Compressible Flow Modeling in a Constant Area Duct

Instructor: Neil Hicks, P.E.

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PDH Online | PDH Center
5272 Meadow Estates Drive
Fairfax, VA 22030-6658
Phone & Fax: 703-988-0088
www.PDHonline.org
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Compressible Flow Modeling In a Constant Area Pipe

Neil Hicks PE

Course Content

Figure 1 shows a system that is designed to power a gearbox through the use of a turbine by expansion of a constant mass flow rate helium gas. In this system the designer wants to control the mass flow rate using a regulator and a sonic orifice. It is imperative that the orifice controls the mass flow to achieve the required turbine output for the gearbox.

The required turbine input conditions are given at the inlet of the turbine. The job of the engineer is to properly size the pipe diameter to achieve the turbine inlet pressure and temperature of the helium gas. The regulator has been set to maintain an inlet orifice pressure of 1500 psia and the orifice has been sized to choke and control the mass flow rate at 4.00 lbm/sec. The maximum backpressure from the downstream system resistance is 1050 psia. Any pressure above that will unchoke the orifice and reduce the mass flow rate. To stay conservative, the engineer’s goal will be to achieve a backpressure or orifice out recovery pressure of no greater than 950 psia.

The following instructions will review compressible flow properties of a gas assuming the flow will be adiabatic and with frictional losses (Fanno Flow). In addition, a Fanno Flow algorithm will be derived to determine the required pipe diameter for the system described in Figure 1.
Adiabatic Flow with Friction In a Constant Area Pipe

Friction cannot be ignored in a long constant area pipe. In addition, if the pipe is insulated or the velocities of the gases are high enough that gas temperature is not effected by heat transfer from or to the surroundings, then modeling can be performed assuming adiabatic flow. Adiabatic flow cannot be assumed in long unsulated pipes in which the velocity of the gas is slow and the gas temperature will be affected by the heat transfer.

The following Figure 2 and Table 1 summarizes the effects of Fanno flow properties in a constant area pipe. The reader should review Figure 2 and Table 1 until he/she has a complete understanding. It is imperative at this point to understand static pressure, total pressure, and temperature always decreases when velocity of a gas entering a pipe is subsonic, and therefore the density decreases. Because the density of the gas is decreasing then the velocity of the gas particles must increase to maintain a constant flow rate. If the velocity of the gas becomes equal to the speed of sound, then the flow rate will choke at the end of the pipe, and the pipe can take control of the total flow rate by unchoking the flow-controlling orifice in the case of Figure 1 system. Thus the operator of the system will lose the capability of knowing the exact flow rate based on the orifice upstream pressure, temperature, and orifice Cd.

![Figure 2: Schematic Ts Diagram Fanno Flow Line for a Constant Area Duct](image)

Where
- \( T \) = Static Temperature
- \( P \) = Static Pressure
- \( P_0 \) = Stagnation Pressure
- \( \rho \) = Density
### Property

<table>
<thead>
<tr>
<th>Property</th>
<th>Duct Entry Subsonic M&lt;1</th>
<th>Duct Entry Supersonic M&gt;1</th>
<th>Obtained from:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stagnation Temperature, To</td>
<td>Constant</td>
<td>Constant</td>
<td>Energy Equation</td>
</tr>
<tr>
<td>Entropy, s</td>
<td>Increases</td>
<td>Increases</td>
<td>Second Law</td>
</tr>
<tr>
<td>Stagnation Pressure, Po</td>
<td>Decreases</td>
<td>Increases</td>
<td>Entropy increases</td>
</tr>
<tr>
<td>Temperature, T</td>
<td>Decreases</td>
<td>Increases</td>
<td>Shape of Fanno line</td>
</tr>
<tr>
<td>Static Pressure, P</td>
<td>Decreases</td>
<td>Increases</td>
<td>Equation of state, and effects on ρ, T</td>
</tr>
<tr>
<td>Density, ρ</td>
<td>Decreases</td>
<td>Increases</td>
<td>Continuity equation and effect on V</td>
</tr>
<tr>
<td>Velocity, V</td>
<td>Increases</td>
<td>Decreases</td>
<td>Energy Equation and trend of T</td>
</tr>
<tr>
<td>Mach Number, M</td>
<td>Increases</td>
<td>Decreases</td>
<td>Trends of V, T, and definition of M</td>
</tr>
</tbody>
</table>

Table 1: Summary of Effects of Friction on a Compressible Gas Flowing In a Constant Area Pipe (Table copied from Introduction to Fluid Mechanics by Fox & McDonald 4th Edition)

Figure 2 also shows what happens if the velocity of the gas entering a pipe is supersonic. Friction will cause the pressure and temperature to increase in the direction of flow. The density will increase and the velocity will decrease to maintain the constant mass flow rate. Eventually the velocity in the pipe becomes sonic. In this course, the discussion and quiz will refer to modeling compressible flow with the velocity of the gas entering a constant area pipe at subsonic conditions.

