PDHonline Course H141 (1 PDH)

Hydraulic Calculations Using Mathcad - Part 1: Simple Pipelines

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COURSE CONTENT

INTRODUCTION

The purpose of this course is to introduce the student the use of Mathcad for solving simple pipeline problems quickly and efficiently. Mathcad is unique in its free form input style. Because of the ease with which Mathcad documents can be understood it is currently one of the most popular applications for doing engineering calculations on the market.   In today's economy any tool which improves the accuracy and efficiency of engineering calculations will improve a company's bottom line. In this course we will illustrate how to solve a simple pipeline problem using a Mathcad solve block. Along the way suggestions will be provided to help the user create the most efficient and useful Mathcad files.

PIPELINE HYDRAULICS

Pipelines are a common sight in the world today, carrying everything from waste water to oil. The need to determine the appropriate size of a pipeline to carry a specific discharge or to compute the discharge from a pipeline is often one of the first assignments given to a new engineering graduate. The solution to such problems often involves one or more of the equations listed below.

The energy equation:
$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2 \cdot g} - h_L - h_{minor} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2 \cdot g}$$

The Reynolds number equation:
$$N_R = \frac{\rho \cdot D \cdot v}{\mu}$$

The Colebrook White Equation:
$$\frac{1}{\sqrt{f}} = -2 \cdot \log\left(\frac{\frac{\varepsilon}{D}}{3.7} + \frac{2.51}{N_R \cdot \sqrt{f}}\right)$$

The continuity equation: $Q = A_1 \cdot v_1 = A_2 \cdot v_2$

The Darcy Weisbach Equation: $h_L = f \cdot \dfrac{L}{D} \cdot \dfrac{v^2}{2 \cdot g}$ used to compute pipeline friction loss

Equations for various minor losses (valves, bends, diffusers, confusors, generally of the form: $h_{minor} = K \cdot \dfrac{v^2}{2 \cdot g}$

All of the equations above are in Mathcad.   The equals sign should be noted. Mathcad has several different forms of equals signs depending on what operation is being carried out. This form of equals sign is obtained by clicking on view =====> toolbars ======> Boolean and clicking on the equals sign in the upper left corner. This form of equals is used when simply typing equations as part of an explanation as done here or when doing certain symbolic or numerical calculations as you will see later.   The remaining variables are defined below:

p_1 **is the pressure at point 1 in the system**

p_2 **is the pressure at point 2 in the system**

γ **is the unit weight of water**

z_1 **and** z_2 **are the elevation of point 1 and point 2 relative to a user chosen datum**

L is the pipe length

D is the pipe diameter

ρ **is the density of water**

f is the Darcy Weisbach friction factor

v_1 **and** v_2 **are the velocities at points 1 and 2**

K is a minor loss coefficient

ε **the equivalent roughness of the pipe material**

THE SYSTEM

In order to understand how we apply these equations using Mathcad we must first define a "system" . The system I have chosen is shown below. We want to know the pressure at point 2 for a specified flow rate. However to do this we will need to find the Darcy Weisbach friction factor and the Reynolds number as well.
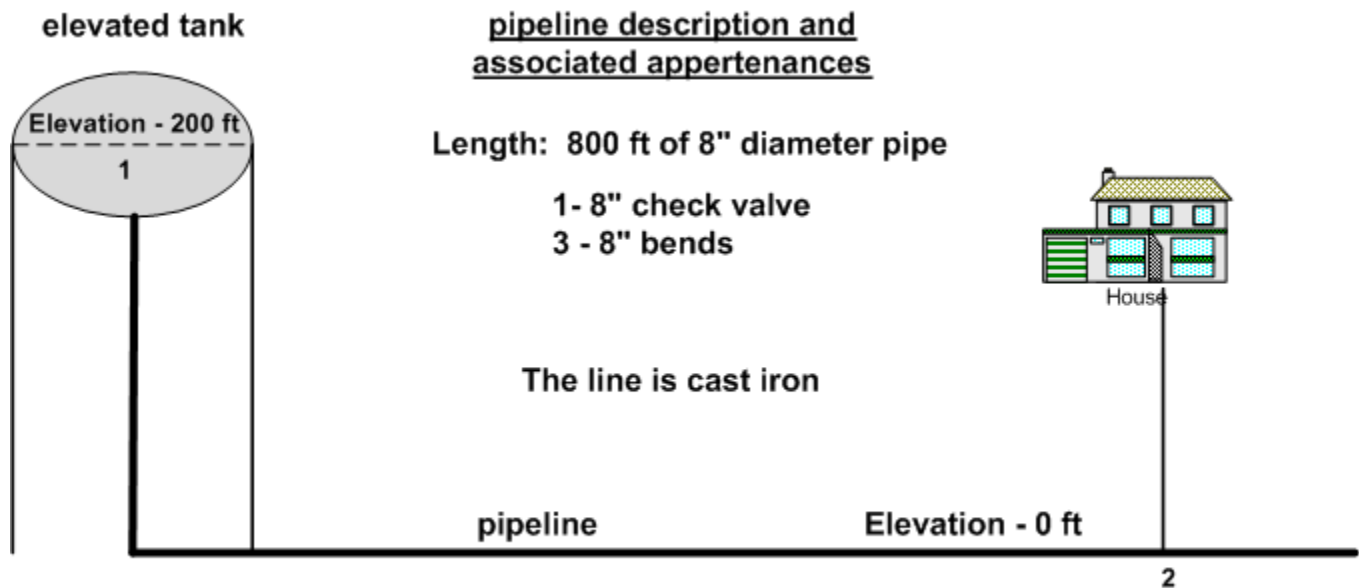
elevated tank

pipeline description and associated appertenances

Elevation - 200 ft

1

Length: 800 ft of 8" diameter pipe

1- 8" check valve
3 - 8" bends

House

The line is cast iron

pipeline   Elevation - 0 ft

2

Figure 1 – The System

A SIMPLE PROBLEM

The schematic above shows the hydraulic components required to convey water from an elevated tank to a point in an 8 inch diameter pipeline.   Now, if you examine most current textbooks and design manuals problems such as these are often solved used some type of manual, trial and error approach involving the Moody diagram. Such procedures are tedious, time consuming, and prone to error.   That is not the case here. Our approach here will be to first determine the "unknowns", that is what we need to determine in order to solve the problem, obviously we want the pressure p at the residence for a specified

flow rate, $Q := 1200 \cdot \dfrac{gal}{min}$. However, a little study shows that, in addition, we will need to

find the velocity in the pipe, Darcy Weisbach friction factor for the pipe, and the Reynolds Number. Thus we will need to solve three independent equations containing the variables simultaneously. One of the equations is nonlinear and one, the Colebrook White Equation, is *transcendental* meaning that f appears on both sides of the equation and cannot be isolated. This would be a daunting problem to solve manually, here we will use a *Mathcad Solve Block.* A Mathcad solve block is a numerical algorithm designed to solve multiple equations for multiple unknowns. It can use one of several user chosen numerical methods to do this however, the Levenberg Marquardt algorithm is usually the most robust. How to select the numerical method to use will be described below.

KNOWN VALUES AND A WORD ABOUT UNITS

Thus far we have not said much about the use of units in Mathcad. Units and the errors arising from them cause numerous mistakes when manually solving even simple engineering problems. This results in lost time, money, and even lawsuits in extreme cases. Excel is unit ignorant and offers little help. Mathcad however is capable of utilizing several different unit systems and *enforces dimensional consistency thus all but eliminating unit conversions and errors resulting from units.* Below we enter the know values for this problem with units.

The elevation of point 1, the water surface in the elevated tank: $z_1 := 200 \cdot ft$. Here I have

attached units of feet however Mathcad does all needed unit conversions so if I want this elevation in meters I simply type $z_1 = 200 \cdot ft$ backspace over ft, and type "m", the

Mathcad unit for meters, $z_1 = 60.96 \, m$ Mathcad has done the unit conversion. If

dimensional consistency is not present Mathcad will issue an error to that effect. To see the possible unit systems click Tools =====> Worksheet options ======> Unit System. To see the available units in Mathcad click on Insert =======> Unit

Known values with units:

I have found that maximum computational efficiency results if every known value is assigned a variable name early in the file and to the maximum extent possible have no numerical values in the file. This avoids having to search for each occurrence of a value in case of an error or if a change in a problem parameter is necessary or desirable. (suppose you had worked this problem manually, then were told to change the pipe diameter !)

pressure at point 1 : $p_1 := 0 \cdot psi$ **velocity at point 1:** $v_1 := 0 \cdot \dfrac{ft}{sec}$

elevations : $z_1 := 200 ft$, $z_2 := 0 \cdot ft$

properties of water: unit weight $\gamma := 62.4 \cdot \dfrac{lbf}{ft^3}$, density $\rho := \dfrac{\gamma}{g}$

absolute viscosity $\mu := 2.735 \cdot 10^{-5} \cdot \dfrac{lbf \cdot sec}{ft^2}$

pipeline length : , $L_8 := 1200 \cdot ft$

Loss coefficients : , $K_{check} := 10$, $K_{bend} := 1.5$

equivalent roughness of pipe materials : $\varepsilon := 0.00085 \cdot ft$,

pipe diameters: $D_8 := 8 \cdot in$

flow rate : $Q := 1200 \cdot \dfrac{gal}{min}$

The "relative roughness" is defined as $\dfrac{\varepsilon}{D_8} = 1.275 \times 10^{-3}$

Given the pipe diameter we can compute its area: $A_8 := \dfrac{\pi \cdot D_8^{\,2}}{4}$, **and the velocity**

$$v_8 := \dfrac{Q}{A_8} = 7.659 \cdot \dfrac{ft}{sec}$$

THE STRUCTURE OF A SOLVE BLOCK

A solve block starts with the word "Given" typed in Math mode (the cursor is blue, it's red in text mode). The equations to be solved are typed below "Given" using the symbolic equals sign (cntl =). The number of equations must match the number of unknowns. Descriptive text can be placed inside the solve block as well. Our unknowns are the pressure at point 2, the D-W friction factor and the Reynolds number.

SOLVE BLOCK STARTS HERE

Given   IMPORTANT - "Given" must be typed in Math mode

INITIAL GUESSES:

A Mathcad solve block requires an initial guess for each of the unknowns being sought, these are supplied below

$$p_2 := 1 \cdot psi \qquad f_8 := .03 \qquad N_R := 10^6$$

All that is important here is that the units be correct, they do not need to be in the same unit system, Mathcad handles all unit conversions.

The energy equation written between the water surface in the elevated tank and the point in the 8" pipe at which the pressure and flow are desired. The line friction loss and minor losses are included, but no pump term is included because there is no pump in the system.

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2 \cdot g} - f_8 \cdot \frac{L_8}{D_8} \cdot \frac{v_8^2}{2 \cdot g} - K_{check} \cdot \frac{v_8^2}{2g} - 3 \cdot K_{bend} \cdot \frac{v_8^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_8^2}{2 \cdot g}$$

The Colebrook - White equation is the mathematical equivalent of the Moody Diagram, note that it is transcendental in nature.

$$\frac{1}{\sqrt{f_8}} = -2 \cdot \log\left[\frac{\left(\frac{\varepsilon}{D_8}\right)}{3.7} + \frac{2.51}{N_R \cdot \sqrt{f_8}} \right]$$

$$\text{The Reynolds Number } N_R = \frac{\rho \cdot D_8 \cdot v_8}{\mu}$$

The solution set to this problem is written as a "vector" containing the three unknowns we are looking for. This is accomplished by clicking on Insert ======> matrix and then filling in the desired number of rows and columns, in this case 3 rows, 1 column

The variable names holding the solution values (on the left) should be different than the variable names of the actual unknowns.

$$\begin{pmatrix} p_{2soln} \\ f_{8soln} \\ N_{Rsoln} \end{pmatrix} := Find\left(p_2, f_8, N_R\right)$$

In order to select the numerical search algorithm to be used to search for the solution click on the word "Find" on the right hand side of the statement above: Find ======> nonlinear. The user then has the choice of clicking on the Levenberg-Marquardt, conjugate-gradient, or quasi-Newton methods. I have found the Levenberg-Marquardt to be the most robust at finding solutions.

The solution values are shown below, because this is a numerical procedure there is no absolute guarantee they are correct as would be the case if an analytical solution had been possible. For this reason it is desirable substitute the solution values into the governing equations and verify that an identity results.

$$p_{2soln} = 65.227 \cdot psi \qquad f_{8soln} = 0.022 \qquad N_{Rsoln} = 3.621 \times 10^5$$

CHECK THE GOVERNING EQUATIONS

In doing this I usually use what I call the "ratio form" of an identity. That is, I set the left side over the right side which will result in 1.0 if solution values are correct.

$$\frac{\dfrac{p_1}{\gamma} + z_1 + \dfrac{v_1^2}{2 \cdot g} - f_{8soln} \cdot \dfrac{L_8}{D_8} \cdot \dfrac{v_8^2}{2 \cdot g} - K_{check} \cdot \dfrac{v_8^2}{2g} - 3 \cdot K_{bend} \cdot \dfrac{v_8^2}{2g}}{\dfrac{p_{2soln}}{\gamma} + z_2 + \dfrac{v_8^2}{2 \cdot g}} = 1$$

$$\frac{\dfrac{1}{\sqrt{f_{8soln}}}}{\left(-2 \cdot \log\left(\dfrac{\dfrac{\varepsilon}{D_8}}{3.7} + \dfrac{2.51}{N_{Rsoln} \cdot \sqrt{f_{8soln}}}\right)\right)} = 1 \qquad\qquad \frac{N_{Rsoln}}{\left(\dfrac{\rho \cdot D_8 \cdot v_8}{\mu}\right)} = 1$$

FURTHER EXAMINATION OF THE SOLUTION

$$Q := A_8 \cdot v_8 = 1.2 \times 10^3 \cdot \frac{gal}{min}$$

In the problem the resulting flow rate is $Q = 1.2 \times 10^3 \cdot \dfrac{gal}{min}$. We know that if the flow rate

is zero the static line pressure will be: $p_{static} := \gamma \cdot z_1 = 86.667 \cdot psi$. The pressure at

point 2 is $p_{2soln} = 65.227 \cdot psi$. Therefore the pipeline losses and minor losses should

approximately equal the difference $p_{static} - p_{2soln} = 21.44 \cdot psi$ as shown below.

The computed sum of the pipe friction loss and minor losses equals:

$$Energy\_loss := \left(f_{8soln} \cdot \frac{L_8}{D_8} \cdot \frac{v_8^2}{2 \cdot g} + K_{check} \cdot \frac{v_8^2}{2g} + 3 \cdot K_{bend} \cdot \frac{v_8^2}{2g} \right) \cdot \gamma = 21.045 \cdot psi$$

This results in a loss rate of $\dfrac{Energy\_loss}{L_8} = 0.018 \cdot \dfrac{psi}{ft}$

Also, we can demonstrate that the minor losses are not so minor after all, they represent nearly 30% of the total system loss:

$$\left(K_{check} \cdot \frac{v_8^2}{2g} + 3 \cdot K_{bend} \cdot \frac{v_8^2}{2g} \right) \cdot \gamma = 5.728 \cdot psi$$

The solution value for the Darcy Weisbach friction falls in the transition zone of the Moody diagram implying that f depends on BOTH the relative roughness and the Reynolds number. If we had assumed *wholly turbulent flow* in which case f would depend only on the relative roughness our resulting value of f would have been underestimated by a small amount. In a number of texts, particularly early ones the assumption of wholly turbulent flow was often made in order to simplify calculations. However, given the widespread use of PVC and other plastic materials for pipe this practice is not advised since these materials often have relative roughness values that routinely place the flow in the transition zone. This is illustrated in the problem in Appendix 1.
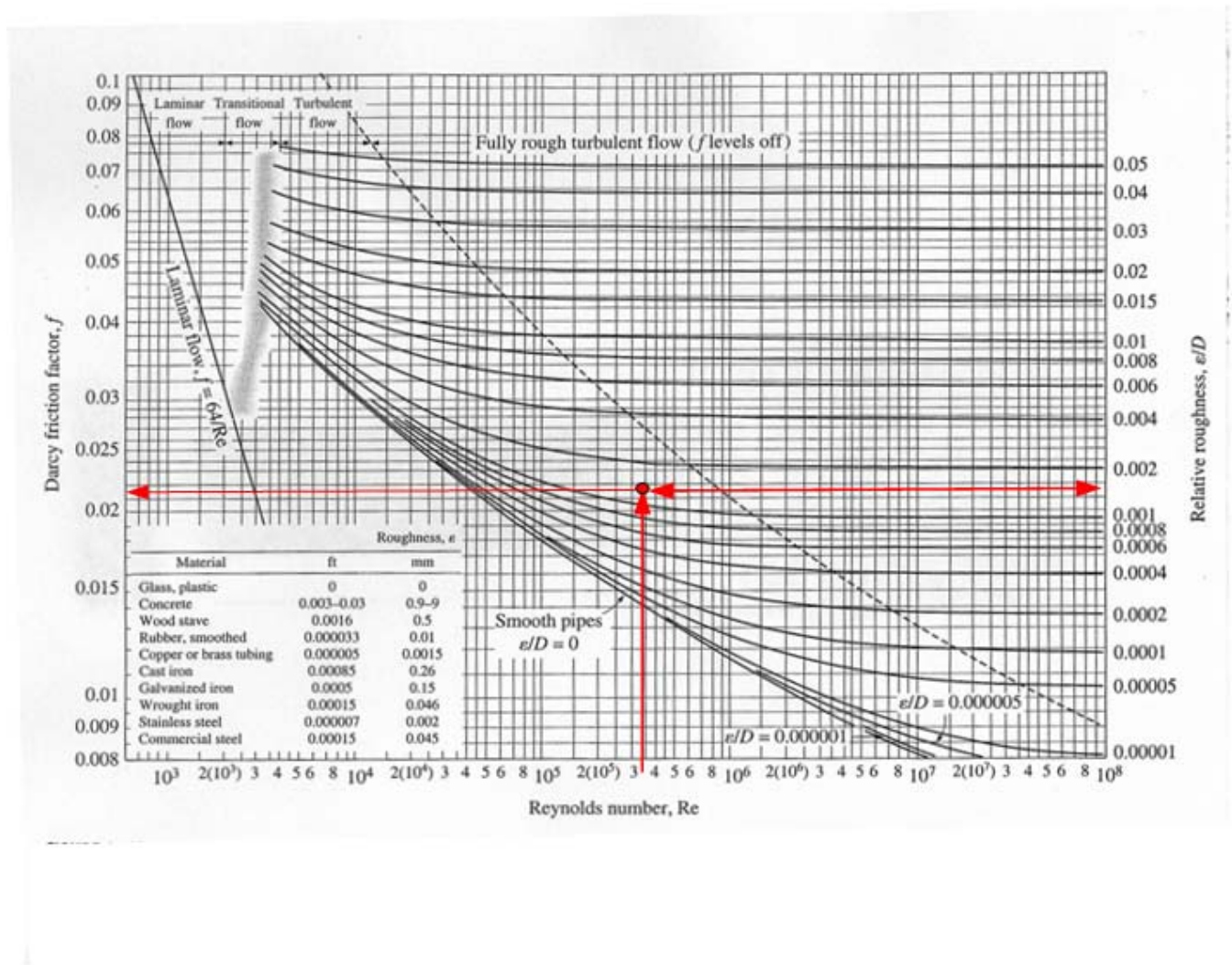
MOODY DIAGRAM

Extending Mathcad

At this point a number of "what if " scenarios can be tried by simply changing any one of the parameter values. Examples of such changes might be:

      1. change the liquid viscosity to examine the effect of temperature

   2. change the pipe diameter, length, pipe material (equivalent roughness)

3. **change the tank elevation**

4. **Change the flow rate**

5. **Change the pressure in the tank (to simulate a hydro pneumatic
    pressure tank, for example)**

