PDHonline Course H146 (4 PDH)

Hydraulic Engineering

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Hydraulic Engineering

Session 3

By

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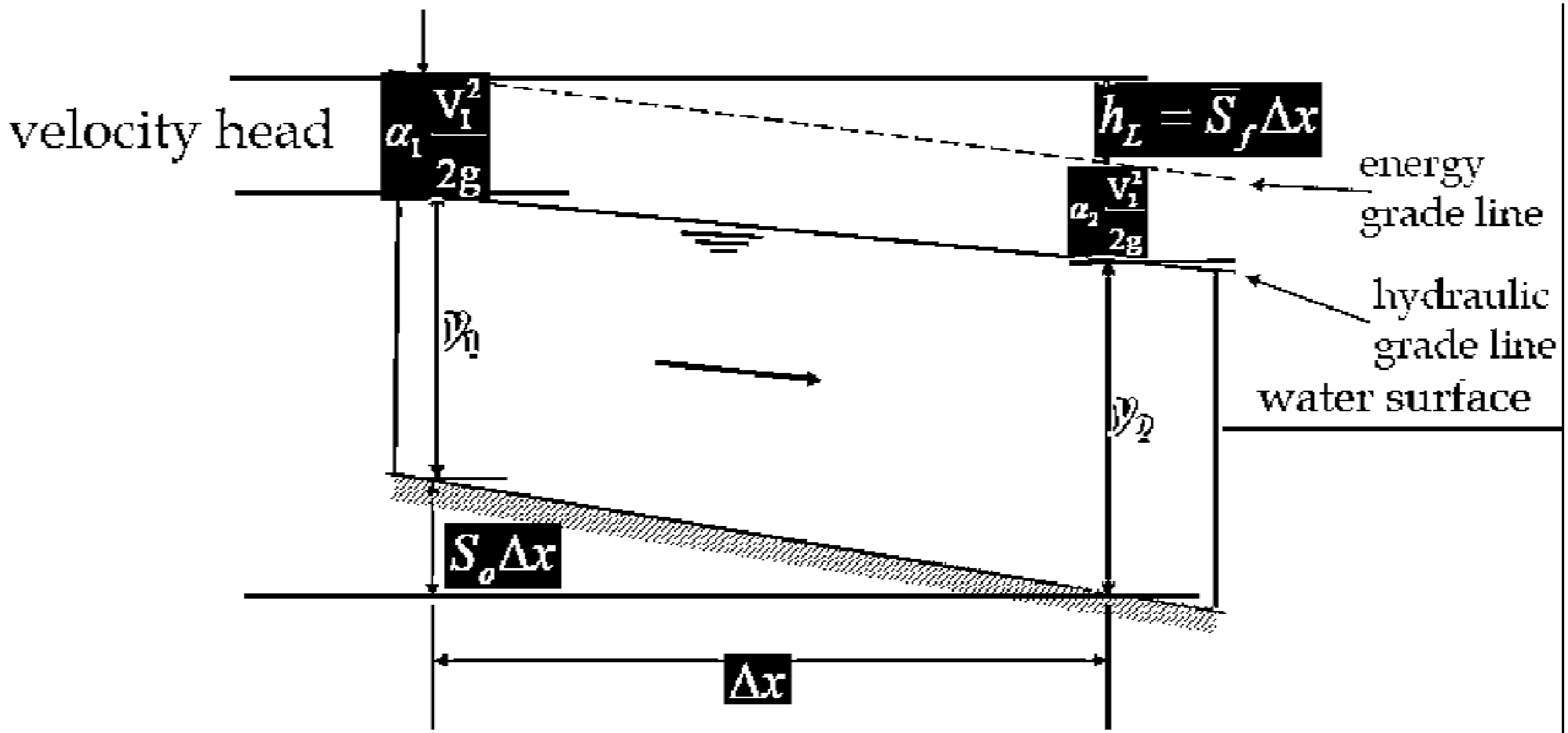
Session Goals

- Understand the gradually varied flow (GVF);
- Understand the surface slope;
- Understand the bed slope;
- Understand the water surface profiles;
- Calculate the water surface curve length.