Figure 3 is a diagram of gas flowing in a constant area pipe. The key to predicting gas properties using FANNO flow is knowing either the inlet or the end condition pressure, temperature, and mass flow rate of the gas. In the case of this course, the engineer knows the pipe end conditions.

Reviewing Figure 3, one can see that L* is some where in space downstream (hopefully) of the outlet of the pipe. L* is a point where the flow velocity will be predicted to reach the speed of sound and the mach number will be one (sonic). If $L^*_{max}$ 1 or 2 is calculated or predicted to be a length less than $L_1$ (pipe length), then the sonic point is inside the pipe and the flow will become sonic at the exit. Typically this is called Fanno choking. As explained in the preceding text, once the pipe becomes choked it could possibly take control of the mass flow rate. It is not uncommon to have more than one
choking points in a system, but a problem can occur if the pipe choking creates enough resistance to unchoke the upstream flow-controlling device. If the upstream flow-controlling device is instrumented to measure upstream pressure and temperature, then the pressure can be regulated to change the mass flow rate using energy equations. Unchoking such a device due to excessive resistance negates the upstream flow rate predicting capability.

Using the Figure 3 as a reference and the equations described in the following algorithm, one can easily solve for conditions at point 1 or point 2 by simply calculating the mach number at either point using the L*max1 or L*max2 respectfully. The remainder of the text will provide step-by-step directions to demonstrate this logic using the described problem in Figure 1.
Solving for the Diameter

Step 1: Since the conditions are known for point 2 of Figure 3, then the analysis will start by solving for the Mach number. However before deriving conditions at point 2, an initial guess of the pipe diameter is required. Several guesses may need to be iterated through the algorithm before achieving the proper pressure drop. For the first guess, the author will use a nominal pipe diameter of 1.25 inch for a seamless stainless steel pipe with a wall thickness of 0.120 inches and an inside diameter of 1.01 inch.

\[ A = \frac{\pi D_i^2}{4} = 0.7853982D_i^2 \quad (1) \]

\[ \rho = \frac{p}{ZRT} \quad (2) \]

where \( Z = 1 \) if the gas is assumed Ideal. However for this exercise the author chose to treat the problem as Real Gas and \( Z \) was found using NIST.

\[ \text{velocity} = \frac{m}{\rho A} = 183.3465 \frac{m}{\rho D_i^2} \quad (3) \]

\[ \dot{V} = \frac{m*60}{\rho_{std}} \quad (4) \]

\[ \text{Mach}# = \frac{v}{\sqrt{kRg_cT}} = \frac{0.1762268v}{\sqrt{kRT}} \quad (5) \]

where \( k \) is the ratio of specific heats \( \frac{c_p}{c_v} \)

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**Known Outlet Conditions 2**

- **P2** = 805 psia
- **T2** = 533 R
- **mdot2** = 4.000000 lbm/s
- **L1** = 60 in
- **Dh** = 1.01 in
- **Gas Type** = 8

**Calculated Inlet Conditions 2**

- **Gas** = Helium
- **Gas Constant** = 386.040 (ft-lbf)/(lbm-R)
- **density2** = 0.545266 lbm/ft^3
- **A1** = 0.801 in^2
- **vdot1** = 22776.092 scfm
- **velocity2** = 1318.504 ft/s
- **k** = 1.667

**Mach 2** = 0.3967

Table 2: Input at Point 2 and Calculated Conditions (reference Figure 1)
Step 2: Once the mach number is known at point 2, the following conditions can be determined at the imaginary sonic point \( L^* \) of Figure 3 using the following algorithm:

\[
Re = \frac{\rho v D}{\mu} \quad (6)
\]

where \( \mu = 0.000012 \text{ lbm/ft - sec for helium} \)

\[
f = \left[ 1.14 - 2 \log_{10} \left( \frac{\varepsilon}{D_h} + \frac{21.25}{Re^{0.9}} \right) \right]^{-2} \quad (7)
\]

where \( \varepsilon = 0.000984 \) for seamless smooth stainless steel duct

\[
T^* = \frac{(T)(2)[1 + (0.5)(k - 1)M^2_i]}{1 + k} \quad (8)
\]

\[
p^* = \frac{(P_i)(M_i)}{\sqrt{1 + k}} \quad \sqrt{2[1 + (0.5)(k - 1)M^2_i]} \quad (9)
\]

\[
\frac{fL^*_\text{max}_2}{D_h} = \left[ \frac{1 - M^2_i}{kM^2_i} + \frac{1 + k}{2k} \ln \left\{ \frac{(1 + k)M^2_i}{2[1 + (0.5)(k - 1)M^2_i]} \right\} \right] \quad (10)
\]

\[
L^*_\text{max}_2 = \left( \frac{D_h}{f} \right) \left[ \frac{1 - M^2_i}{kM^2_i} + \frac{1 + k}{2k} \ln \left\{ \frac{(1 + k)M^2_i}{2[1 + (0.5)(k - 1)M^2_i]} \right\} \right] \quad (11)
\]

