PDHonline Course H146 (4 PDH)

Hydraulic Engineering

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Hydraulic Engineering

Session 2

By

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Session Goals

- Understand the mass (continuity)Equation;
- Understand the Bernoulli’s Equation;
- Understand the Momentum Equation;
- Understand the energy Equation;
- Understand Open-channel flow;
- Design the hydraulic structures.

Continuity Equation

- Review
  - The mass equation is an expression of the conservation of mass principle;
  - The conservation of mass relation for a closed system undergoing a change is expressed as m_{sys} = const. or $dm_{sys}/dt = 0$, which is a statement of the obvious that the mass of the system remains constant during a process;
  - For a control volume (CV) or open system, mass balance is expressed in the rate form as

Conservation of mass: $$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt}$$

Continuity Equation

Conservation of mass: $$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{CV}}}{dt}$$

- where m_{in} and m_{out} are the total rates of mass flow into and out of the control volume, respectively, and dm_{CV}/dt is the rate of change of mass within the control volume boundaries;
- In fluid mechanics, the conservation of mass relation written for a differential control volume is usually called the continuity equation.

Continuity Equation

The **conservation of mass principle** for a control volume can be expressed as: The net mass transfer to or from a control volume during a time interval t is equal to the net change (increase or decrease) in the total mass within the control volume during time (t).

$$\begin{pmatrix}\text{Total mass entering}\\ \text{the CV during } \Delta t\end{pmatrix} - \begin{pmatrix}\text{Total mass leaving}\\ \text{the CV during } \Delta t\end{pmatrix} = \begin{pmatrix}\text{Net change in mass}\\ \text{within the CV during } \Delta t\end{pmatrix}$$

or

$$m_{in} - m_{out} = \Delta m_{CV} \qquad \text{(kg)}$$

The Equations above are often referred to as the **mass balance** and are applicable to any control volume undergoing any kind of process.

Steady Flow Process

During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant). Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.

Steady flow: $$\sum_{in} \dot{m} = \sum_{out} \dot{m} \qquad \text{(kg/s)}$$

When dealing with steady-flow processes, we are not interested in the amount of mass that flows in or out of a device over time; instead, we are interested in the amount of mass flowing per unit time, that is, the **mass flow rate**

The total rate of mass entering a control volume is equal to the total rate of mass leaving it

Steady Flow Process

- Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet);
- For these cases, we denote the inlet state by the subscript 1 and the outlet state by the subscript 2, and drop the summation signs

Steady flow (single stream): $\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$

Steady Flow Process

EXAMPLE:

Water Flow through a Garden Hose Nozzle A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit. If it takes 50 s to fill the bucket with water, determine (a) the volume and mass flow rates of water through the hose, and (b) the average velocity of water at the nozzle exit.

Assumptions:

1. Water is an incompressible substance;
2. Flow through the hose is steady;
3. There is no waste of water by splashing.

Properties: We take the density of water to be 1000 kg/m^3 1 kg/L.

Analysis: (a) Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are:

$$\dot{V} = \frac{V}{\Delta t} = \frac{10\text{ gal}}{50\text{ s}}\left(\frac{3.7854\text{ L}}{1\text{ gal}}\right) = \mathbf{0.757\ L/s}$$

$$\dot{m} = \rho\dot{V} = (1\text{ kg/L})(0.757\text{ L/s}) = \mathbf{0.757\ kg/s}$$

Steady Flow Process

(b) The cross-sectional area of the nozzle exit is:

$$A_e = \pi r_e^2 = \pi(0.4 \text{ cm})^2 = 0.5027 \text{ cm}^2 = 0.5027 \times 10^{-4} \text{ m}^2$$

The volume flow rate through the hose and the nozzle is constant. Then the average velocity of water at the nozzle exit becomes

$$V_e = \frac{\dot{V}}{A_e} = \frac{0.757 \text{ L/s}}{0.5027 \times 10^{-4} \text{ m}^2}\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) = \mathbf{15.1 \text{ m/s}}$$

Energy Equation

In fluid mechanics, it is found convenient to separate mechanical energy from thermal energy and to consider the conversion of mechanical energy to thermal energy as a result of frictional effects as mechanical energy loss. Then the energy equation becomes the mechanical energy balance.

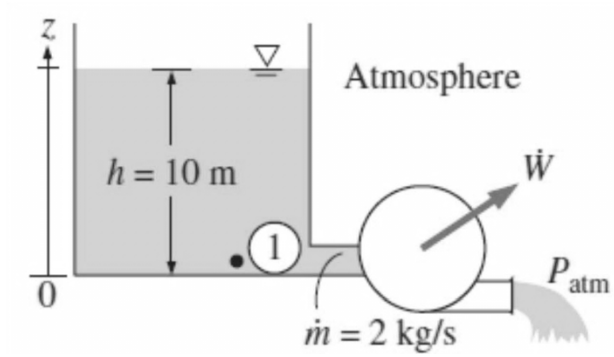
Energy Equation

- The mechanical energy can be defined as the form of energy that can be converted to mechanical work completely and directly by an ideal mechanical device;
- Kinetic and potential energies are the familiar forms of mechanical energy.

Energy Equation

Therefore, the mechanical energy (**e_{mech}**) of a flowing fluid can be expressed on a unit mass basis as:

$$e_{mech} = \frac{P}{\rho} + \frac{V^2}{2} + gz$$



In the absence of any changes in flow velocity and elevation, the power produced by an ideal hydraulic turbine is proportional to the pressure drop of water across the turbine.

Energy Equation

Most processes encountered in practice involve only certain forms of energy, and in such cases it is more convenient to work with the simplified versions of the energy balance. For systems that involve only mechanical forms of energy and its transfer as shaft work, the conservation of energy principle can be expressed conveniently as:

$$E_{\text{mech, in}} - E_{\text{mech, out}} = \Delta E_{\text{mech, system}} + E_{\text{mech, loss}}$$

where E_{mech}, loss represents the conversion of mechanical energy to thermal energy due to irreversibility such as friction. For a system in steady operation, the mechanical energy balance becomes

$$E_{\text{mech, in}} = E_{\text{mech, out}} + E_{\text{mech, loss}}$$

Bernoulli's Equation

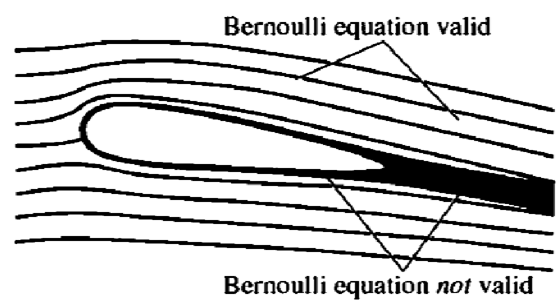
The Bernoulli equation is concerned with the conservation of kinetic, potential, and flow energies of a fluid stream and their conversion to each other in regions of flow where net viscous forces are negligible and where other restrictive conditions apply.

The energy equation is a statement of the conservation of energy principle.

Bernoulli's Equation

The Bernoulli equation is an approximate relation between pressure,velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible (as shown in the Figure below). Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics.

The *Bernoulli equation* is an approximate equation that is valid only in *in-viscid regions* of flow where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of *boundary layers* and wakes.



Bernoulli's Equation

Bernoulli Equation is derived from the mechanical energy equation:

$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \qquad \text{(kJ/kg)}$$

Since we are dealing with steady flow system with out the effect of the mechanical work and the friction on the system the first terms become zero.

$$\textit{Steady, incompressible flow:} \quad \frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$$

This is the famous **Bernoulli equation**, which is commonly used in fluid mechanics for steady, incompressible flow along a streamline in inviscid regions of flow.

Bernoulli's Equation

Bernoulli Equation can also be written between any two points on the same streamline as:

Steady, incompressible flow:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Bernoulli’s Equation limitations

1. **Steady flow** The first limitation on the Bernoulli equation is that it is applicable to steady flow;
2. **Frictionless flow** Every flow involves some friction, no matter how small, and frictional effects may or may not be negligible;
3. **No shaft work** The Bernoulli equation was derived from a force balance on a particle moving along a streamline;
4. **Incompressible flow** One of the assumptions used in the derivation of the Bernoulli equation is that ρ= constant and thus the flow is incompressible;
5. **No heat transfer** The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.

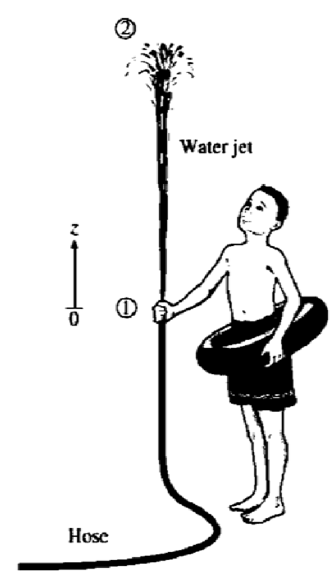
Bernoulli's Equation limitations

Bernoulli equation $P/\rho + V^2/2 + gz = C$ is applicable along a streamline, and the value of the constant C, in general, is different for different streamlines. But when a region of the flow is irrotational, and thus there is no vorticity in the flow field, the value of the constant C remains the same for all streamlines, and, therefore, the Bernoulli equation becomes applicable across streamlines as well.

Bernoulli's Equation

EXAMPLE:

Spraying Water into the Air Water is flowing from a hose attached to a water main at 400 kPa gauge. A child places his thumb to cover most of the hose outlet, causing a thin jet of high-speed water to emerge. If the hose is held upward, what is the maximum height that the jet could achieve?



Bernoulli's Equation

This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($V_1 = 0$) and we take the hose outlet as the reference level ($z_1 = 0$). At the top of the water trajectory $V_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli equation simplifies to:

Bernoulli's Equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g}\nearrow^0 + z_1\nearrow^0 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}\nearrow^0 + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} = \frac{P_{atm}}{\rho g} + z_2$$

Solving for z_2 and substituting,

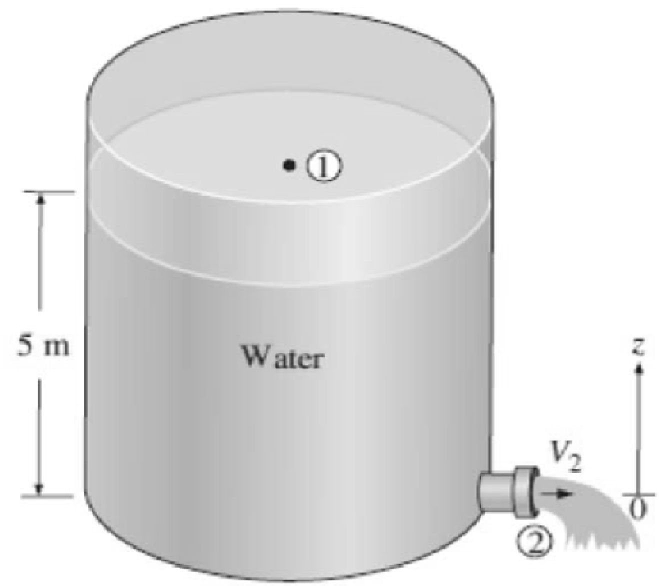
$$z_2 = \frac{P_1 - P_{atm}}{\rho g} = \frac{P_{1,\,gage}}{\rho g} = \frac{400 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right)\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)$$

$$= 40.8 \text{ m}$$

Bernoulli's Equation

EXAMPLE:

Water Discharge from a Large Tank A large tank open to the atmosphere is filled with water to a height of 5 m from the outlet tap. A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet. Determine the water velocity at the outlet.



Bernoulli's Equation

Solution:

This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of water so that:

- $P_1 = P_{atm}$ (open to the atmosphere),
- $V_1 = 0$ (the tank is large relative to the outlet),
- $z_1 = 5$ m and $z_2 = 0$ (we take the reference level at the center of the outlet).
- $P_2 = P_{atm}$ (water discharges into the atmosphere).

Bernoulli's Equation

Solution:

Then the Bernoulli equation simplifies to:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81\ \text{m/s}^2)(5\ \text{m})} = 9.9\ \text{m/s}$$

The relation $V = \sqrt{2gz}$ is called the **Toricelli equation**.

Closed Conduit Flow

Closed Conduit Flow

Assumptions:

- Steady flow
- Pressure is hydrostatic in both cross sections
    - o pressure changes are due to elevation only $p = \gamma h$
- Section is drawn perpendicular to the streamlines (otherwise the Kinetic energy term is incorrect)
- Constant density at the cross section

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Friction Losses

Friction loss: $h_f = f \frac{L}{D} \frac{V^2}{2g}$ Darcy-Weisbach

- Proportional to the length of the pipe
- Proportional to the square of the velocity (almost)
- Increases with surface roughness
- Is a function of density and viscosity
- Is independent of pressure

Pipe roughness (*f*)

pipe material	pipe roughness
glass, drawn brass, copper	0.0015
commercial steel or wrought iron	0.045
asphalted cast iron	0.12
galvanized iron	0.15
cast iron	0.26
concrete	0.18-0.6
rivet steel	0.9-9.0
corrugated metal	45
PVC	0.12

Laminar and Turbulent Flow

Laminar Flow Friction

$$V = \frac{\gamma D^2}{32\mu}\frac{h_l}{L}$$ Hagen-Poiseuille

$$h_f = \frac{32\mu LV}{\rho g D^2}$$

$$h_f = \frac{128\mu LQ}{\pi \rho g D^4}$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$ Darcy-Weisbach

$$\frac{32\mu LV}{\rho g D^2} = f\frac{L}{D}\frac{V^2}{2g}$$

$$f = \frac{64\mu}{\rho VD} = \frac{64}{Re}$$

Slope of -1 on log-log plot

Laminar and Turbulent Flow

Turbulent Flow Friction

- Hydraulically smooth pipe law (von Karman, 1930)

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{\mathrm{Re}\sqrt{f}}{2.51}\right)$$

- Rough pipe law (von Karman, 1930)

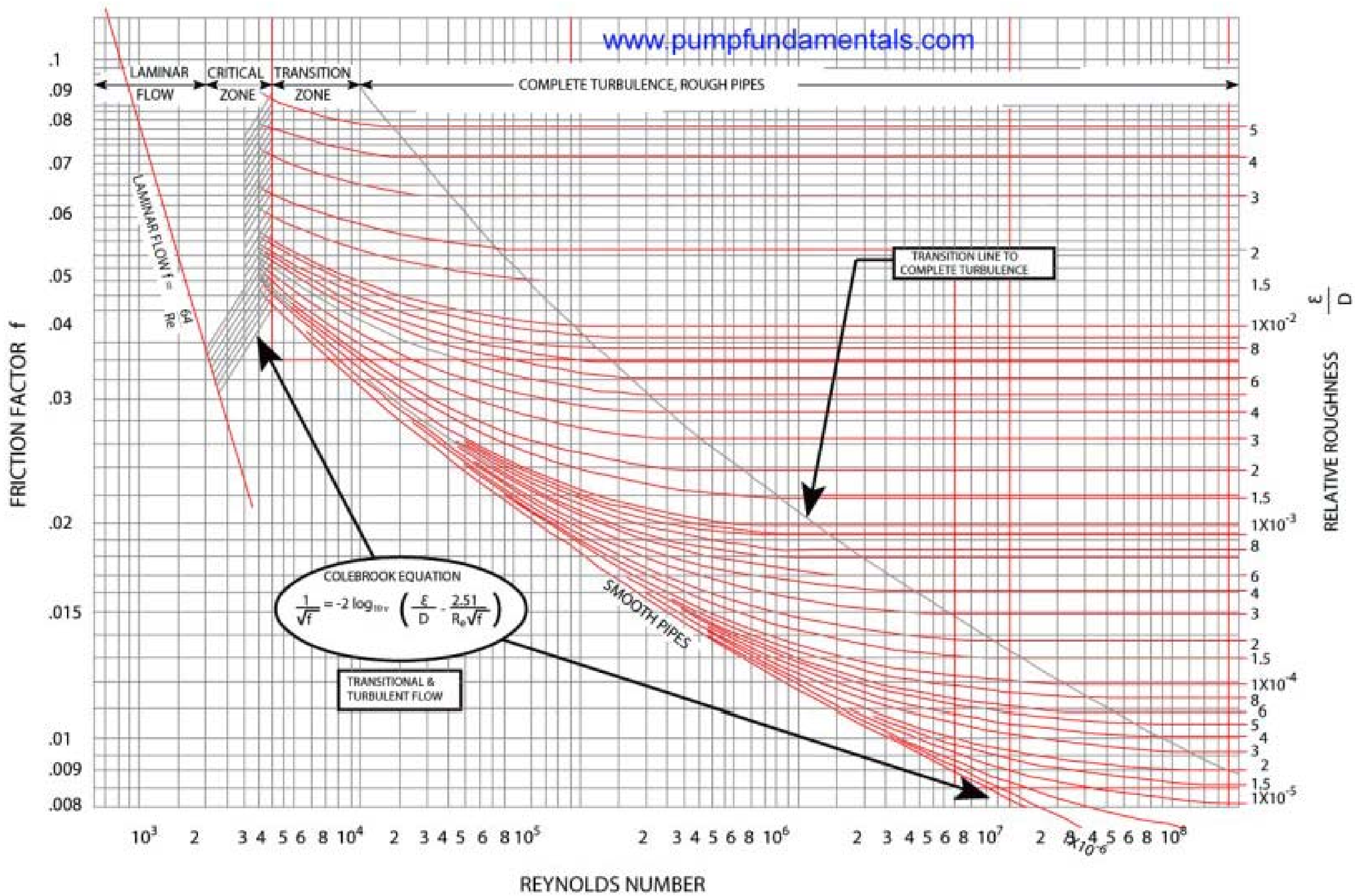
$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{3.7D}{\varepsilon}\right)$$

- Transition function for both smooth and rough pipe laws (Colebrook)

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

(These Equations used to draw the Moody diagram)

Laminar and Turbulent Flow



Moody diagram

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Hazen-Williams friction coefficient

- Commonly used in commercial and industrial settings;

$$h_f = \frac{RLQ^n}{D^m}$$

R = (f)friction factor

- Hazen-Williams exponential friction formula

$$R = \begin{cases} \dfrac{4.727}{C^n} \text{ USC units} \\ \dfrac{10.675}{C^n} \text{ SI units} \end{cases}$$

$$S = \frac{h_f}{L} = \frac{10.67\ Q^{1.85}}{C^{1.85}\ d^{4.87}}$$

S = Hydraulic slope

h_f = head loss in meters (water) over the length of pipe

C = Hazen-Williams coefficient

Hazen-Williams coefficient

C	Condition
150	PVC
140	Extremely smooth, straight pipes; asbestos cement
130	Very smooth pipes; concrete; new cast iron
120	Wood stave; new welded steel
110	Vitrified clay; new riveted steel
100	Cast iron after years of use
95	Riveted steel after years of use
60-80	Old pipes in bad condition

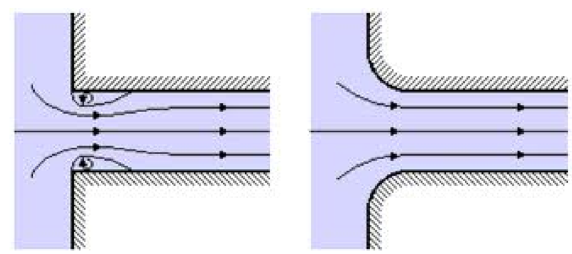
Hazen-Williams vs. Darcy-Weisbach

- Both equations are empirical;
- Darcy-Weisbach is rationally based, dimensionally correct, and preferred;
- Hazen-Williams can be considered valid only over the range of gathered data;
- Hazen-Williams can’t be extended to other fluids without further experimentation.

Minor (Secondary) Losses (h_l)

- Main sources: fittings, valves, changes;
- Most minor losses can not be obtained analytically, so they must be measured;
- Minor losses are often expressed as a loss coefficient, K, times the velocity head.

$$h_l = K\frac{V^2}{2g}$$



Non-Circular Conduits

- A is cross sectional area
- P is wetted perimeter
- R_h is the "Hydraulic Radius" (Area/Perimeter)
- Don't mix up with radius!

$$R_h = \frac{A}{P} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4}$$

For a pipe

$$D = 4R_h$$

$$h_f = f\frac{L}{4R_h}\frac{V^2}{2g}$$

Uniform Open Channel Flow

- Liquid (water) flow with free surface to atmosphere
- Relevant for:
  - ✓ natural channels: rivers, streams
  - ✓ engineered channels: canals, sewer lines or culverts (partially full), storm drains
- Hydraulic engineers looking at:
  - ✓ Location of free surface
  - ✓ Velocity distribution
  - ✓ Discharge - stage (depth) relationships
  - ✓ Optimal channel design