**Anyone that has completed this course material and has an elementary knowledge of
pipeline hydraulics can see that the solution technique used here can easily be extrapolated
to multiple pipe sizes in series or parallel. In addition, pumps can be incorporated in the
system if an equation for the pump characteristic is known or can be developed from data.
Such systems may be explored in future courses.**

Appendix 1

The effect of the wholly turbulent flow assumption on pipe discharge

PROBLEM: Find the water flow for the following system.
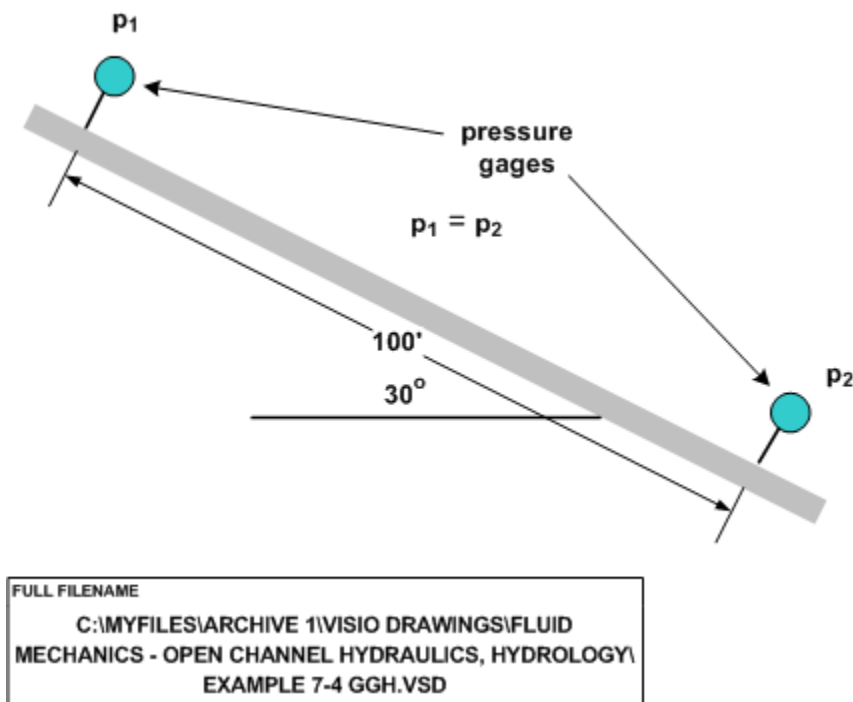
Water flow steadily down the inclined, 3/4 in diameter, copper pipe shown in the Figure. Two pressure gages 100' apart indicate <u>identical pressures</u>. Find the volumetric flow rate. Work the problem assuming fully developed, turbulent flow as well as using a complete solution, without assuming complete turbulence. NOTE that the pressure readings are identical. This occurs because energy loss due to friction is exactly balanced by the change in elevation head. In the absence of a pump this could not occur if the pipe was horizontal. If the pipe were horizontal AND the pressure drop was zero there would be no flow. Note that we cannot predict this using precisely zero and the equations below because we end up with V = 0 which results in a zero in the denominator of one of the terms in the C-W equation.

<mark>KNOWN VALUES</mark>

$$\text{Length} := 100 \cdot \text{ft} \quad \varepsilon := .000005 \cdot \text{ft} \quad \nu := 1.22 \cdot 10^{-5} \cdot \frac{\text{ft}^2}{\text{sec}} \quad \theta := 30 \cdot \text{deg} \quad \gamma := 62.4 \cdot \frac{\text{lbf}}{\text{ft}^3}$$

$$D := .75 \cdot \text{in}$$

$$A := \frac{\pi \cdot D^2}{4} = 0.442 \, \text{in}^2$$

$$\text{relative roughness :} \quad \frac{\varepsilon}{D} = 8 \times 10^{-5}$$

vertical distance between gages: $\text{Length} \cdot \sin(\theta) = 50 \cdot \text{ft}$

Write Bernoulli's equation between the pressure gages:

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2 \cdot g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2 \cdot g}.$$

In doing so we see that the difference in elevation between the ends of the pipe apparently balances the friciton loss.

$$\frac{\Delta p}{\gamma} = (z_2 - z_1) + h_L = 0$$

$h_L := \text{Length} \cdot \sin(\theta)$ **from Bernoulli's equation below**

Complete Solution, no assumption of wholly turbulent flow

Now, since the pipe diameter remains constant and, according to the problem statement, the pressure readings are identical the head loss term becomes:

$$h_L = \left(z_1 - z_2\right) + \frac{\Delta p}{\gamma} = \text{Length} \cdot \sin(\theta) + \frac{\Delta p}{\gamma}.$$ That is the total energy loss is the sum of

the change in elevation and the friction loss within the pipe.

The friction loss is also obtainable using the Darcy Weisbach equation: $h_L = f \cdot \dfrac{L}{D} \cdot \dfrac{V^2}{2 \cdot g}$.

The immediate problem is that we don't know f or V so we can't get Q. We need more equations. We can make use of the Colebrook-White equation to get f:

$$f = \frac{0.25}{\left[\log\left[\dfrac{\left(\dfrac{\varepsilon}{D}\right)}{3.7} + \dfrac{2.51}{N_R \cdot \sqrt{f}}\right]\right]^2}$$ **Colebrook - White equation**

However, the Colebrook White equations presents 2 problems of its own - (1.) It's a transcendental equation and must be solved iteratively and (2.) It involves the Reynolds number which involves V, which we do not know.

In summary then have 3 independent equations and three unknowns, f, V and N_R. We can use a solve block to obtain a solution.

The headloss is NOT unknown, but obtained from the difference in elevation of the two ends of the pipe: $h_L = 50 \cdot \text{ft}$

initial guesses for unknowns: $f := .012$, $V := .3 \cdot \dfrac{\text{ft}}{\text{sec}}$ and $N_R := 10^6$

<mark>SOLVE BLOCK - 3 equations - 2 of them nonlinear, 1 of those transcendental</mark>

Given

$$h_L = f \cdot \frac{Length}{D} \cdot \frac{V^2}{2 \cdot g}$$ **Darcy Weisbach equation**

$$f = \frac{0.25}{\left[\log\left[\frac{\left(\frac{\varepsilon}{D}\right)}{3.7} + \frac{2.51}{N_R \cdot \sqrt{f}} \right] \right]^2}$$ **Colebrook White equation**

$$N_R = D \cdot \frac{V}{\nu}$$ **Reynolds number**

$$\begin{pmatrix} f_{soln} \\ V_{soln} \\ N_{Rsoln} \end{pmatrix} := Find(f, V, N_R)$$

RESULTS

$$f_{soln} = 0.0212 \quad N_{Rsoln} = 4.991 \times 10^4$$

$$V_{soln} = 2.97 \frac{m}{s} \qquad \frac{\varepsilon}{D} = 8 \times 10^{-5}$$

$$A := \frac{\pi \cdot D^2}{4} \qquad Q := A \cdot V_{soln}$$

$$Q = 13.41544 \frac{gal}{min}$$ **volumetric flow rate**

The volumetric flow rate varies with the angle of inclination as shown for zero pressure drop

inclination, degrees		flowrate, gpm	
.001 deg		0.029	
10 deg		7.4	
20 deg		10.8	
30 deg		13.4	
50 deg		17.02	
90 deg		19.7	

Assume Complete Turbulence

IF we assume completely turbulent flow then the value of the friction factor, f, depends only on the relative roughness of the pipe. This being the case, we obtain a value of .0125 for f and a corresponding value of $Q = 13.415 \cdot \dfrac{gal}{min}$ for the flowrate.

$f := .0125$ **from the Moody diagram, corresponding to relative roughness of the pipe**