Gradually Varied Flow (GVF)



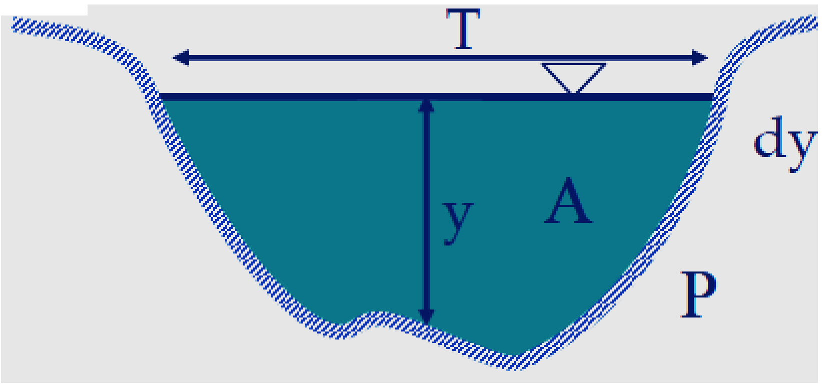
Bottom slope (S_o) not necessarily equal to surface slope (S_f)

Gradually Varied Flow (GVF)

Energy equation for nonuniform, steady flow with basic assumption that the change in energy with distance is equal to the friction loses

$$y_1 + \frac{V_1^2}{2g} + S_0 \Delta x = y_2 + \frac{V_2^2}{2g} + S_f \Delta x$$

T=dA/dy



$$\Delta y = y_2 - y_1$$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{d}{dx}\left(\frac{V^2}{2g}\right)\Delta x$$

$$\Delta y = S_0 \Delta x - S_f \Delta x - \frac{d}{dx}\left(\frac{V^2}{2g}\right)\Delta x$$

Gradually Varied Flow (GVF)

By dividing the equation by dx:

$$\frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) = S_0 - S_f$$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 + d\left(V^2/2g\right)/dy}$$

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right)$$

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{-2Q^2}{2gA^3} \cdot \frac{dA}{dy}$$

Gradually Varied Flow (GVF)

$$\frac{d}{dy}\left(\frac{V^2}{2g}\right) = \frac{-Q^2}{gA^2 D} = -F_r^{\ 2}$$

Therefore, the governing equation for gradually varied flow:

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^{\ 2}}$$

Where the variation of depth *y* with the channel distance x is shown to be a function of bed slope S_0, Friction Slope , S_f and the flow Froude number F_r .

Gradually Varied Flow (GVF)

Gives change of water depth with distance along channel

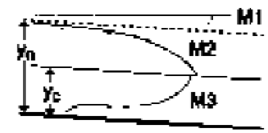
For GVF calculation please note:

- S_o and S_f are positive when sloping down in direction of flow
- y is measured from channel bottom
- $dy/dx = 0$ means water depth is constant
- $y = y_n$ is when $S_o = S_f$

Water Surface Profiles

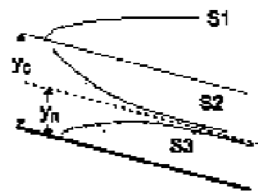
There are five slope classifications:

- Mild slope ($y_n > y_c$)
  - in a long channel subcritical flow will occur



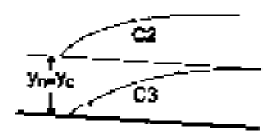
Mild Slope

- Steep slope ($y_n < y_c$)
  - in a long channel supercritical flow will occur



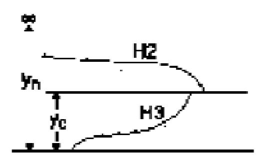
Steep Slope

- Critical slope ($y_n = y_c$)
  - in a long channel unstable flow will occur



Critical Slope

- Horizontal slope ($S_o = 0$)
  - y_n undefined (infinity)



