/T/L^3 = 1/T$):

$$S_s \frac{\partial h}{\partial t} + W = \frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) \quad (2\text{-}20)$$

where

$W$ = volumetric flux per unit volume [1/T]

Assuming that the material is homogeneous, i.e. $K$ does not vary with position, Equation 2-19 can be written as:

$$S_s \frac{\partial h}{\partial t} = K_x \frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) + K_y \frac{\partial}{\partial y}\left(\frac{\partial h}{\partial y}\right) + K_z \frac{\partial}{\partial z}\left(\frac{\partial h}{\partial z}\right) \quad (2\text{-}21)$$

If the material is both homogeneous and isotropic, i.e. $K_x = K_y = K_z$, then Equation 2-21 becomes:

$$S_s \frac{\partial h}{\partial t} = K\left[\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial h}{\partial z}\right)\right]$$

or, combining partial derivatives:

$$S_s \frac{\partial h}{\partial t} = K\left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right] \quad (2\text{-}22)$$

Using the definitions for storage coefficient, ($S = bS_s$), and transmissivity, ($T = Kb$), where $b$ is the aquifer thickness, Equation 2-22 becomes:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T}\frac{\partial h}{\partial t} \quad (2\text{-}23)$$

If the flow is steady-state, the hydraulic head does not vary with time and Equation 2-23 becomes:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (2\text{-}24)$$

Equation 2-24 is known as the *Laplace equation*.

*b. Unconfined aquifer*. In an unconfined aquifer, the saturated thickness of the aquifer changes with time as the hydraulic head changes. Therefore, the ability of the aquifer to transmit water (the transmissivity) is not constant:

$$\frac{\partial}{\partial x}\left(K_x h \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y h \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z h \frac{\partial h}{\partial z}\right) = S_y \frac{\partial h}{\partial t} \quad (2\text{-}25)$$

where

$S_y$ = specific yield [dimensionless]

For a homogeneous, isotropic aquifer, the general equation governing unconfined flow is known as the *Boussinesq equation* and is given by:

$$\frac{\partial}{\partial x}\left(h \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(h \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(h \frac{\partial h}{\partial z}\right) = \frac{S_y}{K}\frac{\partial h}{\partial t} \quad (2\text{-}26)$$

If the change in the elevation of the water table is small in comparison to the saturated thickness of the aquifer, the variable thickness $h$ can be replaced with an average thickness $b$ that is assumed to be constant over the aquifer. Equation 2-26 can then be linearized to the form:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S_y}{Kb}\frac{\partial h}{\partial t} \qquad (2\text{-}27)$$

## 2-17. Aquifer Diffusivity

Aquifer diffusivity is a term commonly used in surface water/groundwater interaction and is defined as the ratio of transmissivity to storage coefficient (T/S). Equation 2-27 can be written as:

$$\frac{T}{S}\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right) = \frac{\partial h}{\partial t} \qquad (2\text{-}28)$$

where

$S$ = storage coefficient [dimensionless]

Equation 2-28 demonstrates the direct relationship between the promulgation of a groundwater flood wave (and pressure wave) and aquifer diffusivity. Equation 2-28 is applicable to homogeneous, isotropic aquifers under either confined or unconfined (where the change in aquifer thickness is insignificant) conditions.

## 2-18. Flow Lines and Flow Nets

*a. Definition.* Two-dimensional, steady flow which can be described by the Laplace equation (Equation 2-24) can be solved by a graphical construction of a flow net. A flow net is a network of curves called streamlines and equipotential lines. A streamline is an imaginary line that traces the path that a particle of groundwater would follow as it flows through an aquifer. In an isotropic aquifer, streamlines are perpendicular to equipotential lines. If there is anisotropy in the plane of flow, then the streamlines will cross the equipotential lines at an angle dictated by the degree of anisotropy. An equipotential line represents locations of equal potentiometric head (Section 2-6). A flow net is a family of equipotential lines with sufficient orthogonal flow lines drawn so that a pattern of square figures (or elements) results. While different elements may be different in size, the change in flow and change in hydraulic head is the same for all elements. Except in cases of the most simple geometry, the figures will not truly be squares.