$$h_L = f \cdot \frac{Length}{D} \cdot \frac{V^2}{2 \cdot g}$$

$$V := \frac{1}{f \cdot Length} \cdot 2^{\frac{1}{2}} \cdot \left(f \cdot Length \cdot h_L \cdot D \cdot g \right)^{\frac{1}{2}}$$
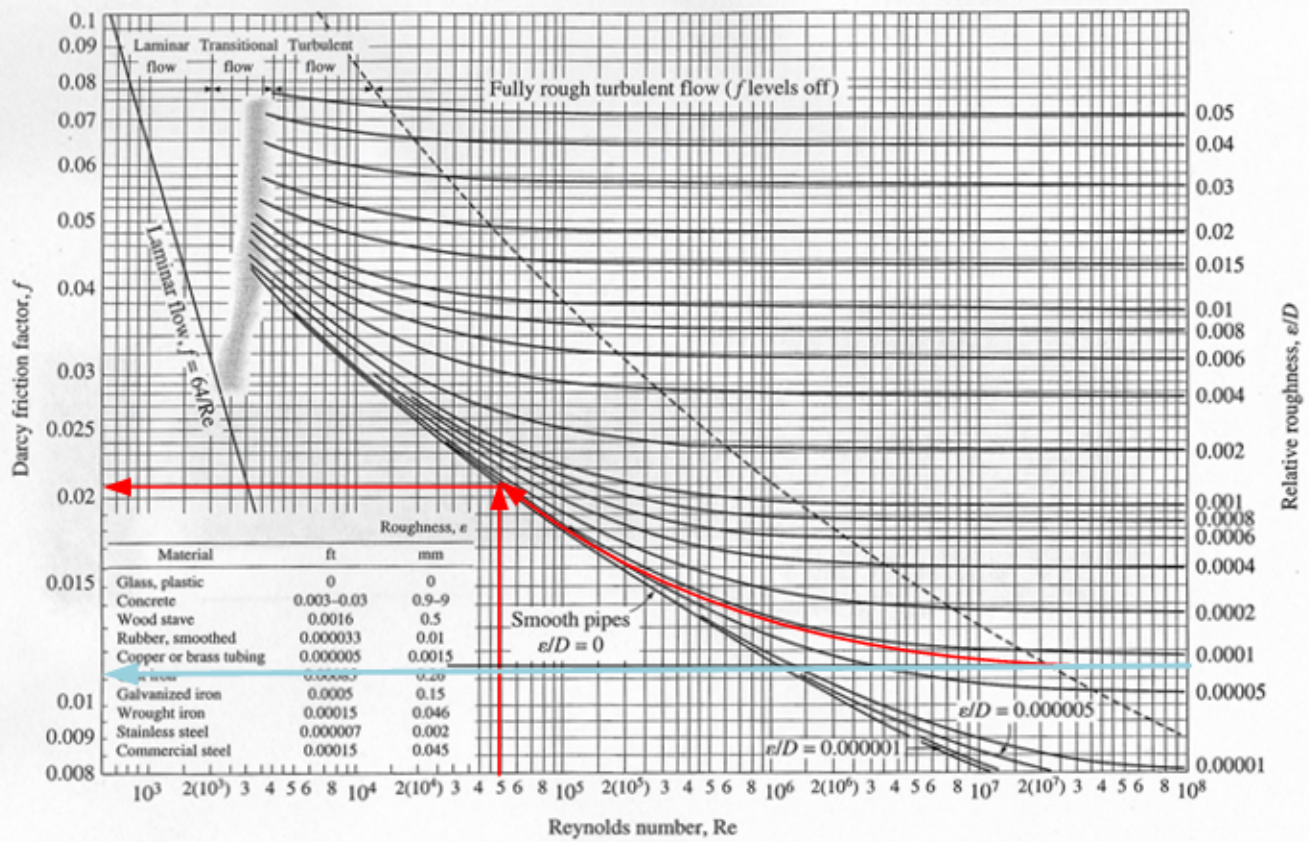
$$V = 12.683 \cdot \frac{ft}{sec}$$

$$Q_{turbulent} := A \cdot V \quad Q_{turbulent} = 17.465 \cdot \frac{gal}{min} \quad Q = 13.415 \cdot \frac{gal}{min}$$

$$\frac{Q_{turbulent}}{Q} = 1.302$$

The completely turbulent approach gives a flow about 30% higher than the complete solution

FULL FILENAME

C:\MYFILES\ARCHIVE 1\VISIO DRAWINGS\FLUID MECHANICS - OPEN CHANNEL HYDRAULICS, HYDROLOGY\MOODY DIAGRAM PDH COURSE.VSD