Open Channel Flow

- Uniform Flow at Normal depth y_o
  - ✓ Discharge-Depth relationships
- Channel flow transitions:
  - ✓ Control structures (sluice gates, weirs…)
  - ✓ Rapid changes in bottom elevation or cross section
- Flow type (Critical, Subcritical and Supercritical)
- Rapidly Varied Flow
  - ✓ Hydraulic Jump
- Gradually Varied Flow
  - ✓ Classification of flows
  - ✓ Surface profiles

Flow Classification

- Steady and Unsteady (w.r.t time)
  - ✓ Steady: velocity at a given point does not change with time
- Uniform, Gradually Varied, and Non-uniform (w.r.t. distance):
  - ✓ Uniform: velocity at a given time does not change within a given length of a channel
  - ✓ Gradually varied: gradual changes in velocity with Distance
- Laminar and Turbulent
  - ✓ Laminar: flow appears to be as a movement of thin layers on top of each other
  - ✓ Turbulent: packets of liquid move in irregular paths

Open Channel coefficients

- **Chezy Equation (1768)**:

$$V = C\sqrt{R_h S_f}$$

where C = Chezy coefficient

$$V = \sqrt{\frac{2g}{\lambda}}\sqrt{S_f R_h}$$

$$60\frac{\sqrt{m}}{s} < C < 150\frac{\sqrt{m}}{s}$$

where 60 is for rough and 150 is for smooth

also a function of **R** (like *f* in Darcy-Weisbach)

Open Channel coefficients

- **Manning Equation (1891)**:

Most available hydraulic equations for open channels relate the section averaged mean velocity (V) to hydraulic radius (R) and hydraulic gradient (S). In wide open channels, R can be approximated by the mean flow depth (d), which is equal to the flow area (A) divided by the top width (T). In the absence of local hydraulic controls, the hydraulic gradient is usually equal to the channel slope for high in-bank flows.

Some equations also include a roughness parameter to account for the different nature of flow resistance offered by a range of channels. One of the most commonly applied equations is the Manning equation, which can be expressed as :

$$V = 1/n\, d^{2/3}\, S^{1/2}$$

where 'n' is a roughness coefficient.

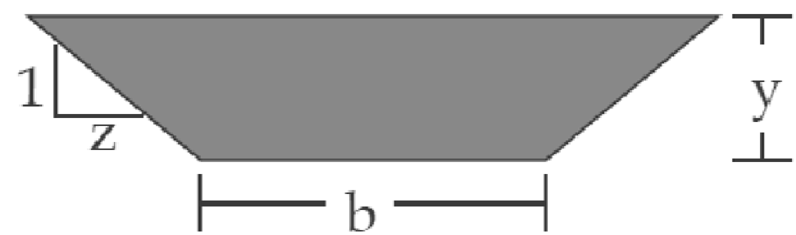
Trapezoidal Open Channel

- Derive P = f(y) and A = f(y) for a trapezoidal channel
- How would you obtain y = f(Q)?

$$A = yb + y^2 z$$

$$P = 2\left[y^2 + (yz)^2\right]^{1/2} + b$$

$$P = 2y\left[1 + z^2\right]^{1/2} + b$$



We can use trial and error method or Matlab

Flow in round Conduits

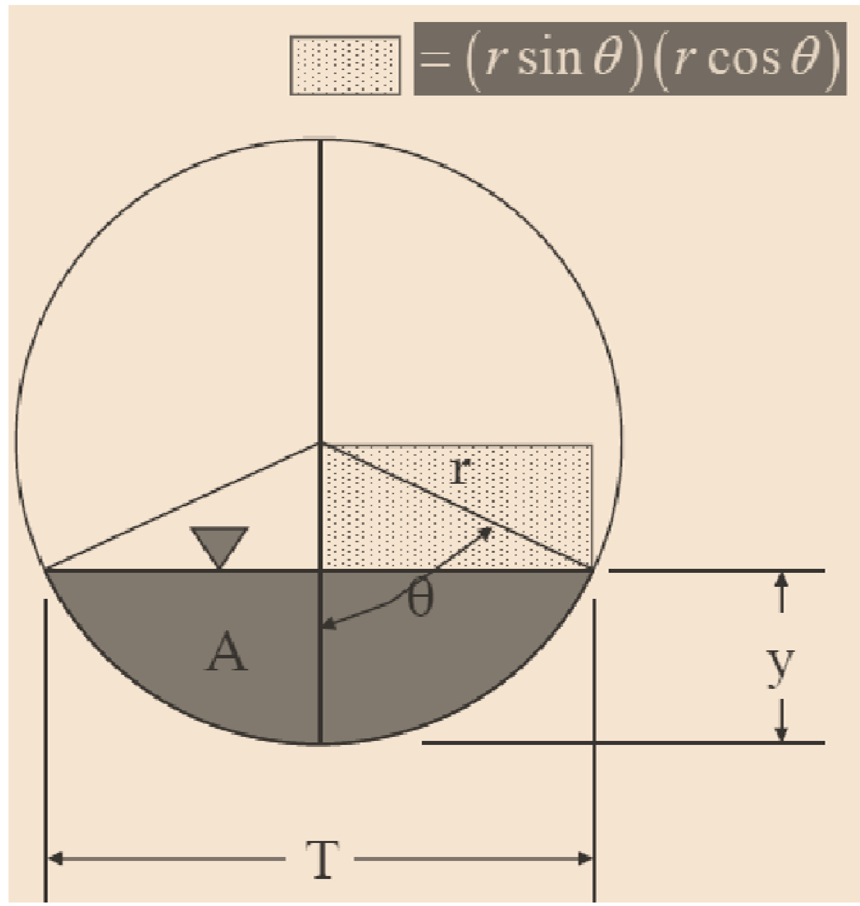
$$\theta = \arccos\left(\frac{r-y}{r}\right)$$

$$A = r^2(\theta - \sin\theta\cos\theta)$$

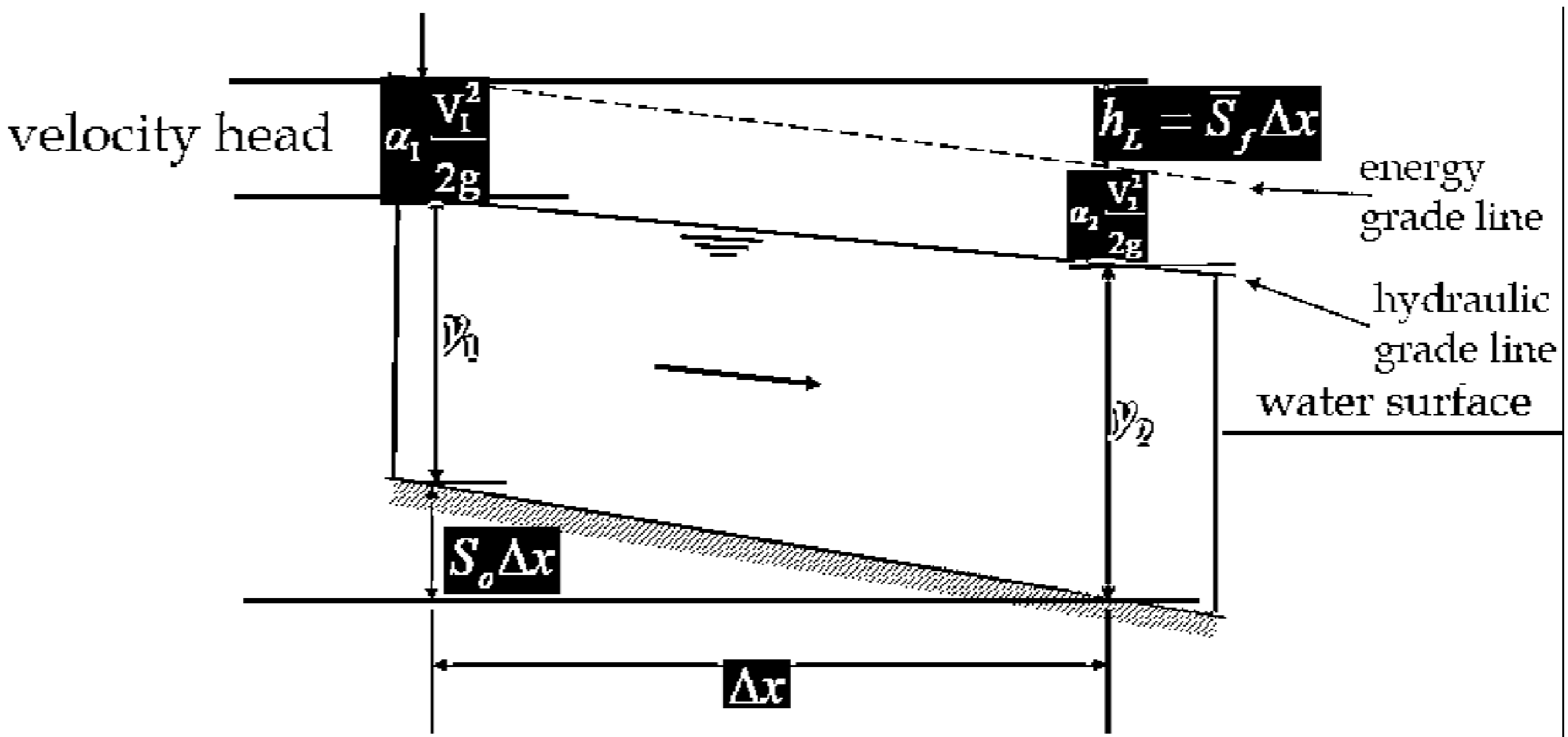
$$T = 2r\sin\theta$$

$$P = 2r\theta$$

Maximum discharge when y = 0.938d



Energy in Open Channels



Bottom slope (S_o) not necessarily equal to surface slope (S_f)

Energy in Open Channels

$$\frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_L$$

Pipe flow

z - measured from horizontal datum

From diagram on previous slide...

$$y_1 + S_o \Delta x + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

Turbulent flow ($\alpha \cong 1$)

y - depth of flow

Energy Equation for Open Channel Flow

$$y_1 + \frac{V_1^2}{2g} + S_o \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

Specific Energy Curve

The sum of the depth of flow (Potential energy, y) and the velocity head (Kinetic energy) is the specific energy:

$$E = y + \frac{v^2}{2g}$$

When energy is measured with respect to another fixed datum – Total energy

$$E = y + z + \frac{v^2}{2g}$$

The specific energy between two sections is equal if there are no head losses and the channel bottom is horizontal :

$$E_1 = \frac{v_1^2}{2g} + y_1 = E_2 = \frac{v_2^2}{2g} + y_2$$

Otherwise, E1- dy= E2

Specific Energy Curve

In a channel with constant discharge, Q:

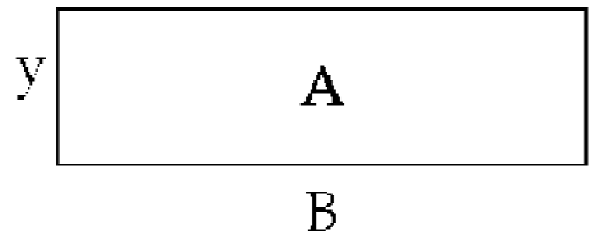
$$Q=A_1V_1=A_2V_2$$

$$E = y + \frac{Q^2}{2gA^2}$$ Where $A=f(y)$

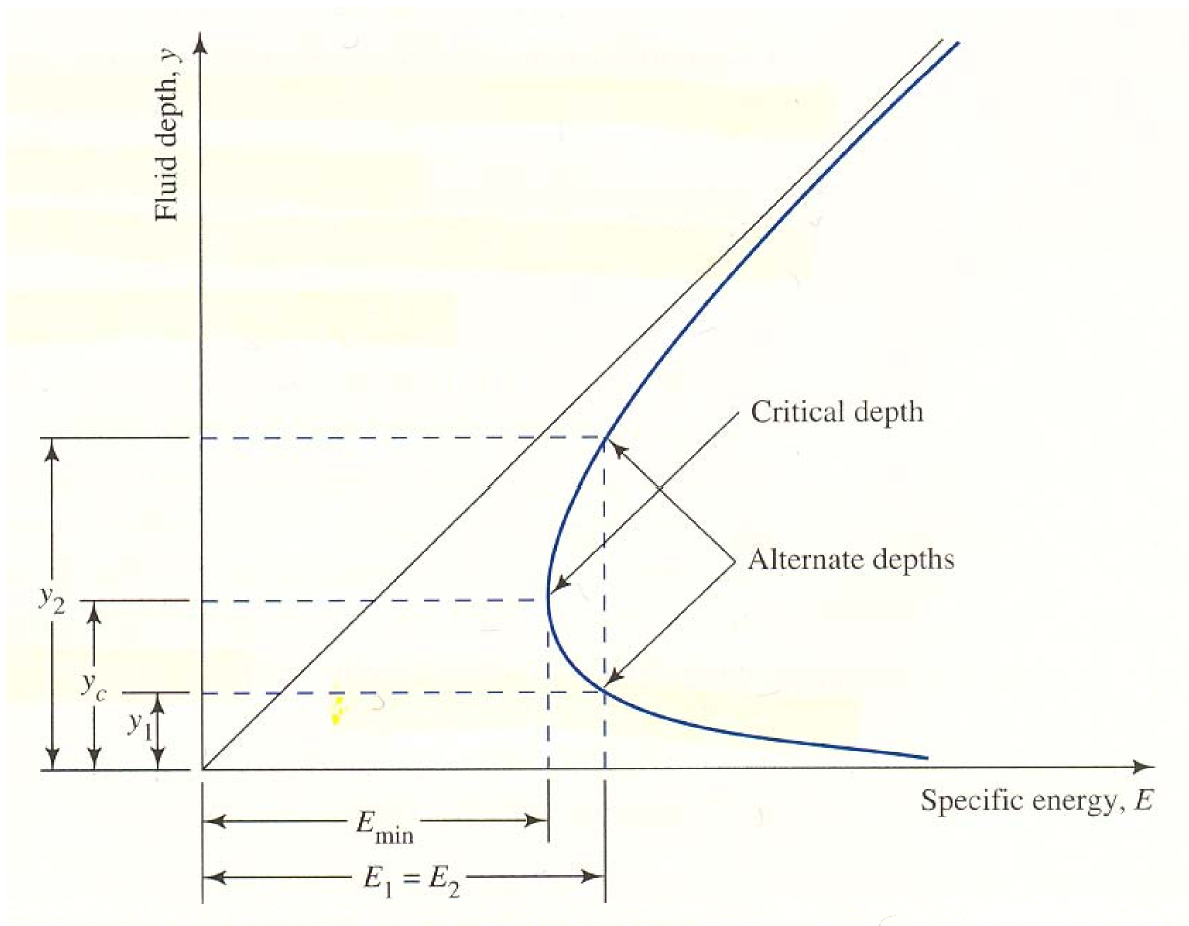
Consider rectangular channel (A=By) and Q=qB

$$E = y + \frac{q^2}{2gy^2}$$

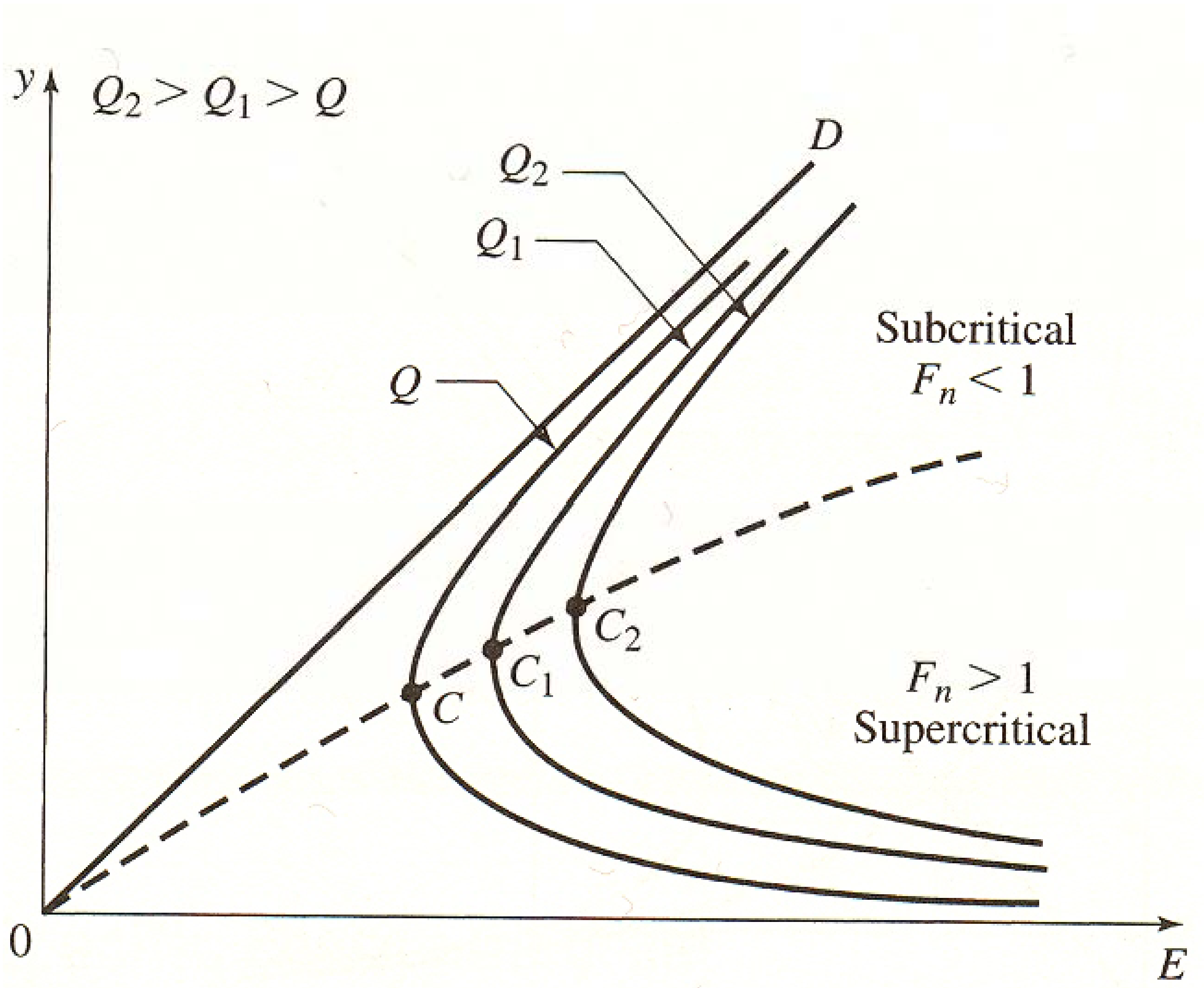
q is the discharge per unit width of channel



Specific Energy Curve



Specific Energy Curve



Hydraulic Structures

Hydraulic structures are anything that can be used to divert, restrict, stop, or otherwise manage the natural flow of water. They can be made from materials ranging from large rock and concrete to obscure items such as wooden timbers or tree trunks. A dam, for instance, is a type of hydraulic structure used to hold water in a reservoir as potential energy, just as a weir is a type of hydraulic structure which can be used to pool water for irrigation, establish control of the bed (grade control) or, as a new innovative technique, to divert flow away from eroding banks or into diversion channels for flood control.

http://chl.erdc.usace.army.mil/hydraulicstructures

Assigenment

A large upstream reservoir, with conditions as shown below, is connected to a 5 m^2 tank via a pipe that is 2000 m long, 0.381 m in diameter and has a Darcy-Weisbach friction factor of 0.015.

(a) What is the initial flow Q_0 in the system if the water level in the downstream tank is considered constant at the 75 m level? Approximately how long will it take for this downstream water level to reach 79 m?

A large upstream reservoir, with conditions as shown below, is connected to a 5 m^2 tank via a pipe that is 2000 m long, 0.381 m in diameter and has a Darcy-Weisbach friction factor of 0.015.

(a) What is the initial flow Q_0 in the system if the water level in the downstream tank is considered constant at the 75 m level? Approximately how long will it take for this downstream water level to reach 79 m?

Assigenment

A canal is designed to carry a flow of 5.4 m^3/s. Its trapezoidal section has a side slope of 3 horizontal to 1 vertical, it is built on a 1.5% slope (longitudinal), and it has a Manning's roughness n of 0.018.

(a) When the channel carries the design flow, the water depth is half the bottom width. Determine the water depth and channel width. Also determine the resulting water depth when the channel carries both half and twice the design flow.

(b) For the three flow values in (a), estimate the critical depth in the channel and determine whether the flows are supercritical or subcritical. Is there any special significance to this evaluation? That is, in what ways would determination of critical depth be important?