Table 3 gives the derived results of the conditions at \( L^*_\text{max}_2 \). Note that \( L^*_\text{max}_2 \) is positive showing the sonic point is downstream of the pipe. Having this data enables us to determine \( L^*_\text{max}_1 \) which will be explained in step 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T^* )</td>
<td>420.682</td>
</tr>
<tr>
<td>( P^* )</td>
<td>283.736 psia</td>
</tr>
<tr>
<td>Darcy friction factor</td>
<td>0.019599031</td>
</tr>
<tr>
<td>( L^*_\text{max}_2 )</td>
<td>99.021 in</td>
</tr>
<tr>
<td>Line Length Check</td>
<td>Not Choked</td>
</tr>
<tr>
<td>( fL_2/Dh )</td>
<td>1.164</td>
</tr>
<tr>
<td>( fL^*_2/Dh )</td>
<td>1.921</td>
</tr>
</tbody>
</table>

Table 3: Calculated Conditions at Imaginary Sonic Point \( L^*_\text{max}_2 \)
Step 3: The following step derives \( L^{*}\text{max}_1 \) by simply adding the length of the pipe to \( L^{*}\text{max}_2 \) derived in step 2.

\[
L^{*}\text{max}_1 = L_1 + L^{*}\text{max}_2 \quad (12)
\]

Step 4: Once \( L^{*}\text{max}_1 \) is calculated the following equation is used to derive the Mach number at point 1. This process can be simplified greatly by using some type of solver program or a spreadsheet. In table 4, the Mach number is derived by iterating a mach number until the difference between the calculated \( L^{*}\text{max}_1 \) and iterated \( L^{*}\text{max}_1 \) is equal to zero within six significant digits.

\[
L^{*}\text{max}_1 = \left( \frac{D_h}{f} \right) \left( \frac{1 - M_{i}^{2}}{kM_{i}^{2}} \right) + \left( \frac{1 + k}{2k} \right) \ln \left( \frac{(1 + k)M_{i}^{2}}{2[1 + (0.5)(k-1)M_{i}^{2}]} \right) \quad (13)
\]

Table 4: Calculated Mach number at Point 1 Pipe Inlet

<table>
<thead>
<tr>
<th>Calculated Inlet Conditions 1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated ( L_1 + L^{<em>}\text{max}_2 = L^{</em>}\text{max}_1 )</td>
<td>159.021</td>
<td>in</td>
</tr>
<tr>
<td>Guess Mach 1 =</td>
<td>0.3391</td>
<td></td>
</tr>
<tr>
<td>Use Solver</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iterated ( L^{*}\text{max}_1 ) =</td>
<td>159.021</td>
<td>in</td>
</tr>
<tr>
<td>Difference between Calculated and Iterated 4fl2*/Dh =</td>
<td>0.000000</td>
<td>iterates to 0000</td>
</tr>
</tbody>
</table>

Step 5: Now that the Mach number is known at the inlet of the pipe, the conditions at the inlet can be derived. Most importantly, the pressure and pressure drop can be determined using a 1.25 in diameter tubing.

\[
P_1 = \frac{P^*}{M_{i}} \sqrt{\frac{1 + k}{2[1 + (0.5)(k-1)M_{i}^{2}]} } \quad (14)
\]

\[
T_1 = \frac{T^*(1 + k)}{2[1 + (0.5)(k-1)M_{i}^{2}]} \quad (15)
\]

Table 5: Pipe Inlet Conditions

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 ) =</td>
<td>948.105</td>
<td>psia</td>
</tr>
<tr>
<td>( T_1 ) =</td>
<td>540.256</td>
<td>R</td>
</tr>
</tbody>
</table>

Conditions shown at Table 5 show that using a 1.25 inch stainless steel tubing with an inside diameter of 1.01 inch provides a downstream orifice recovery pressure of 948 psia. Our goal was a maximum pressure of 950 psia. That is 0.20% difference in pressure, and well within operational requirements for this sample problem.
Course Summary

Fanno flow modeling is used extensively in one-dimensional modeling of gases when the Mach number is greater than 0.3, the process is assumed adiabatic, and pressure loss due to friction is considered. However, it is not uncommon to use Fanno flow modeling for velocities lower than Mach 0.3 as long as the system adheres to the same assumptions. Installing this algorithm into a program or a spreadsheet reduces the workload greatly when iterating several different pipe geometry configurations. In addition, alternating inlet conditions can lead to a quick understanding of the effect of pressure and temperature on the gas density and gas velocity, which in turn affects choking the flow inside the pipe or choking the flow outside of the pipe at some imaginary point L*. Once that understanding is understood, then designing or analyzing compressible systems is simplified greatly.

Reference: