PDHonline Course K154 (4 PDH)

Practical Multi-Factor Test Design and Analysis

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Practical Multi-Factor Test Design and Analysis

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1. Introduction

Process and product testing is necessary for establishing improvements. Due to the expense of testing on production lines and equipment efficient testing to determine these improvements is necessary. Testing single factor (one at a time) can result in missing interactions and actually require more test runs to find an optimum operating point.

Practical Multi-Factor Test Design and Analysis provides a simplified approach to statistical test design and analysis for process and product improvements. This course provides the foundation for testing multiple factors at low and high values in combinations to discern primary and interactive effects. The sequence for proper test design and analysis are provided as follows:

- Understanding the process – Process Mapping.
- Understanding the Product and Process descriptive statistics.
- Construction of the 2^k test design and Analysis of Variance (ANOVA). The ANOVA is used to determine primary and interacting factors which impact product quality, process yield and waste.
- Test execution.
- Analysis of Results. Multiple EXCEL spreadsheets with pre-built ANOVA forms are provided for use of test design and analysis. These forms will be used in the quiz for the course and can be used in future applications.

The first assumption of the test method provided within this training is that the process results are normally distributed. There are means of dealing with a non-normal distribution which are outside the scope of this text, and may be covered in future training.

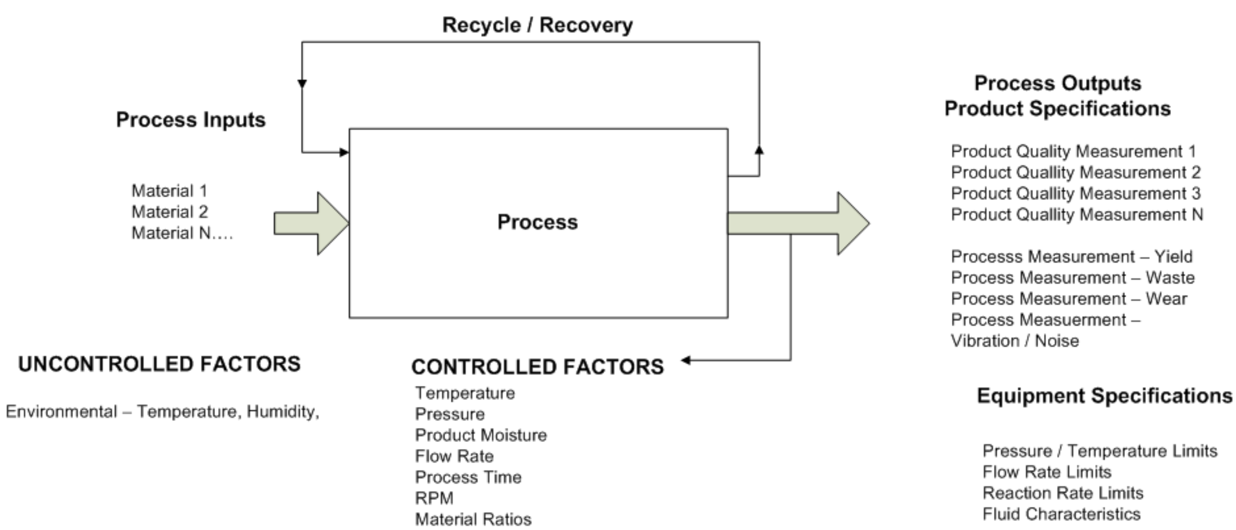
2. Process Mapping

Designing valid experiments of a process requires an in depth understanding of what is occurring in the process. The first step is to map the process in a flow sheet and understand the material flows, energy flows and if present the chemical reactions occurring in the process. This includes not only the product flow but also if present any waste streams, energy loss to the environment, recycle amounts and concentrations. Information sources include any existing flow sheets and Process and Instrumentation (P&I) diagrams along with the final or intermediate product specifications.

Equipment specification sheets are also a critical part of the mapping process. In older "mature" process or equipment this may involve going back through the nameplate data of individual components of the equipment to understand the capabilities and limits of the equipment. This is critical for safety consideration in processes involving temperature and pressure, energy flow or areas where bad things can occur when limits are exceeded.

With this information the process engineer has a clear understanding of the current system of mass and energy flows; the control points used to manipulate the outcome; and how far control variables can safely be moved in the experiment.

In some processes there are uncontrolled factors like the weather conditions in an outdoor process. These should be recorded in any experiment for reference.



3. Descriptive Statistics

All processes have some amount of variation in the results. Multiple measurements of the product will indicate the amount of variation. Specific terms are used to describe the results. A review of the basic terms and functions are provided below:

Mean

The mean (μ) of a measurement is a sum of all the measurements divided by the number of measurements.

$$\text{Mean} = \mu = \sum_{n=1}^{N} Sn/N$$

Where Sn = Value of sample n
N = Total number of Samples

Sample Variance

The sample variance (σ^2) is the average squared deviation from the mean. The equation for sample variance is as follows:

$$\sigma^2 = \sum_{n=1}^{N}(Sn-\mu)^2/(N-1)$$

Where Sn= Value of sample n
N = Total number of samples
μ = Mean value of measurement
σ^2 = Sample Variance

It should be noted in the equation above that the denominator =N-1. This is compensation for not measuring the entire population. A Population Variance has a denominator of N, where a Sample Variance uses a denominator of N-1.

Sample Standard Deviation

The Sample Standard Deviation (σ) is the square root of the Sample Variance

$$\sigma = [\sigma^2]^{½} = [\sum_{n=1}^{N}(Sn-\mu)^2/(N-1)]^{½}$$

Where
- Sn = Value of sample n
- N = Total number of samples
- μ = Mean value of measurements
- σ^2 = Sample Variance
- σ = Sample Standard Deviation

Normal Distribution

The Normal Distribution is the famous bell curve used in everything from adjusting course grades to the test analysis in this course. The characteristic of the Normal Distribution is that the population will be distributed as follows

Percentage of Population	Boundary
68.2 %	$\mu \pm 1\sigma$
95.4 %	$\mu \pm 2\sigma$
99.7 %	$\mu \pm 3\sigma$

A measurement with a "low" Standard Deviation will have a high peak at the mean value with a rapid drop off. A measurement with a "high" Standard Deviation will have a lower peak at the mean value with a higher spread above and below the mean value.