Horizontal Slope

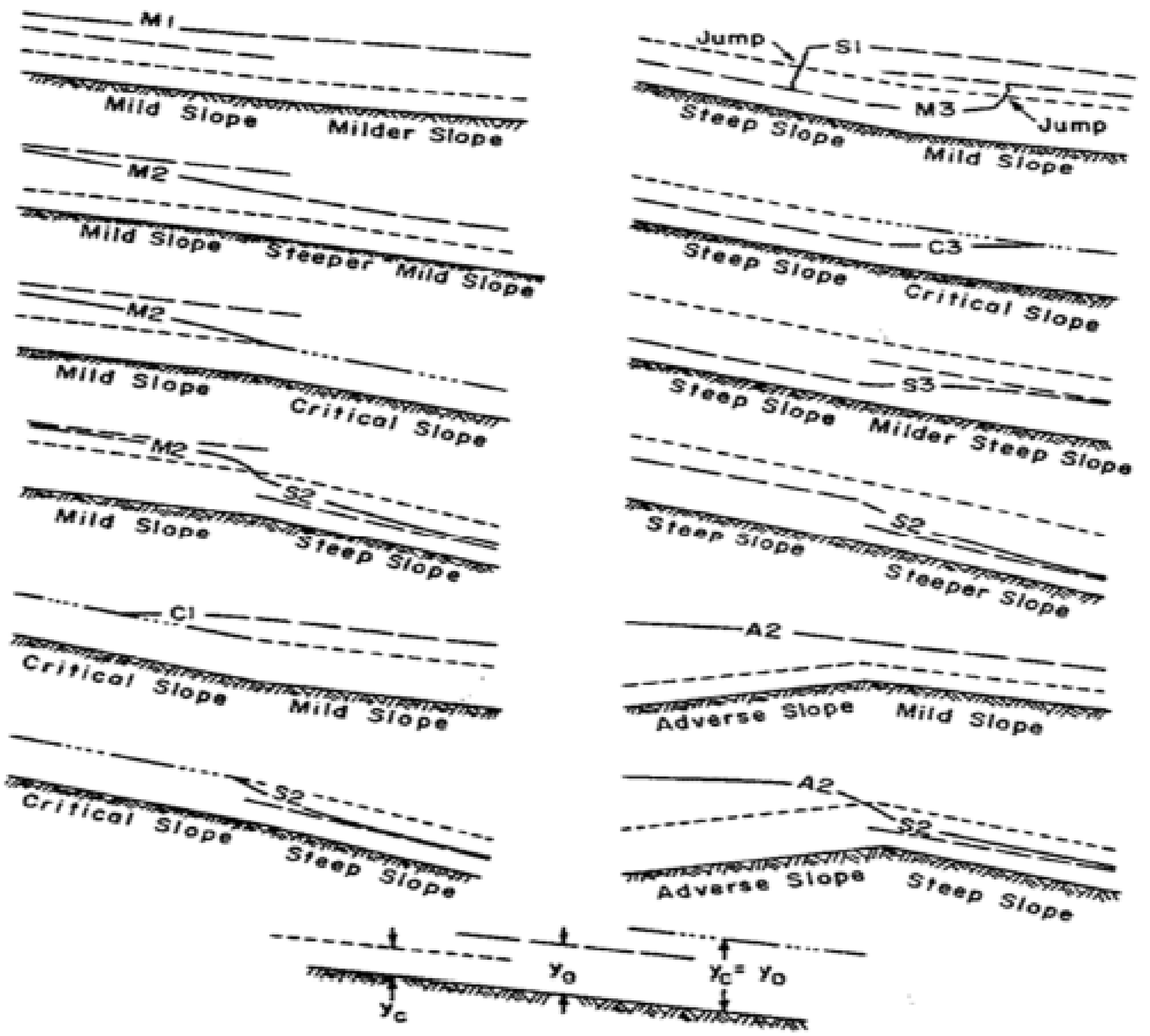
- Adverse slope ($S_o < 0$)
  - y_n undefined (infinity)

Adverse Slope

Water Surface Profiles

When there is a change in cross section or slope or an obstruction to the flow, the qualitative analysis of the flow profile depends on locating the control points, determining the type of water surface profile upstream and downstream of the control points, and then sketching these profiles. It must be remembered that when flow is supercritical ($F_r > 1$), the control depth is upstream and the water surface profile analysis proceeds in the downstream direction. When flow is subcritical ($F_r < 1$), the control depth is downstream and the computations must proceed upstream. Water surface profiles that result from a change in slope of the bed are shown in the next slide:

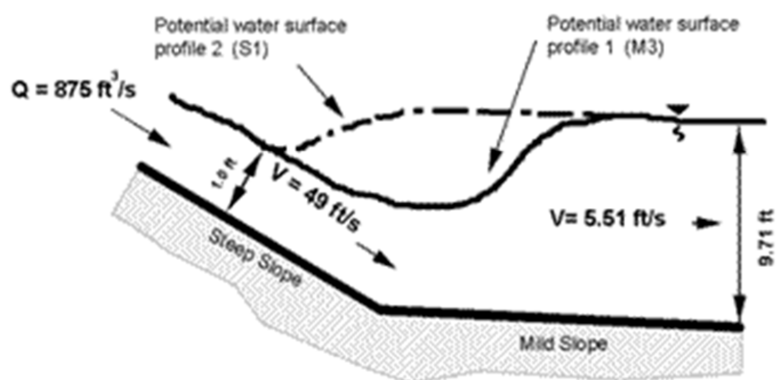
Water Surface Profiles



Water Surface Profiles

Example:

A 16 ft wide, rectangular channel goes from a very steep grade to a mild slope. The design discharge is 875 ft3/s and the normal depth and velocity on the steep slope were calculated to be 1.0 ft and 49 ft/s, respectively. On the mild slope, the normal depth and velocity were calculated to be 9.71 ft and 5.51 ft/s, respectively. Determine the type of flow occurring in both channels. If a hydraulic jump occurs, evaluate the depth downstream of the hydraulic jump, the location of the jump, and the water surface profile classification.



Water Surface Profiles

Find:

1. Find the critical depth, yc, on the steep slope.

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$$

q = Q/B where B is the channel width

$$y_c = \left[\frac{(Q/B)^2}{g}\right]^{\frac{1}{3}} \qquad y_c = \left[\frac{(875/16)^2}{32.2}\right]^{\frac{1}{3}} = 4.52\,\text{ft}$$

On the steep slope, the normal depth is 1.0 ft. Since $y < y_c$, supercritical flow occurs on the steep slope. Note that the unit discharge (q) is the same for the mild slope and hence, y_c, is the same for the steep and mild slope sections. On the mild slope, the normal depth is 9.71 ft. Since $y > y_c$, subcritical flow occurs on the mild slope. Therefore, a hydraulic jump should occur.

2. Next, determine if the jump will occur on the steep slope or on the mild slope.

To determine if the hydraulic jump occurs on the steep or mild slope, calculate the sequent depth (y_2) for the steep slope y_1 depth using the hydraulic jump equation. If y_2 from the hydraulic jump is larger than the normal depth y_0 from Manning's equation on the mild slope, then there will be an M3 curve on the mild slope until the y_2 equals the critical depth. If y_2 is smaller than y_0 on the mild slope, then the jump may occur on the steep slope and an S1 curve will occur to connect with the normal depth at the control section.

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8\,Fr_1^2} - 1\right)$$

$$Fr = \frac{V}{\sqrt{gy}} \qquad Fr = \frac{49}{\sqrt{32.2\,(1.0)}} = 8.64$$

$$y_2 = \frac{1.0}{2}\left(\sqrt{1 + 8(8.64)^2} - 1\right) = 11.72\ \text{ft}$$

Compare parameters and determine the type of water surface classification.

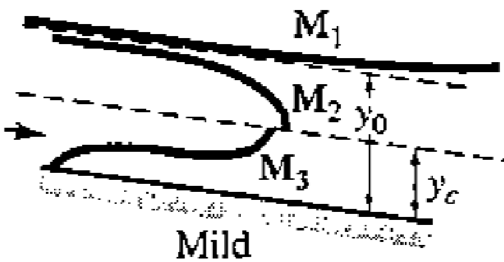
Since the sequent depth (y_2) is greater than the mild slope normal depth (i.e., 11.72 > 9.71), the mild slope channel will have an M3 curve until the hydraulic jump occurs.

Water Surface Profiles

Normal depth

Sluice gate

Steep slope

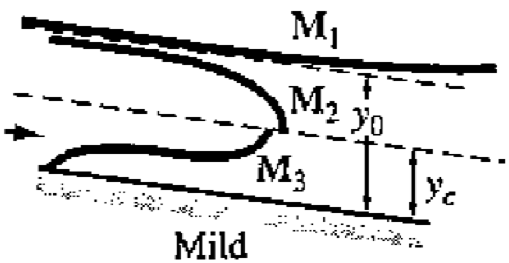


Obstruction

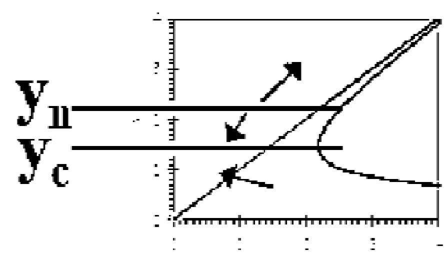
Steep slope (S_2)

Hydraulic Jump

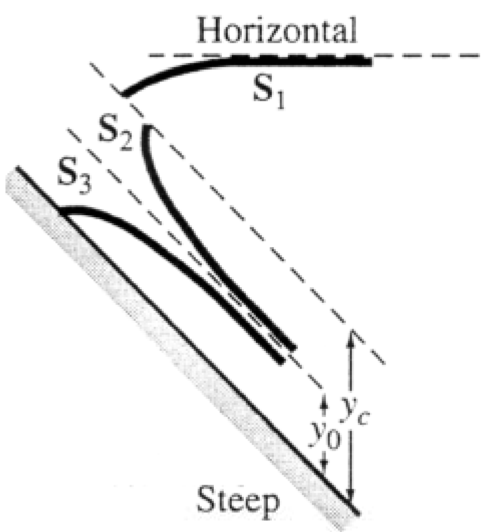
$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$



$S_0 - S_f$	$1 - Fr^2$	dy/dx
+	+	+
-	+	-
-	-	+

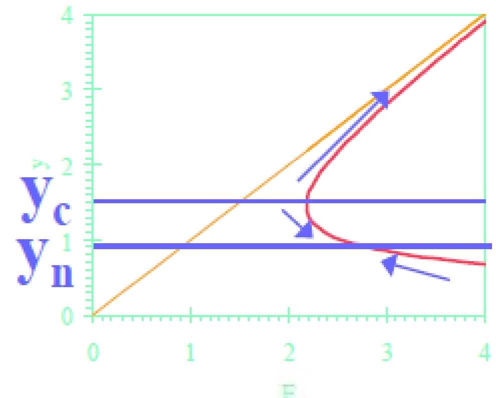


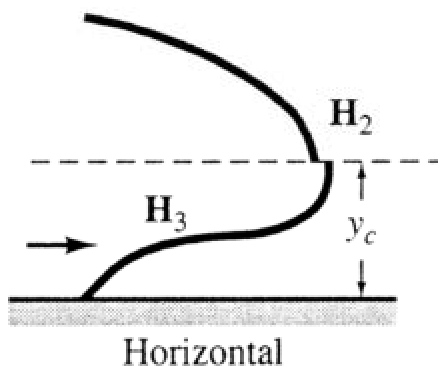
Water Surface Profiles

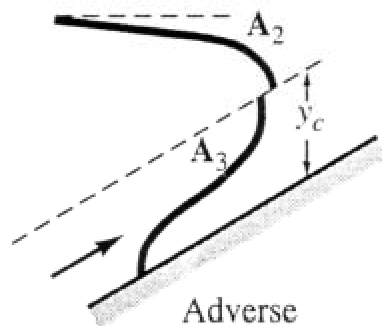


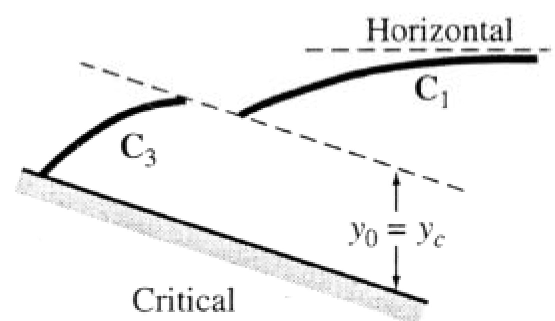
$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

	$S_0 - S_f$	$1 - Fr^2$	dy/dx
1	+	+	+
2	+	-	-
3	-	-	+





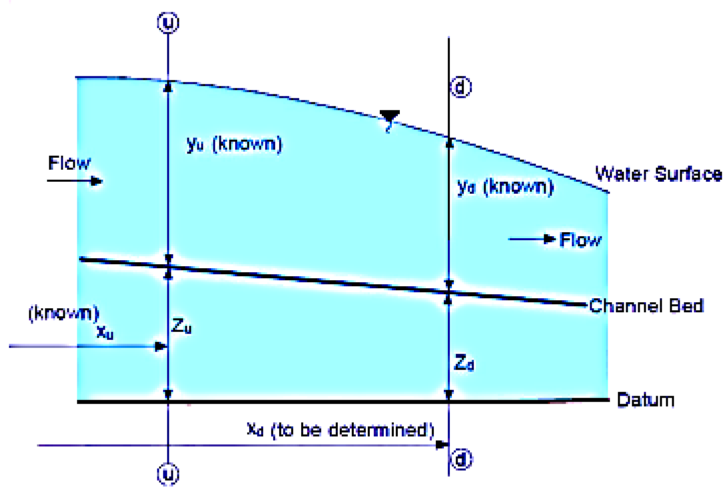




Computation of GVF

In the Direct Step method, the location where the specified depth, y_d occurs is determined, given the location for the occurrence of depth, y_u. Consider the channel shown in figure In this channel, say depth y_u occurs at a distance x_u from the reference point. Discharge, Q, channel bottom slope, S_0, the roughness coefficient, n and cross - sectional shape parameters (which relate A, P and R to y) are also known

The problem now is to determine the location x_d



Computation of GVF

Example A very long trapezoidal canal has $b = 18$ ft, $m = 2.0$, $S_0 = 0.001$, and $n = 0.020$, and it carries $Q = 800$ cfs. The canal terminates at a free fall. Calculate the water surface profile.

To solve this problem, we need to predict the type of profile. First, we should calculate y_c and y_n and determine whether the channel is mild, critical or steep.

$y_n = 5.16$ ft and

$y_c = 3.45$ ft. Because, $y_n > y_c$, the channel is mild.

Computation of GVF

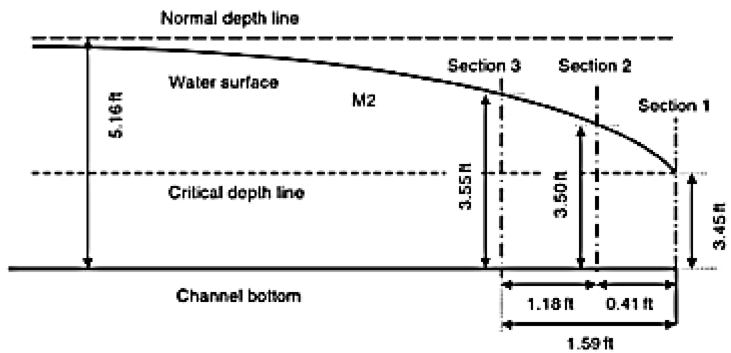


FIGURE 4.12 Direct step method example for subcritical flow

We know that far away from the free fall, the flow will tend to be normal. In a mild channel, the normal flow is subcritical. It is known that when subcritical flow approaches a free fall, critical depth occurs near the brink. For practical purposes, we assume that y_c occurs at the brink. Thus in this example an M2 profile will occur, and the flow depth will change between 5.16 ft and 3.45 ft as shown in Figure 4.12.

The calculations are best performed in tabular form as shown in Table 4.1, where the entries in the first column denote the section numbers. Because the flow is subcritical, the calculations will start at the downstream end of the channel and will proceed upstream. Referring to Figure 4.12, let us consider the most downstream reach – the reach between Sections 1 and 2. For this reach, we already know that $y_D = y_c = 3.45$ ft. This value is entered in column 2 of Table 4.1 for Section 1. Then, by using the expressions given for trapezoidal channels in Table 1.1,

$$A_D = (b + my_D)y_D = [18.0 + (2.0)(3.45)](3.45) = 85.91 \text{ ft}^2$$

$$P_D = b + 2y_D\sqrt{1+m^2} = 18.0 + 2(3.45)\sqrt{1+(2.0)^2} = 33.43 \text{ ft}$$

$$R_D = A_D/R_D = (85.91)/(33.43) = 2.57 \text{ ft}$$

Computation of GVF

$$V_D = Q/A_D = (800)/(85.91) = 9.313\ \text{fps}$$

$$S_{fD} = \frac{n^2}{k_n^2}\frac{V_D^2}{R_D^{4/3}} = \frac{(0.020)^2(9.313)^2}{(1.49)^2(2.57)^{4/3}} = 0.00444$$

$$E_D = y_D + \frac{V_D^2}{2g} = (3.45) + \frac{(9.313)^2}{2(32.2)} = 4.79666\ \text{ft}$$

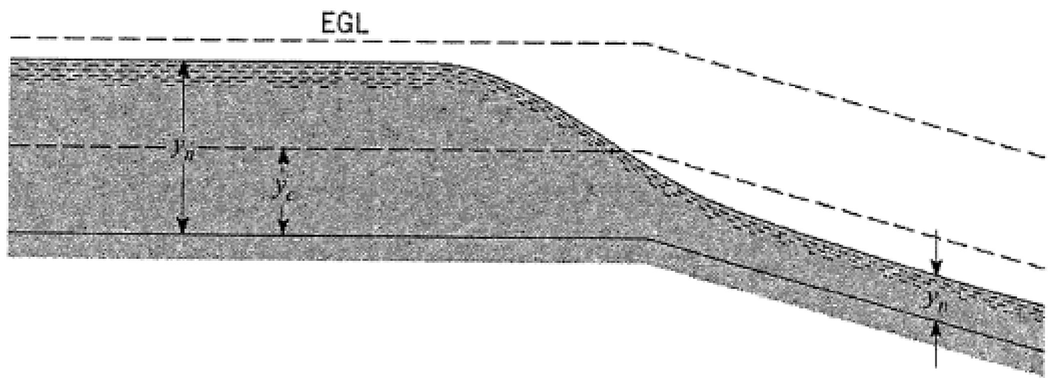
We are now ready to choose a value for y_U. We know that for an M2 curve, $y_U > y_D$ and $y_n > y_U > y_c$. Thus we choose $y_U = 3.50$ ft. This value is entered in column 2 as the flow depth for Section 2. Knowing the flow depth, we can calculate all the flow characteristics at the upstream

TABLE 4.1 Summary of direct step method calculations for subcritical flow

	Variables for section i						Variables for reach between sections i and $i-1$					
i	y (ft)	A (ft²)	P (ft)	R (ft)	V (fps)	E (ft)	$\Delta E = E_D - E_U$ (ft)	S_f	S_{fm}	$S_0 - S_{fm}$	ΔX (ft)	$\Sigma\Delta X$ (ft)
1	3.45	85.905	33.429	2.570	9.313	4.79666		0.00444				0
2	3.50	87.500	33.652	2.600	9.143	4.79801	−0.00135	0.00421	0.00433	−0.00333	0.41	0.41
3	3.55	89.105	33.876	2.630	8.978	4.80167	−0.00366	0.00400	0.00411	−0.00311	1.18	1.59
4	3.60	90.720	34.100	2.660	8.818	4.80750	−0.00583	0.00380	0.00390	−0.00290	2.01	3.60
5	3.65	92.345	34.323	2.690	8.663	4.81538	−0.00788	0.00361	0.00371	−0.00271	2.91	6.51
6	3.70	93.980	34.547	2.720	8.512	4.82518	−0.00980	0.00344	0.00353	−0.00253	3.88	10.39
7	3.75	95.625	34.771	2.750	8.366	4.83680	−0.01162	0.00327	0.00336	−0.00236	4.93	15.32
8	3.80	97.280	34.994	2.780	8.224	4.85014	−0.01334	0.00312	0.00320	−0.00220	6.08	21.40
9	3.85	98.945	35.218	2.810	8.085	4.86509	−0.01495	0.00297	0.00304	−0.00204	7.32	28.71
10	3.90	100.620	35.441	2.839	7.951	4.88158	−0.01649	0.00283	0.00290	−0.00190	8.67	37.38
11	3.95	102.305	35.665	2.869	7.820	4.89951	−0.01793	0.00270	0.00277	−0.00177	10.14	47.52
12	4.00	104.000	35.889	2.898	7.692	4.91881	−0.01930	0.00258	0.00264	−0.00164	11.76	59.28
13	4.05	105.705	36.112	2.927	7.568	4.93941	−0.02060	0.00246	0.00252	−0.00152	13.53	72.81
14	4.10	107.420	36.336	2.956	7.447	4.96124	−0.02183	0.00236	0.00241	−0.00141	15.48	88.29
15	4.15	109.145	36.559	2.985	7.330	4.98423	−0.02299	0.00225	0.00230	−0.00130	17.64	105.93
16	4.20	110.880	36.783	3.014	7.215	5.00833	−0.02410	0.00215	0.00220	−0.00120	20.03	125.96
17	4.25	112.625	37.007	3.043	7.103	5.03347	−0.02515	0.00206	0.00211	−0.00111	22.70	148.66

Assignment

In the sketch below, the wide rectangular channel shown has a Mannings $n = 0.024$ (everywhere) and the upstream depth $y_n = 2$ m. The upstream portion of the channel has a slope $S_1 = 0.003$ and the downstream portion a slope of $S_2 = 0.03$. Determine the flow per unit width in the channel and specify, as completely and quantitatively as reasonable, the profile of the water surface.



Assignment

In the 10.0 m wide rectangular channel shown below, the original upstream depth y_1 is 2.5 m, the original downstream depth is $y_2 = 0.5$ m and the flow 10.0 m^3/s. The discharge is suddenly decreased by 50%, creating two surge waves, one that propagates upstream and a second one downstream from the gate. Estimate the height of both surge waves y_3 and y_4 and their associated speeds of propagation.