*b. Boundary conditions.*

(1) All boundary conditions of the flow domain must be known prior to the construction of the flow net. Three types of boundary conditions are possible: a no-flow boundary, a constant-head boundary, and a water-table boundary. Along a no-flow boundary streamlines will run parallel, and equipotential lines will intersect the boundary at right angles. A constant-head boundary (such as large lake) represents an equipotential line and streamlines will intersect at a right angle while adjacent equipotential lines will run parallel to the boundary.

(2) A flow net is presented in Figure 2-9 for the two-dimensional, steady-state flow in a homogeneous, isotropic aquifer between two reservoirs with different hydraulic heads.

*c. Analysis of results.* The completed flow net can be used to determine the quantity of water flowing through the domain. For the system shown in Figure 2-9, the flow per unit thickness in one element is:

$$q_1 = K \Delta z \frac{\Delta h}{\Delta x} \qquad (2\text{-}29)$$

But Δx = Δz, so Equation 2-29 becomes:

$$q_1 = K \Delta h = K \frac{\Delta h_T}{N_d} \qquad (2\text{-}30)$$

where

$N_d$ = the number of equipotential drops

The total flow per unit thickness is equal to the sum of the flow through each flow tube:

$$q_T = q_1 + q_2 \qquad (2\text{-}31)$$

Since the flow through each flow tube is equal, $q_1 = q_2$ and:

$$q_T = N_f q_1 \qquad (2\text{-}32)$$

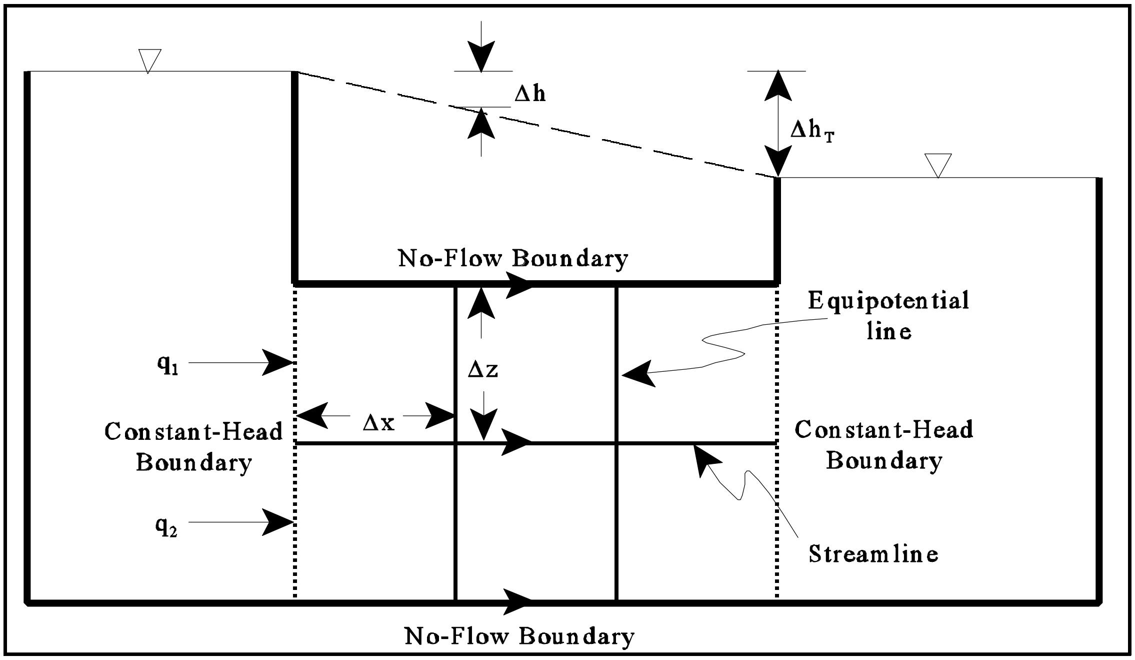


**Figure 2-9. Basic flow net**

where

$N_f$ = the number of flow tubes in a domain

Substituting Equation 2-30 into Equation 2-32 gives:

$$q_T = K \Delta h_T \frac{N_f}{N_d} \tag{2-33}$$

Thus, the flow per unit thickness, as well as total flow, through this simplified system can be derived as a function of hydraulic gradient (K), the drop in hydraulic head ($\Delta h_t$), and the number of equipotential drops ($N_d$) and flow tubes ($N_f$) in the domain.

## 2-19. Estimating Capture Zones of Pumping Wells

*a. General.* A capture zone consists of the upgradient and downgradient areas that will drain into a pumping well. If the water table (or potentiometric surface) is flat, the capture zone is circular. However, in most cases the water table (or potentiometric surface) is sloping. Calculating capture zones of wells aids in the design of pump-and-treat groundwater remediation systems, and well-head protection zones. Figure 2-10 illustrates a typical capture zone. This section presents analytical methods for estimating the capture zones of pumping wells under steady-state conditions. Steady-state conditions are approximated after the well has been pumping for some time, and the change in water levels (with time) in the well and its zone of influence are judged insignificant. Assumptions included in the methods presented are as follows:

(1) The aquifer is homogeneous, isotropic, and infinite in horizontal extent.

(2) Uniform flow (steady-state) conditions prevail.

(3) A confined aquifer has uniform transmissivity and no leakage.

(4) An unconfined aquifer has a horizontal lower confining layer with no leakage and no recharge from precipitation.

(5) Vertical gradients are negligible.

(6) The well is screened through the full saturated thickness of the aquifer and pumps at a constant rate.

*b. Confined steady-state flow.* Assume the well in Figure 2-10 is located at the origin (0,0) of the x,y

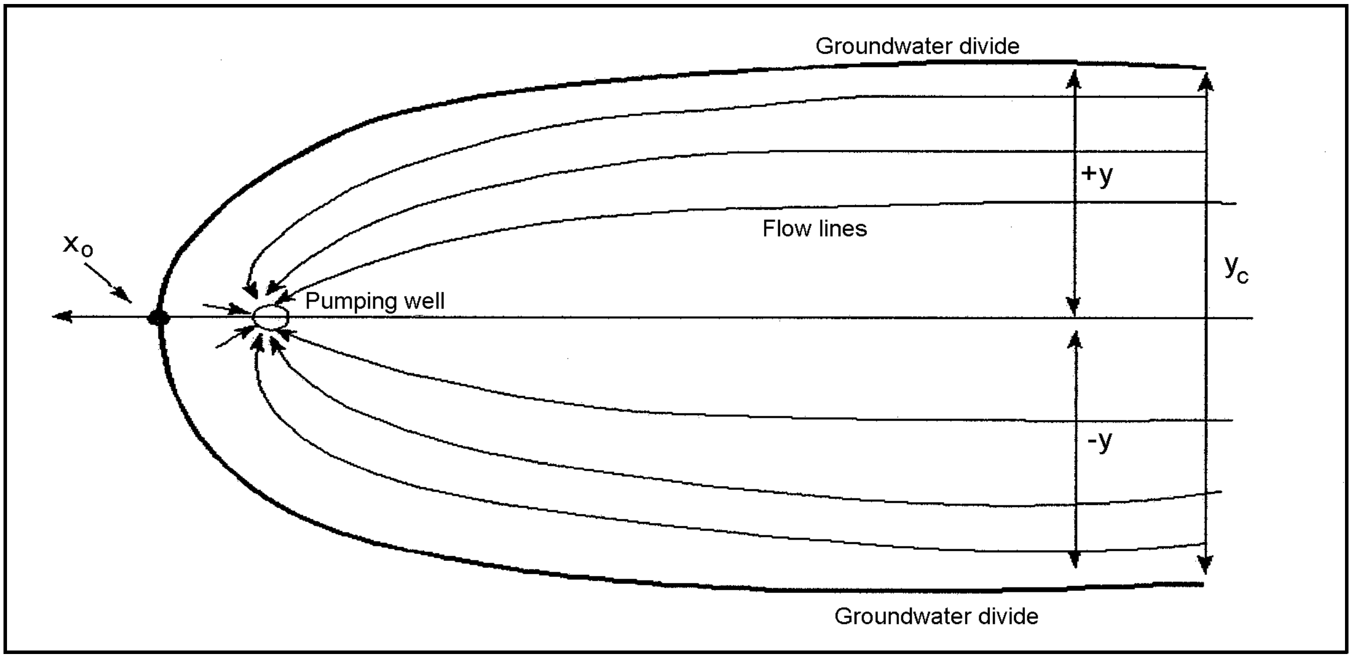


**Figure 2-10. Capture zone of a pumping well in plan view. The well is located at the origin (0,0) of the x,y plane**

plane. The equation to describe the edge of the capture zone (groundwater divide) for a confined aquifer when steady-state conditions have been reached is (Grubb 1993):

$$x = \frac{-y}{\tan(2\pi Kbiy/Q)} \qquad (2\text{-}34)$$

where

$Q$ = pumping rate [$L^3$/T]

$K$ = hydraulic conductivity [L/T]

$b$ = aquifer thickness [L]

$i$ = hydraulic gradient of the flow field in the absence of the pumping well (dh/dx) and tan (*) is in radians.

The distance from the pumping well downstream to the stagnation point that marks the end of the capture zone is given by:

$$x_0 = -Q/(2\pi Kbi) \qquad (2\text{-}35)$$

The maximum width of the capture zone as the distance (x) upgradient from the pumping well approaches infinity is given by (Todd 1980):

$$y_c = Q/Kbi \qquad (2\text{-}36)$$

*c. Unconfined steady-state flow.* Assume the well in Figure 2-10 is located at the origin (0,0) of the x,y plane. The equation to describe the edge of the capture zone (groundwater divide) for an unconfined aquifer when steady-state conditions have been reached is (Grubb 1993):

$$x = \frac{-y}{\tan[\pi K(h_1^2 - h_2^2)y/QL]} \qquad (2\text{-}37)$$

where

$Q$ = pumping rate [$L^3$/T]

$K$ = hydraulic conductivity [L/T]

$h_1$ = upgradient head above lower boundary of aquifer prior to pumping

$h_2$ = downgradient head above lower boundary of aquifer prior to pumping

$L$ = distance between $h_1$ and $h_2$ and tan (*) is in radians.

The distance from the pumping well downstream to the stagnation point that marks the end of the capture zone is given by:

$$x_0 = -QL/(\pi K(h_1^2 - h_2^2)) \qquad (2\text{-}38)$$

The maximum width of the capture zone as the distance (x) upgradient from the pumping well approaches infinity is given by:

$$y_c = 2(QL)/(K(h_1^2 - h_2^2)) \qquad (2\text{-}39)$$

*d. Example problem.*

(1) Background. City planners are concerned about the potential contaminants from a toxic spill to contaminate the municipal water supply. The municipal well (which pumps at 19,250 $m^3$/day) is screened in a confined aquifer located 30 m to 80 m below the surface. Aquifer materials consist of coarse sands with a hydraulic conductivity of about 80 m/day. The well has been pumping for several years and conditions approach steady state. Groundwater flow in the aquifer trends toward the north. The potentiometric surface of the aquifer (measured before pumping commenced) drops approximately 1 m for every 200 m. The location of the well relative to the toxic spill is illustrated by Figure 2-11. Estimate the spatial extent of the capture zone of the pumping well.

(2) Solution. We are given:

the aquifer is confined
$K$ = 80 m/day
$b$ = 50 m
$i$ = 1/200 = 0.005
$Q$ = 19,250 $m^3$/day

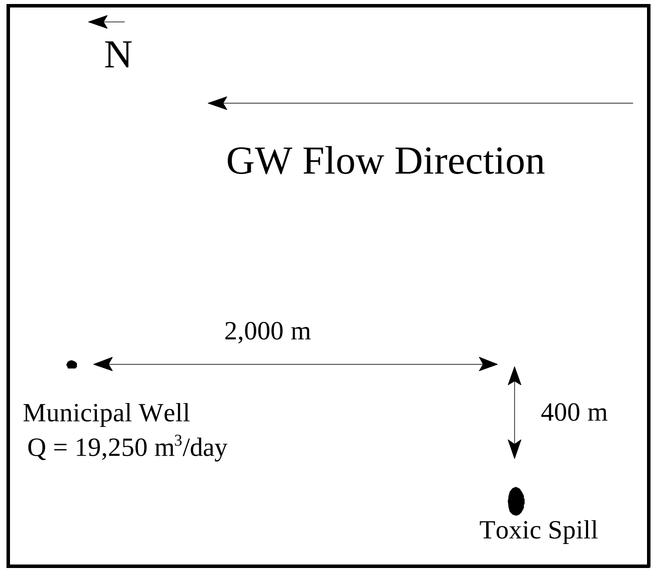


**Figure 2-11. Location of toxic spill relative to pumping well**

- Find maximum extent of capture zone:

  $y_c = Q/Kbi = 19250/(80)(50)(0.005) = 960$ m or ± 480 m from the x axis.