ANOVA

ANOVA stands for Analysis of Variance. It is a statistical method of analyzing results of the designed test to find meaningful changes. By comparing the variation in results between different settings (Signal) to those within repetitions of settings (Noise) a difference can be detected.

The analysis assumes the null hypothesis of $H_0=H_1$, where H_0 is the result of treatment 0 and H_1 is the result of treatment 1. If a Statistically Significant Difference (SSD) is calculated based on the F Ratio then $H_0 \neq H_1$ and the null hypothesis is disproved.

Degrees of Freedom

The Degrees of Freedom is a description of the number of values in a statistic that are free to vary.

In general, if a factor is tested at two levels it contributes one degree of freedom to the analysis. Each factor in a multi-factor test run at two levels each will therefore contribute one degree of freedom each.

Each interaction in the multi-factor test also contributes a degree of freedom. In general, it will be the product of the DoF of the primary Factors.

Each measurement or observation is made up of the primary effects, interactive effects and error. As a result, the number of Degrees of Freedom for the Error term make up a large part of the total DoF of the test. The Error term DoF will be a product of the number of Replications -1 times the number of test treatments. For example, a test of two factors at two levels results in 4 test treatments. If four replications are run the Error DoF is (n-1) X (T) = (4-1) X (4) = 12

The DoF for the Error term is important to understand as it is used to determine the noise level. It is the calculated effects and the ratio to the noise level that determines the statistical significance of the changes tested.

Factorial Test

The Factorial Test is a specific design and the focus of this course. The specific design is to take a number of factors (Controlled Variables) and test at two levels for each factor. The normal terminology is to refer to this class of testing as 2^k tests, where k is the number of factors involved, and defines the number of test conditions.

Replication is also a requirement of this type of test in that it recognizes the failure of "once in a row" results. The minimum amount of replication required is two, and the spreadsheets provided for analysis allow for up to 4 replications at each condition.

The number of runs required for testing factors at two levels each with replication are as follows:

Number of runs required for each factor at two levels with Replication				
Factors	Test Treatments	2 Replications	3 Replications	4 Replications
2	4	8	12	16
3	8	16	24	32
4	16	32	48	64

Contrast

The contrast is the difference in the results measured when the factor is at the high level and the low level. Mathematically it is the sum of the results of the factor at the + (high) value minus the sum of factor at the – (low) value.
C= Sum (+) – Sum(-)

Sum of Squares

The Sum of Squares is the contrast squared divided by the number of treatments times the number of replications.

$SS= C^2/(Tn)$

Mean Squares

The Mean Squares is the Sum of Squares divided by the Degrees of Freedom.

F Ratio

The F Ratio is the ratio for the calculated Mean Squares for a factor or interaction divided by the mean squares of the error term. It is the signal to noise ratio for the factor or interaction. The F stands for the Frequency Distribution. A ratio closer to 1 indicates a low likelihood of any impact of the factor or interaction.

Probability

The probability used in this test is in the probability of a F Ratio which is calculated based on the degrees of freedom of the two values used in the calculation – i.e. the factor or interaction and the error term. The probability can be generated in an Excel spread sheet or looked up in tables to determine the critical value.

Significance or Statistically Significant Difference

The critical value described in the probability table look up is based on the confidence level of declaring a difference. While these are available at many levels, the more commonly used value is 0.05. If the calculated F Ratio is greater than the critical value from the table (F_{CRIT}) then there is a Statistically Significant Difference (SSD).

Similarly, when the probability is directly calculated if the value is equal to or less than 0.05 then there is a Statistically Significant Difference.

The way this would be interpreted in a case where the F Ratio is equal to F_{CRIT} or the Probability is calculated as 0.05 is “There is a 95% probability of the factor changing the response.”

4. Factorial Test Design

This course is focused on the design, execution and analysis of 2^k factorial experiments, where k is the number of test factors, and each factor will be tested at two levels. As implied in the name, the number of runs required is defined in the test definition – two factor testing will require 4 test combinations, three factor testing will require 8.

The advantage of factorial testing is that interactions of multiple factors can be clearly defined through the analysis of results. A disadvantage is that a clear result is not provided until all combinations have been run at least twice, as replication of each combination is required in order to establish the variation within a setting combination for comparison to changes between the combinations. The test design sequence then becomes:

1. Based on the review and understanding of the process, assign a trial name and provide a clear definition of the objective of the test.
2. Define the factors to be tested and the proposed levels for each and clearly define and document each. The low value for each factor is defined as the -1 level, the high as +1 level.
3. Define the metrics to be used for the analysis, and note any special considerations for the measurements, calibration or sample requirements.
4. Based on a review of the amount of variation in the current variations select the number of replications for the test conditions. A minimum of two replications is required for the test methods presented in this course. The spread sheets provided are set up with up to 4 replications available.
5. Plan the test sequence for randomization and blocking. Do not repeat the same order of set up in the replications and ensure that as much as possible the incoming materials are for the same lots to minimize the incoming variability.
6. Review the test plan with all the participants in the test. This is critically important in a processing line that involves multiple operators in stages across the line. It is important that all operators understand the objectives, limits and allowable changes within the test.

5. Factorial Test Execution

After performing the plan review execute the test according to the plan. As the test is executed note any items of interest in the test log to aid in the review and explanation of results.

Copy the appropriate test design sheet for each metric defined for the test and enter the results as the data is generated. It is worthwhile to have any notes or subjective observations by the operators / technicians added in record of the test and results as the tests are performed. These can be helpful in the next step of forming conclusions and recommendations.

6. Factorial Test Analysis

The ANOVA performed on the resultant data is an indication of the statistical significance of the primary and interactive effects. When the replications are complete

1. Determine the Contrast for each factor
   Contrast A = C_A =Sum (A+) – Sum (A-)
   Contrast B = C_B =Sum(B+) – Sum (B-)
   Contrast AB = C_{AB} =Sum(AB+) – Sum(AB-)

2. Determine the Degrees of Freedom for the factors, error and total

   Since each factor is tested at two levels, each factor contributes 1 DoF, this also applies to each interaction.
   DoF Factors=DoF_A=DoF_B=DoF_{AB}=1
   DoF Error = DoF_E= (n-1)* T
   DoF Total = DoF_T= Tn-1

3. Determine the Sum of Squares for each factor
   SS= $C^2/(Tn)$ Where T = number of treatments
   n= number of Replications

   SSA= $C_A^2/(Tn)$
   SSB= $C_B^2/(Tn)$
   SSAB= $C_{AB}^2/(Tn)$

4. Determine the Sum of Squares for the total and Error

   $SS_T=DoF_T$ x Variation(all data)$=DoF_T$ x σ^2

   The sum of squares for the Error term is the difference between the total sum of squares and the sum of all the test factors and interactions:

   $SS_E = SS_T - [SS_A + SS_B + SS_{AB} \ldots]$

5. Determine the Mean Squares for each factor and error, which is the ratio of the Sum of Squares divided by the Degrees of Freedom for each factor and the error term.

   $MS_A=SS_A/DoF_A$
   $MS_B=SS_B/DoF_B$
   $MS_{AB}=SS_{AB}/DoF_{AB}$
   $MS_E=SS_E/DoF_E$

6. Calculate the F Ratio for each of the factors and interactions. This is the ratio of the Mean Squares of the factor or interaction to the Mean Squares of the Error. In essence, it is the Signal to Noise ratio. The higher the ratio the more likely the significance of the factor.

   $FR_A= MS_A/MS_E$
   $FR_B= MS_B/MS_E$
   $FR_{AB}= MS_{AB}/MS_E$

7. Determine the probability of the calculated F Ratio. This is the probability for the value of the F statistic of a normal distribution based on the degrees of freedom for the factor and the degrees of freedom of the error. If the calculated probability value is less than 0.05 then a Statistically Significant Difference has been detected.

8. Validate the Test Results

Validation is done in two parts. The first is a statistical review through a plot of the residuals. The residuals can be generated by creating a linear model based on the calculated effects and plotting the difference in the predicted and actual values for each run. The simplest method is to take the mean value for a treatment and calculating the residual as the difference of the actual run, $R_1= (T_{R1}-T_M)$, $R_2= (T_{R2}-T_M)$, $R_3= (T_{R3}-T_M)$…

The residuals are then ranked in ascending order and the cumulative probability is calculated

$P_k=(k-.5)/N$

Where
P_k= Cumulative Probability
k= Rank order of Data
N=Number of Data Points

A graph of the Cumulative Probability (0-1) and Residuals should result in a roughly linear line if the results are normally distributed and there are no errors in the data collection or recording.

The next step in validation is to review the results considering the current understanding of the process and ensure that it makes sense. If there is no logical explanation considering material balances, chemical, mechanical or electrical operations then dig deeper. It may be necessary to repeat the test and add tracking and monitoring of more variables to confirm the results and find the root cause which will lead to sustainable gains.

9. Review for Conclusions and Recommendations

The ANOVA results and model validation provide the information for detecting a statistical difference in results. When a statistical difference it detected, it is up to the process engineer to determine if this is a practical and worthwhile change. A 2% improvement in the yield of the process may not be enough to pay for the changes in capital and/or operating costs associated with the change if the material costs are low and there are no environmental impacts or charges associated with the waste or recycle.

The results should then be reviewed with the operators and participants of the test. This can help solidify thoughts and ideas on the results, impacts and aid in defining the next steps, which can be to develop and implementation plan, find other methods or repeat testing.

7. Example 1 – Milling Vibration 2^2

7.1 There is an opportunity to modify a substrate milling process to reduce the surface variation. It is known that the surface roughness of the resulting cuts are related to the vibration, so the test metric will be vibration data which can be gathered directly on the machine.

7.2 Two factors will be included in the factorial test, bit diameter and spindle RPM. Based on the slot size a $^1/_8$-inch diameter and a ¼ -inch diameter bit will be tested. The current machine capabilities allow operation at 500 and 1000 RPM.

7.3 The metric used will be the vibration measured during the milling process. The milling machine will be fitted with a 3-axis accelerometer and the RMS value of the vibration vector recorded for each test condition.

7.4 The test is of two factors which requires 4 test conditions. Based on the ease of testing 4 replications will be run, resulting in a total of 16 runs.

7.5 The incoming substrate is from a single supply lot. The run order is selected for each set to ensure that the replication order does not repeat, and all 4 test conditions are run each day. The resulting test plan is shown below.

7.6 Following the review of the test design, sequence and plans with the operators and technicians the test is conducted.

Trial Name	Vibration Reduction						
Trial Objective	Reduce vibration during milling process. Test bit size and RPM.						
FACTORS		**A**	**B**	**RUN ORDER**			
LEVELS	-	bit 1/8	500 RPM				
	+	Bit 1/4	1000 RPM	**REP 1**	**REP 2**	**REP 3**	**REP 4**
TREATMENT		A	B				
1		-	-	1	4	2	3
2		+	-	2	3	4	1
3		-	+	3	2	1	4
4		+	+	4	1	3	2
Metrics							
Vibration	Fluke 810 vibration tester, sensor mounted on board at position A7.						
Run Instructions							
	Use Substrate Lot 21175 for all tests.						
Run Notes							

7.7 Example 1 – Analysis of Results.

7.7.1 The measured vibration of the substrate board is entered in the spreadsheet as the data is generated. The results of the analysis are shown below:

Trial Name	Vibration Reduction										
Trial Objective	Reduce vibration during milling process. Test bit size and RPM.						METRIC				
Factors	A	B				BOARD VIBRATION					
-	bit 1/8	500 RPM				REP 1	REP 2	REP 3	REP 4		
+	Bit 1/4	1000 RPM									
SET UP	A	B	AB		tc					SUM	MEAN
1	-1	-1	1		(1)	18.2	18.9	12.9	14.4	64.4	16.1
2	1	-1	-1		a	27.2	24	22.4	22.5	96.1	24.025
3	-1	1	-1		b	15.9	14.5	15.1	14.2	59.7	14.925
4	1	1	1		ab	41	43.9	36.3	39.9	161.1	40.275
Total Treatments	4				n	4					
					N	16				381.3	23.83125
	A	B	AB	ERROR	TOTAL						
CONTRAST	133.1	60.3	69.7								
EFFECTS	16.6375	7.5375	8.7125								
+ MEAN	32.15	27.6	28.1875								
- MEAN	15.5125	20.0625	19.475								
SUM OF SQUARES	1107.225625	227.255625	303.630625	71.7225	1709.834375						
DF	1	1	1	12	15						
MEAN SQUARES	1107.225625	227.255625	303.630625	5.976875							
F RATIO	185.2515947	38.02248248	50.8008993								
PROBABILITY	1.17E-08	4.83E-05	1.20E-05								
SIGNIFICANCE	**SSD**	**SSD**	**SSD**								

Description of Results

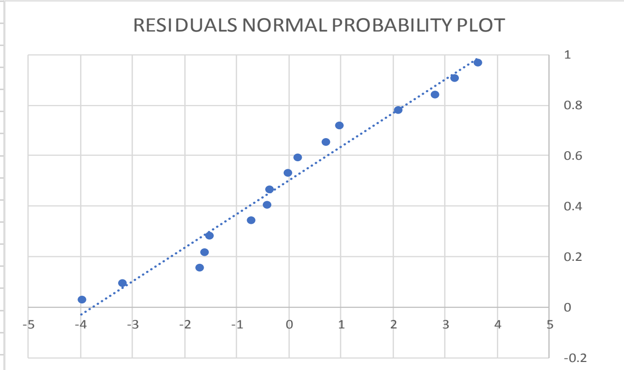
- Name, Objective and Factors as described in the test design.
- Setup Treatments
  - 1: A- (1/8 bit), B- (500 RPM), AB interaction=-1x-1=1
  - 2: A+ (1/4 bit), B- (500 RPM), AB interaction= 1x-1=-1
  - 3: A- (1/8 bit), B+ (1000RPM), AB interaction= -1x1=-1
  - 4: A+ (1/4 bit), B+ (1000RPM), AB interaction= 1x1=1
- REP1 – 4 : Vibration measurement taken under each treatment.
- Sum: The sum of the 4 readings taken under each setup treatment.
- Mean: The mean value of the readings taken for each setup treatment.
- Contrast: C=Sum (Factor +)- Sum (Factor -) This is calculated for each factor and the interaction of the factors.
- Effects: This is the calculated difference in moving from the – to the + condition for each factor and interaction. The calculation is E=2C/Tn where C=Contrast; T=Treatments (2^2=4); and n=Replications. In this example the calculated effect will be 2C/16.
- +Mean, -Mean: Mean value of the tests when the factor is at the +1 or -1 level.

- Sum of Squares: The Sum of Squares is the Contrast squared divided by the product of treatments and number of replications. $SS=C^2/(Tn)$
- DF: Degrees of Freedom. For the factors, the DF is the number of levels - 1. The total test DF is the number of tests -1. The Error term is the difference between the total and sum of factors, i.e. 15-3=12.
- Mean Squares: MS=SS/DF. This is essentially the signal level for the factors or interactions. The error term MS_E is essentially the noise level within the test.
- F Ratio: The F Ratio is the signal to noise ratio, which is the mean squares of the factor divided by the mean square Error. $F=MS_F/MS_E$
- Probability: This value is comes from the normal probability curve for the value of the F Ratio and the number of degrees of freedom for the factor and error. It is a measure of the probability of that ratio showing up randomly.
- Significance: on the spreadsheet, this is set to show a statistically significant difference at the 95% confidence level, i.e. when the probability value is below 0.05 there is declared a Statistically Significant Difference. (SSD). In this example it indicates that the bit size, RPM both introduce a Statistically Significant Difference as well as a SSD in the interaction of the two.

7.8 Validation of the results

The residuals calculated indicate a roughly linear plot on a normal probability plot with no outlying data points.

#	TR	TM	TR-TM	RANK(k)	Pk (k-.5)/N
1R1	18.2	16.1	2.1	13	0.78125
1R2	18.9	16.1	2.8	14	0.84375
1R3	12.9	16.1	-3.2	2	0.09375
1R4	14.4	16.1	-1.7	3	0.15625
2R1	27.2	24.025	3.175	15	0.90625
2R2	24	24.025	-0.025	9	0.53125
2R3	22.4	24.025	-1.625	4	0.21875
2R4	22.5	24.025	-1.525	5	0.28125
3R1	15.9	14.925	0.975	12	0.71875
3R2	14.5	14.925	-0.425	7	0.40625
3R3	15.1	14.925	0.175	10	0.59375
3R4	14.2	14.925	-0.725	6	0.34375
4R1	41	40.275	0.725	11	0.65625
4R2	43.9	40.275	3.625	16	0.96875
4R3	36.3	40.275	-3.975	1	0.03125
4R4	39.9	40.275	-0.375	8	0.46875



Review

The results indicate that the lower diameter bit (1/8”) combined with the lower RPM results in a lower amount of overall vibration. This is a logical result considering the milling process. The decision is to implement the change.

8. Example 2 – Milling Vibration 2^3

8.1 As was done in Example 1, this will be a test to modify a substrate milling process to reduce the surface variation. As before, the test metric will be vibration data which can be gathered directly on the machine.

8.2 Three factors will be included in the factorial test, bit diameter; spindle RPM and substrate lot. Based on the slot size a $^1/_8$-inch diameter and a ¼ -inch diameter bit will be tested. The current machine capabilities allow operation at 500 and 1000 RPM.

8.3 As in Example 1, the metric used will be the vibration measured during the milling process. The milling machine will be fitted with a 3-axis accelerometer and the RMS value of the vibration vector recorded for each test condition.

8.4 The test is of three factors which requires 8 test treatments. In order to minimize the run count only 2 replications will be run, resulting in a total of 16 runs.

8.5 The incoming substrate is made up of two supply lots. The run order is selected for each set to ensure that the replication order does not repeat, and all 8 test treatments are run each day. The resulting test plan is shown below.

8.6 Following the review of the test design, sequence and plans with the operators and technicians the test is conducted.

Trial Name	Vibration Reduction								
Trial Objective	Reduce vibration during milling process. Test bit size and RPM.								
FACTORS		A	B	C		RUN ORDER			
LEVELS	-	bit 1/8	45 RPM	Lot 21175					
	+	Bit 1/4	90 RPM	Lot 24875		REP 1	REP 2	REP 3	REP 4
SET UP		A	B	C					
1		-	-	-		1	4		
2		+	-	-		2	3		
3		-	+	-		3	2		
4		+	+	-		4	1		
5		-	-	+		5	8		
6		+	-	+		6	7		
7		-	+	+		7	6		
8		+	+	+		8	5		
9		-	-	-					
10		+	-	-					
11		-	+	-					
12		+	+	-					
13		-	-	+					
14		+	-	+					
15		-	+	+					
16		+	+	+					
Metrics									
Vibration	Fluke 810 vibration tester, sensor mounted on board at position A7.								
Run Instructions									
	Substrate Lots as defined in test plan								
	Run 2 replications of each Treatment for a total of 8 runs.								

8.7 Example 2 – Analysis of Results

8.7.1 The measured vibration of the substrate board is entered in the spreadsheet as the data is generated. The results of the analysis are shown below:

Trial Name	Vibration Reduction														
Trial Objective	Reduce vibration during milling process. Test bit size and RPM.									METRIC					
Factors	A	B		C						VIBRATION					
-	bit 1/8	45 RPM		Lot 21175						REP 1	REP 2	REP 3	REP 4		
+	Bit 1/4	90 RPM		Lot 24875											
TREATMENT	A	B	AB	C	AC	BC	ABC	I	tc					SUM	MEAN
1	-1	-1	1	-1	1	1	-1	1	(1)	18.2	12.9			31.1	15.55
2	1	-1	-1	-1	-1	1	1	1	a	27.2	22.4			49.6	24.8
3	-1	1	-1	-1	1	-1	1	1	b	15.9	15.1			31	15.5
4	1	1	1	-1	-1	-1	-1	1	ab	41	36.3			77.3	38.65
5	-1	-1	1	1	-1	-1	1	1	c	18.9	14.4			33.3	16.65
6	1	-1	-1	1	1	-1	-1	1	ac	24	22.5			46.5	23.25
7	-1	1	-1	1	-1	1	-1	1	bc	14.5	14.2			28.7	14.35
8	1	1	1	1	1	1	1	1	abc	43.9	39.9			83.8	41.9
Total Treatments	8								n	2					
									N	16				381.3	23.83125
	A	B	AB	C	AC	BC	ABC	ERROR	TOTAL						
CONTRAST	133.1	60.3	69.7	3.3	3.5	5.1	14.1								
EFFECTS	16.6375	7.5375	8.7125	0.4125	0.4375	0.6375	1.7625								
+ MEAN	32.15	27.6	28.1875	24.0375	24.05	24.15	24.7125								
- MEAN	15.5125	20.0625	19.475	23.625	23.6125	23.5125	22.95								
SUM OF SQUARES	1107.225625	227.255625	303.630625	0.680625	0.765625	1.625625	12.425625	56.225	1709.834						
DF	1	1	1	1	1	1	1	8	15						
MEAN SQUARES	1107.225625	227.255625	303.630625	0.680625	0.765625	1.625625	12.425625	7.028125							
F RATIO	157.5421076	32.33517119	43.20222321	0.096843041	0.108937305	0.231302801	1.767985771								
PROBABILITY	1.52E-06	4.62E-04	1.74E-04	7.64E-01	7.50E-01	6.43E-01	2.20E-01								
SIGNIFICANCE	SSD	SSD	SSD												

Description of Results

- Name, Objective and Factors as described in the test design.
- Description of Factors
    - A - = 1/8 bit, A+=1/4 bit
    - B-= 45 RPM, B+=90 RPM
    - C-= Substrate Lot 21175, C+= Lot 24875
- Setup Treatments
    - 1: A-, B-, C-; AB=+,AC=+,BC=+
    - 2: A+, B-,C-; AB=-,AC=-,BC=+
    - 3: A-, B+,C-; AB=-,AC=+,BC=-
    - 4: A+, B+,C-; AB=+,AC=-,BC=-
    - 5: A-, B-,C+; AB=+,AC=-,BC=-
    - 6: A+, B-,C+; AB=-,AC=+,BC=-
    - 7: A-, B+,C+; AB=-,AC=-,BC=+
    - 8: A+, B+,C+; AB=+,AC=+,BC=+

- REP1 – 2 : Vibration measurement taken under each treatment. Only 2 Replications are done in this test.
- Sum: The sum of the 2 readings taken under each setup treatment.
- Mean: The mean value of the readings taken for each setup treatment.
- Contrast: C=Sum (Factor +)- Sum (Factor -) This is calculated for each factor and the interaction of the factors.
- Effects: This is the calculated difference in moving from the – to the + condition for each factor and interaction. The calculation is E=2C/Tn

where C=Contrast; T=Treatments (2^3=8); and n=Replications. In this example the calculated effect will be 2C/16.

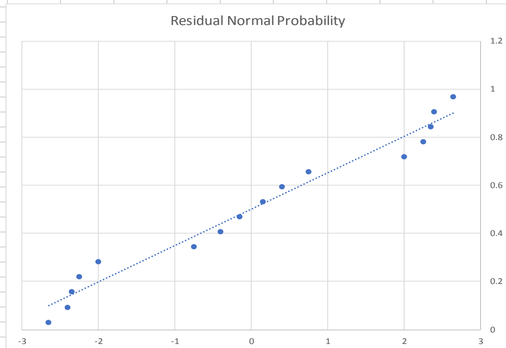
- +Mean, -Mean: Mean value of the tests when the factor is at the +1 or -1 level.
- Sum of Squares: The Sum of Squares is the Contrast squared divided by the product of treatments and number of replications. $SS=C^2/(Tn)$
- DF: Degrees of Freedom. For the factors, the DF is the number of levels - 1. The total test DF is the number of tests -1. The Error term is the difference between the total and sum of factors, i.e. 15-3=12.
- Mean Squares: MS=SS/DF. This is essentially the signal level for the factors or interactions. The error term MS_E is essentially the noise level within the test.
- F Ratio: The F Ratio is the signal to noise ratio, which is the mean squares of the factor divided by the mean square Error. $F=MS_F/MS_E$
- Probability: This value is from the normal probability curve for the value of the F Ratio and the number of degrees of freedom for the factor and error. It is a measure of the probability of that ratio showing up randomly.
- Significance: on the spreadsheet, this is set to show a statistically significant difference at the 95% confidence level, i.e. when the probability value is below 0.05 there is declared a Statistically Significant Difference. (SSD). In this example it indicates that the bit size, RPM both introduce a Statistically Significant Difference as well as a SSD in the interaction of the two. The added factor of the substrate lot does not indicate any significance.

8.8 Validation of Results

8.8.1 A calculation and plot of the results provides an indication of a normal distribution without any great outliers.

#	TR	TM	RESIDUAL	RANK(k)	Pk (k-.5)/N
1R1	18.2	15.55	2.65	16	0.96875
1R2	12.9	15.55	-2.65	1	0.03125
1R3					
1R4					
2R1	27.2	24.8	2.4	15	0.90625
2R2	22.4	24.8	-2.4	2	0.09375
2R3					
2R4					
3R1	15.9	15.5	0.4	10	0.59375
3R2	15.1	15.5	-0.4	7	0.40625
3R3					
3R4					
4R1	41	38.65	2.35	14	0.84375
4R2	36.3	38.65	-2.35	3	0.15625
4R3					
4R4					
5R1	18.9	16.65	2.25	13	0.78125
5R2	14.4	16.65	-2.25	4	0.21875
5R3					
5R4					
6R1	24	23.25	0.75	11	0.65625
6R2	22.5	23.25	-0.75	6	0.34375
6R3					
6R4					
7R1	14.5	14.35	0.15	9	0.53125
7R2	14.2	14.35	-0.15	8	0.46875
7R3					
7R4					
8R1	43.9	41.9	2	12	0.71875
8R2	39.9	41.9	-2	5	0.28125
8R3					
8R4					

RESIDUAL	RANK(k)	(k-.5)/N
-2.65	1	0.03125
-2.4	2	0.09375
-2.35	3	0.15625
-2.25	4	0.21875
-2	5	0.28125
-0.75	6	0.34375
-0.4	7	0.40625
-0.15	8	0.46875
0.15	9	0.53125
0.4	10	0.59375
0.75	11	0.65625
2	12	0.71875
2.25	13	0.78125
2.35	14	0.84375
2.4	15	0.90625
2.65	16	0.96875



8.9 Review

Following the review, the decision is to implement the 1/8 bit at 45 RPM to provide the least vibration in the milling process. The positive results occur with both substrate selections so the selection can be considered robust for the incoming material feed.

9. Example 3 – Non Woven Web Tensile Strength 2^4

9.1 This is a test on a non-woven process line to improve the machine direction tensile strength of the finished web. A brief process description is as follows:

9.1.1 The stock solution is fed into a slanting box forming section where an adjustable vacuum is used to draw off the excess fluid. The vacuum level can be adjusted in a profile along the length of the forming box.

9.1.2 The binding agent for the web is added and the excess removed by vacuum systems.

9.1.3 The web is dried in an air dryer to the target moisture content.

9.1.4 Specification parameters include the weight per square foot of the finished web (basis weight); web thickness (caliper); binder level % and moisture content; machine direction tensile (MDT); cross direction tensile (CDT).

9.1.5 The goal of the test is to increase the resulting machine direction tensile strength while to the specification targets of the other parameters.

9.2 Four factors are selected for testing. The first two are expected to impact the entanglement of the stock, the latter two to impact the binding of the web.

9.2.1 Slant box vacuum profile

9.2.1.1 A- Decreasing vacuum bottom to top

9.2.1.2 A+ Increasing vacuum bottom to top

9.2.2 Former Concentration

9.2.2.1 B- .45 ratio

9.2.2.2 B+ .66 ratio

9.2.3 Saturator Concentration

9.2.3.1 C- .012

9.2.3.2 C+ .021

9.2.4 Dryer Static Pressure

9.2.4.1 D- 0.8

9.2.4.2 D+ 1.2

9.3 The test metric is MDT. Other specification values will be monitored and controlled to target as needed through the test.

9.4 The test is of four factors at two levels each which requires 16 test treatments. In order to minimize the run count only 2 replications will be run, resulting in a total of 32 runs.

9.5 The incoming stock is from a single supply lot. The run order is selected for each set to ensure that the replication order does not repeat, and all 16 test treatments are run each day. The resulting test plan is shown below.

9.6 Following the review of the test design, sequence and plans with the operators and technicians the test is conducted.

Trial Name	WEB FORMATION TESTING								
Trial Objective	INCREASE MACHINE DIRECTION TENSILE STRENGTH								
FACTORS		A SLANT BOX VACUUM PROFILE	B FORMER CONCENTRATION	C SATURATOR CONCENTRATION	D DRYER STATIC PRESSURE	RUN ORDER			
LEVELS	-	DEC BOT TO TOP	0.45	1.20%	0.8				
	+	INC BOT TO TOP	0.66	2.10%	1.2	REP 1	REP 2	REP 3	REP 4
SET UP		A	B	C	D				
1		-	-	-	-	1	16		
2		+	-	-	-	2	15		
3		-	+	-	-	3	14		
4		+	+	-	-	4	13		
5		-	-	+	-	9	8		
6		+	-	+	-	10	7		
7		-	+	+	-	11	6		
8		+	+	+	-	12	5		
9		-	-	-	+	5	12		
10		+	-	-	+	6	11		
11		-	+	-	+	7	10		
12		+	+	-	+	8	9		
13		-	-	+	+	13	4		
14		+	-	+	+	14	3		
15		-	+	+	+	15	2		
16		+	+	+	+	16	1		
Metrics									
MD TENSILE	CD TENSILE								
Run Instructions									
Caliper, basis weight and finished binder level to be maintained on target / within specification conditions									
Alter Conditions and allow line to stabilize before taking test samples.									
Standard SPC / QC procedures - isolate any out of specification material.									
Due to time lags for Saturator Concentration levels the run order will be set to minimize transitions in trial.									
Run Notes									

9.7 Example 3 – Analysis of Results

9.7.1 The measured machine direction tensile strength is entered in the spreadsheet as the data is generated. The results of the analysis are shown below:

Trial Name	WEB FORMATION TESTING																						
	INCREASE MACHINE DIRECTION TENSILE STRENGTH																	METRIC					
Factors	A SLANT BOX VACUUM PROFILE	B FORMER CONCENTRATION		C SATURATOR CONCENTRATION				D DRYER STATIC PRESSURE															
-	DEC BOT TO TOP	0.45		0.012				0.8										REP 1	REP 2	REP 3	REP 4		
+	INC BOT TO TOP	0.66		0.021				1.2															
SET UP	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD	I	tc					SUM	MEAN
1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	1	(1)	11.7	11.46			23.16	11.58
2	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1	1	a	13.26	15.65			28.91	14.455
3	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	1	b	12.2	13.1			25.3	12.65
4	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	ab	10.78	13.31			24.09	12.045
5	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	c	14.9	14.7			29.6	14.8
6	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	ac	16.7	15.25			31.95	15.975
7	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	bc	16.5	14.03			30.53	15.265
8	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	abc	14.9	15.3			30.2	15.1
9	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	1	d	12.67	14.2			26.87	13.435
10	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	ad	12.1	12.1			24.2	12.1
11	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	bd	12.4	13.7			26.1	13.05
12	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	abd	13.5	12.2			25.7	12.85
13	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	cd	13.7	14.8			28.5	14.25
14	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	acd	16.1	15.8			31.9	15.95
15	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	bcd	15.5	15.6			31.1	15.55
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	abcd	15.3	16.6			31.9	15.95
Total Treatments	16																n	2					
																	N	32				450.01	14.06281
	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD	ERROR	TOTAL						
CONTRAST	7.69	-0.17	-9.97	41.35	4.75	3.73	-0.59	2.53	-5.43	6.83	9.31	-0.29	9.79	0.01	-9.15								
EFFECTS	0.480625	-0.010625	-0.623125	2.584375	0.296875	0.233125	-0.03687	0.158125	-0.33938	0.426875	0.581875	-0.01812	0.611875	0.000625	-0.57188								
+ MEAN	14.303125	14.0575	13.75125	15.355	14.21125	14.17938	14.04438	14.141875	13.89313	14.27625	14.35375	14.05375	14.36875	14.06313	13.77688								
- MEAN	13.8225	14.068125	14.374375	12.770625	13.914375	13.94625	14.08125	13.98375	14.2325	13.84938	13.77188	14.07188	13.75688	14.0625	14.34875								
SUM OF SQUARES	1.848003125	0.000903125	3.106278125	53.43195313	0.705078125	0.434778	0.010878	0.200028125	0.921403	1.457778	2.708628	0.002628	2.995128	3.12E-06	2.616328	15.05245	85.49225						
DF	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	16	31						
MEAN SQUARES	1.848003125	0.000903125	3.106278125	53.43195313	0.705078125	0.434778	0.010878	0.200028125	0.921403	1.457778	2.708628	0.002628	2.995128	3.12E-06	2.616328	0.940778							
F RATIO	1.96433471	0.000959977	3.301817976	56.79548844	0.749462712	0.462147	0.011563	0.212619873	0.979405	1.549545	2.879136	0.002794	3.183671	3.32E-06	2.781026								
PROBABILITY	0.180150275	0.97566591	0.087980816	1.19273E-06	0.399441798	0.50634	0.915705	0.65092781	0.337077	0.231128	0.109098	0.958502	0.093351	0.998568	0.114837								
SIGNIFICANCE				**SSD**																			
Instructions																							
Comments																							

The only factor indicating a Statistically Significant Difference is the Saturator concentration, which indicates a move from 12.7 psi at .012 % to 15.3 psi tensile strength at .021%. A check of the other data (no provided in this example) on the run confirms that the caliper, basis weight and finished binder level were maintained at specification for the run and samples provided. The change could be explained as a change in the viscosity of the binder at the higher concentration which improves the binding effect at the fiber intersections.

The other factors do not show any statistical significance or major effect. A plot of the residuals does not show a significant departure from a normal distribution, as shown below:

#	TR	TM	RESIDUAL	RANK(k)	Pk (k-.5)/N
1R1	11.7	11.58	0.12	20	0.609375
1R2	11.46	11.58	-0.12	13	0.390625
1R3					
1R4					
2R1	13.26	14.455	-1.195	3	0.078125
2R2	15.65	14.455	1.195	30	0.921875
2R3					
2R4					
3R1	12.2	12.65	-0.45	10	0.296875
3R2	13.1	12.65	0.45	23	0.703125
3R3					
3R4					
4R1	10.78	12.045	-1.265	1	0.015625
4R2	13.31	12.045	1.265	32	0.984375
4R3					
4R4					
5R1	14.9	14.8	0.1	19	0.578125
5R2	14.7	14.8	-0.1	14	0.421875
5R3					
5R4					
6R1	16.7	15.975	0.725	28	0.859375
6R2	15.25	15.975	-0.725	5	0.140625
6R3					
6R4					
7R1	16.5	15.265	1.235	31	0.953125
7R2	14.03	15.265	-1.235	2	0.046875
7R3					
7R4					
8R1	14.9	15.1	-0.2	11	0.328125
8R2	15.3	15.1	0.2	22	0.671875
8R3					
8R4					
9R1	12.67	13.435	-0.765	4	0.109375
9R2	14.2	13.435	0.765	29	0.890625
9R3					
9R4					
10R1	12.1	12.1	0	16	0.484375
10R2	12.1	12.1	0	16	0.484375
10R3					
10R4					
11R1	12.4	13.05	-0.65	6	0.171875
11R2	13.7	13.05	0.65	25	0.765625
11R3					
11R4					
12R1	13.5	12.85	0.65	26	0.796875
12R2	12.2	12.85	-0.65	6	0.171875
12R3					
12R4					
13R1	13.7	14.25	-0.55	9	0.265625
13R2	14.8	14.25	0.55	24	0.734375
13R3					
13R4					
14R1	16.1	15.95	0.15	21	0.640625
14R2	15.8	15.95	-0.15	12	0.359375
14R3					
14R4					
15R1	15.5	15.55	-0.05	15	0.453125
15R2	15.6	15.55	0.05	18	0.546875
15R3					
15R4					
16R1	15.3	15.95	-0.65	6	0.171875
16R2	16.6	15.95	0.65	26	0.796875
16R3					
16R4					

#	RESIDUAL	RANK(k)	(k-.5)/N
4R1	-1.265	1	0.015625
7R2	-1.235	2	0.046875
2R1	-1.195	3	0.078125
9R1	-0.765	4	0.109375
6R2	-0.725	5	0.140625
11R1	-0.65	6	0.171875
12R2	-0.65	6	0.171875
16R1	-0.65	6	0.171875
13R1	-0.55	9	0.265625
3R1	-0.45	10	0.296875
8R1	-0.2	11	0.328125
14R2	-0.15	12	0.359375
1R2	-0.12	13	0.390625
5R2	-0.1	14	0.421875
15R1	-0.05	15	0.453125
10R1	0	16	0.484375
10R2	0	16	0.484375
15R2	0.05	18	0.546875
5R1	0.1	19	0.578125
1R1	0.12	20	0.609375
14R1	0.15	21	0.640625
8R2	0.2	22	0.671875
3R2	0.45	23	0.703125
13R2	0.55	24	0.734375
11R2	0.65	25	0.765625
12R1	0.65	26	0.796875
16R2	0.65	26	0.796875
6R1	0.725	28	0.859375
9R2	0.765	29	0.890625
2R2	1.195	30	0.921875
7R1	1.235	31	0.953125
4R2	1.265	32	0.984375
1R3			
1R4			
2R3			
2R4			
3R3			
3R4			
4R3			
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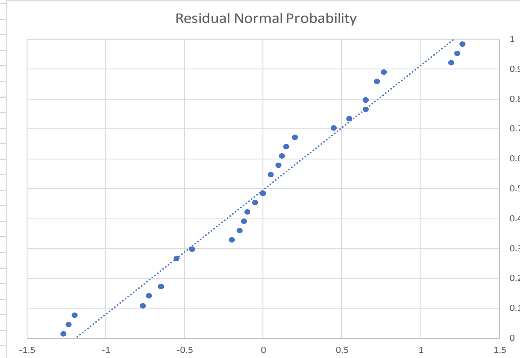

With this testing the decision is made to increase the saturator concentration to .21% for future production runs.

10. Conclusion

The factorial testing and analysis methods presented in this text provide a means of quickly identifying significant factors and interactions in product or process operations. The spreadsheets provided give a means of designing and executing and analyzing the results to provide clear direction and focus for the product or process improvement.

Once the number of factors for the test are selected the proper spreadsheet template for the test can be copied and filled in. The analysis worksheet can be copied as needed for analysis of different responses.

When a Statistically Significant Difference is identified for a factor or interaction it is up to the process engineer to validate the results and perform the additional steps necessary to show the economic or objective feasibility of the change.

11. Use of the Excel Templates

Templates are provided for 2^2, 2^3 and 2^4 test designs. They can be used as follows;

1. Copy the template and save as an xlsx worksheet.
2. Rename the worksheet to the trial name.
3. The calculation on the worksheet are protected to prevent inadvertent changes in the calculations. The locations for entry are unprotected.
4. Open the worksheet and fill in the factors, levels and run order on the design tab.
5. Create additional analysis tabs as necessary for all the factors to be analyzed by right clicking the tab and selecting “copy”.
6. Rename the copied tabs as needed to identify the metric.
7. Enter the metric on the spreadsheet.
8. As the trial is run, enter the results in the proper cells.
9. Check the residuals by copying the calculated values to the plot areas defined.

12. Reference and Further Reading

- Douglas C Montgomery, *Design and Analysis of Experiments*, 2nd edition, John Wiley & Sons, 1984.

- Rickmers and Todd, *Statistics: an Introduction*, 1st edition, McGraw Hill, 1967.