- Find location of stagnation point:

  $x_0 = - y_c/2\pi = -960/2\pi = - 150$ m

- Delineate boundary of the capture zone:

$$x = \frac{-y}{\tan(2\pi Kbiy/Q)} = -y/\tan(0.0065y)$$

| ±y | x |
|---|---|
| 1 | -150 |
| 100 | -130 |
| 200 | -50 |
| 300 | 120 |
| 400 | 670 |
| 450 | 2,045 |
| 480 | 24,000 |

(3) Analysis of results. The capture zone at a distance x = 2,000 m from the pumping well extends 450 m from the horizontal (x) axis. Therefore, initial calculations indicate that a portion of the contaminant plume will contaminate the municipal water supply unless mitigative measures are taken. Further investigation is warranted.

## 2-20. Specialized Flow Conditions

*a. General.* Darcy's Law (Section 2-11) is an empirical relationship which is only valid under the assumption of constant density and laminar flow. These assumptions are not always met in nature. Flow conditions in which Darcy's Law is not necessarily applicable are cited below.

(1) Fractured flow. Fractured-rock aquifers occur in environments in which the flow of water is primarily through fractures, joints, faults, or bedding planes which have not been significantly enlarged by dissolution. Fracturing adds secondary porosity to a soil medium that already has some original porosity. The original porosity consists of pores that are roughly similar in length and width. These pores interconnect to form a tortuous water network for groundwater flow. Fractured porosity is significantly different. The fractures consist of pathways that are much greater in length than width. These pathways provide conduits for groundwater flow that are much less tortuous than the original porosity. At a local scale, fractured rock can be extremely heterogeneous. Effective permeability of crystalline rock typically decreases by two or three orders of magnitude in the first thousand feet below ground surface, as the number of fractures decrease or close under increased lithostatic load (Smith and Wheatcraft 1992).

(2) Karst aquifers. Karst aquifers occur in environments where all or most of the flow of water is through joints, faults, bedding planes, pores, cavities, conduits, and caves, any or all of which have been significantly enlarged by dissolution. Effective porosity in karst environments is mostly tertiary, where secondary porosity is modified by dissolution through pores, bedding planes, fractures, conduits, and caves. Karst aquifers are generally highly anisotropic and heterogeneous. Flow in karst aquifers is often fast and turbulent where Darcy's law rarely applies. Solution channels leading to high permeability are favored in areas where topographic, bedding, or jointing features promote flow localization which focuses the solvent action of circulating groundwater, or well-connected pathways exist between recharge and discharge zones, favoring higher groundwater velocities (Smith and Wheatcraft 1992).

(3) Permafrost. Temperatures significantly below 0° C are required to produce permafrost. The depth and location of frozen water within the soil depends upon many factors such as fluid pressure, salt content of the pore water, the grain size distribution of the soil, soil mineralogy, and the soil structure. The presence of frozen or partially frozen groundwater has a tremendous effect upon flow. As water freezes, it expands to fill pore spaces. Soil that normally conveys water easily becomes an aquitard or aquiclude when frozen. The flow of water in permafrost regions is analogous to fractured flow environments where flow is confined to conduits in which complete freezing has not taken place.

(4) Variable-density flow. Unlike aquifers containing constant-density water, where flow is controlled only by the hydraulic head gradient and the hydraulic conductivity, variable-density flow is also affected by change in the spatial location within the aquifer. Water density is commonly affected by temperature, or total dissolved solids. As water temperature increases, its density decreases. A temperature gradient across an area influences the measurements of hydraulic head and the corresponding hydraulic gradient. Intrinsic hydraulic conductivity is also a function of water temperature (Equation 2-6). Thus, it is important to assess effects of fluid density on hydraulic gradient and hydraulic gradient in all site investigations.

(5) Saltwater intrusion. Due to different concentrations of dissolved solids, the density of the saline water is greater than the density of fresh water. In aquifers hydraulically connected to the ocean, a significant density difference occurs which can discourage mixing of waters and result in an interface between salt water and sea water. The depth of this interface can be estimated by the Ghyben-Herzberg relationship (Figure 2-12, Equation 2-40).

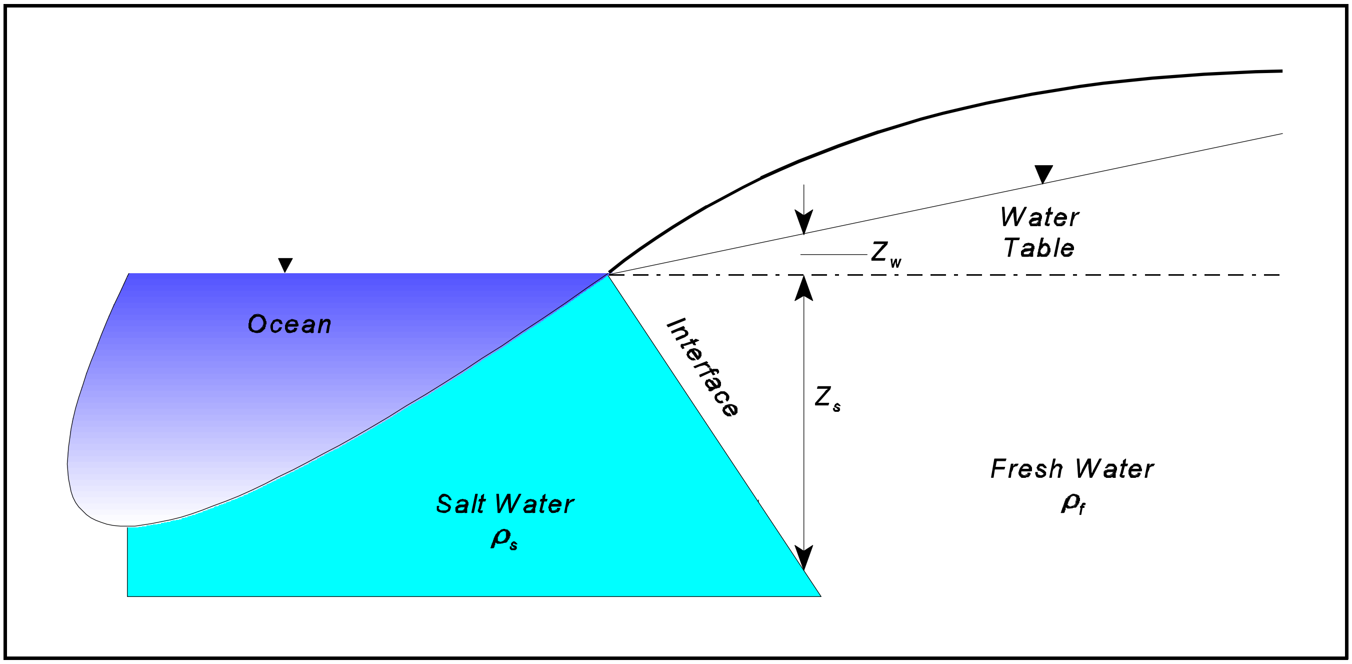


**Figure 2-12. Saltwater-freshwater interface in an unconfined coastal aquifer**

$$z_s = \frac{\rho_f}{\rho_s - \rho_f} z_w \qquad (2\text{-}40)$$

where

$z_s$ = depth of interface below sea level

$z_w$ = elevation of water table above sea level

$\rho_s$ = saltwater density

$\rho_w$ = freshwater density

For the common values of $\rho_w$ = 1.0 and $\rho_s$ = 1.25,

$$z_s = 40 z_w \qquad (2\text{-}41)$$

The Ghyben-Herzberg relationship assumes hydrostatic conditions in a homogeneous, unconfined aquifer. Additionally, it assumes a sharp interface between fresh water and salt water. In reality, there tends to be a mixing of salt water and fresh water in a zone of diffusion around the interface. If the aquifer is subject to hydraulic head fluctuations caused by tides, the zone of mixed water will be enlarged.