Centrifugal Pumps & Fluid Flow
Practical Calculations

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I. INTRODUCTION:

Every day a student or a professional is looking for a short and timely handbook with practical information and comprehensive application for many technical subjects, including this essay of Centrifugal Pumps & Fluid Flow Calculations. Then, this is the main motivation for the preparation of this outline.

Centrifugal pumps are one of the most common components inserted in fluid systems. In order to understand how a fluid system containing process piping and accessories operate, it is necessary to understand the basic concepts of fluid flow and all relationships with centrifugal pumps.

II. FLUID FLOW FUNDAMENTALS:

The basic principles of fluid flow include three concepts: The first is equations of fluid forces, the second is the conservation of energy (First Law of Thermodynamics) and the third is the conservation of mass.

1. Relationship Between Depth and Pressure:

Careful measurements show that the pressure of a liquid is directly proportional to the depth, and for a given depth the liquid exerts the same pressure in all directions.

As shown in figure below, the pressure at different levels in the tank varies and also varies velocities. The force is due to the weight of the water above the point where the pressure is being determined.

Then, pressure is defined to be force per unit area, as shown by the following equations:

\[
\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\text{Weight}}{\text{Area}}
\]

\[
P = \frac{m \cdot g}{A \cdot g_c} = \frac{\rho \cdot V \cdot g}{A \cdot g_c}
\]
Where:

\[ m = \text{Mass, in lbm}; \]
\[ g = \text{Acceleration (earth's gravity), } 32.17 \text{ ft/s}^2 \]
\[ g_c = 32.17 \text{ lbm-ft/lbf.s}^2 \]
\[ A = \text{Area, in ft}^2 \]
\[ V = \text{Volume, in ft}^3 \]
\[ P = \text{Density, in lbm/ft}^3 \]

Since the volume is equal to the cross-sectional area (A) multiplied by the height (h) of liquid, then:

\[ P = \rho \cdot h \cdot g \]

\[ g_c \]

**Example 1:**

If the tank in figure above is filled with water that has a density of 62.4 lbm/ft³, calculate the pressures at depths of 10, 20, and 30 feet.

**Solution:**

\[ P = \rho \cdot h \cdot g \]

\[ g_c \]

\[ P = 62.4 \times 10 \times 32.17 = 624 \text{ lbf/ft}^2 = 4.33 \text{ psi} \] (divided by 144 in² to psi)

\[ P = 62.4 \times 20 \times 32.17 = 1248 \text{ lbf/ft}^2 = 8.67 \text{ psi} \] (divided by 144 in² to psi)

\[ P = 62.4 \times 30 \times 32.17 = 1872 \text{ lbf/ft}^2 = 13.00 \text{ psi} \] (divided by 144 in² to psi)
Example 2:

A cylindrical water tank 40 ft high and 20 ft in diameter is filled with water with a density of 61.9 lbm/ft³.

(a) What is the water pressure on the bottom of the tank?
(b) What is the average force on the bottom?

a) \[ P = \rho \cdot h \cdot g \]
\[ P = \frac{62.4 \times 40 \times 32.17}{32.17} = 2476 \text{ lbf/ft}^2 = 17.2 \text{ psi} \]

b) Pressure = Force = Area

\[ \text{Force} = \text{Pressure} \times \text{Area} = 2476 \text{ lbf/ft}^2 \times (\pi \cdot R^2) = 17.2 \times (\pi \cdot 10^2) = 777858 \text{ lbf}. \]

2. Pascal's Law:

Pascal's law states that when there is an increase in pressure at any point in a confined fluid, there is an equal increase at every other point in the container.

The cylinder on the left shows a cross-section area of 1 sq. inch, while the cylinder on the right shows a cross-section area of 10 sq. inches. The cylinder on the left has a weight (force) on 1 lb acting downward on the piston, which lowers the fluid 10 inches. As a result of this force, the piston on the right lifts a 10 pound weight a distance of 1 inch.

The 1 lb load on the 1 sq. inch area causes an increase in pressure on the fluid. This pressure is distributed equally on every square inch area of the large piston. As a result, the larger piston lifts up a 10 pound weight. The bigger the cross-section area of the second piston, more weight it lifts.

Since pressure equals force per unit area, then it follows that:
F1 / A1 = F2 / A2

1 lb / 1 sq. inch = 10 lb / 10 sq. inches

The Volume formula is:

V1 = V2

Then,

A1.S1 = A2.S2

Or,

A1 / A2 = S2 / S1

It is a simple lever machine since force is multiplied. The mechanical advantage is:

MA = [S1 / S2 = A2 / A1]; can also be = [S1 / S2 = (π. r²) / (π.R²)]; or = [S1 / S2 = r² / R²]

Where:

A = Cross sectional area, in²
S = Piston distance moved, in

For the sample problem above, the MA is 10:1 (10 inches / 1 inch or 10 square inches / 1 square inch).

Example 3:

A hydraulic press, similar the above sketch, has an input cylinder 1 inch in diameter and an output cylinder 6 inches in diameter.

a. Find the estimated force exerted by the output piston when a force of 10 pounds is applied to the input piston.

b. If the input piston is moved 4 inches, how far is the output piston moved?

a. Solution:

F1 / A1 = F2 / A2
A1 = π. r² = 0.7854 sq. in;
A2 = π. R² = 28.274 sq. in
10 / 0.7854 = F2 / 28.274 =

F2 = 360 lb

b. Solution

S1 / S2 = A2 / A1
4 / S2 = 28.274 / 0.7854 = 4 / 36

S2 = 1 / 9 inch
Example 4:

A hydraulic system is said to have a mechanical advantage of 40. Mechanical advantage (MA) is \( F_2 / F_1 \). If the input piston, with a 12 inch radius, has a force of 65 pounds pushing downward a distance of 20 inches, find:

a. the upward force on the output piston;
b. the radius of the output piston;
c. the distance the output piston moves;
d. the volume of fluid that has been displaced;

a. Solution:

\[
MA = \frac{F_2}{F_1} = \frac{40}{65} = \frac{2600}{1200} = 2.15
\]

Upward force = \( F_2 = 2600 \) lb

b. Solution:

Piston radius = 12 inches, then, \( A_1 = \pi r^2 = \pi (12^2) = 452.4 \text{ in}^2 \)

\[
\frac{F_1}{A_1} = \frac{F_2}{A_2}
\]

\[
\frac{65}{452.4} = \frac{2600}{A_2}
\]

\( A_2 = 18096 \text{ in}^2 \)

\[
R^2 = \frac{A_2}{\pi} = \frac{18096}{\pi} = 5760
\]

Output piston radius = ~76 inches

c. Solution:

The input piston displaces 20 inches of fluid, then:

\[
\frac{A_1}{A_2} = \frac{S_2}{S_1}
\]

\[
\frac{452.4}{18096} = \frac{S_2}{20}
\]

Output piston moves, \( S_2 = 0.5 \) inch

d. Solution:

Output Volume = \( A_2 \times S_2 = 18096 \text{ in}^2 \times 0.5 \text{ inch} = 9048 \text{ in}^3 \)

3. Density (\( \rho \)) and Specific Gravity (Sg)

a) Density (\( \rho \)) of a material is defined as mass divided by volume:

\[
\rho = \frac{m \text{ (lb)}}{V \text{ (ft}^3)}
\]

Where:
Density of water = 1 ft³ of water at 32°F equals 62.4 lb.

Then, $\rho_{\text{water}} = 62.4 \text{ lb/ft}^3 = 1000 \text{ Kg/m}^3$

**b) Specific Gravity** is the substance density compared to water. The density of water at standard temperature is:

$\rho_{\text{water}} = 1000 \text{ Kg/m}^3 = 1 \text{ g/cm}^3 = 1 \text{ g/liter}$

So, the Specific Gravity (Sg) of water is 1.0.

**Example 5:**

If the Density of iron is 7850 kg/m³, the Specific Gravity is:

$Sg = \frac{7850 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = 7.85$

### 4. Volumetric Flow Rate

The volumetric flow rate ($Q - \text{ft}^3/\text{s}$) can be calculated as the product of the cross sectional area ($A - \text{ft}^2$) for flow and the average flow velocity ($v - \text{ft/s}$).

$$Q = A \times v$$

**Example 6:**

A pipe with an inner diameter of 4 inches contains water that flows at an average velocity of 14 ft/s. Calculate the volumetric flow rate of water in the pipe.

$$Q = (\pi r^2) \times v =$$

$$Q = (\pi \times 0.16^2 \text{ ft}) \times 14 \text{ ft/s} = 1.22 \text{ ft}^3/\text{s}$$

### 5. Mass Flow Rate:

The mass flow rate is related to the volumetric flow rate as shown in equation below:

$$m = \rho \times V$$

Replacing with the appropriate terms allows the calculation of direct mass flow rate:

$$m = \rho \times (A \times v)$$

**Example 7:**

The water in the pipe, (previous example) had a density of 62.44 lb/ft³ and a velocity of 1.22 ft/s. Calculate the mass flow rate.

$$m = \rho \times V =$$
\[ m = 62.44 \text{ lb/ft}^3 \times 1.22 \text{ ft/s} = \]
\[ m = 76.2 \text{ lb/s} \]

6. Continuity Equation:

The continuity equation is simply a mathematical expression of the principle of conservation of mass. The continuity equation is:

\[ m \text{ (inlet)} = m \text{ (outlet)} \]

\[ (p_1 \times A_1 \times v_1) \text{ inlet} = (p_2 \times A_2 \times v_2) \text{ outlet} \]

\[ (p_1 \times (R_1)^2 \times v_1) \text{ inlet} = (p_2 \times (R_2)^2 \times v_2) \text{ outlet} \]

Example 8:

In a piping process undergoes a gradual expansion from a diameter of 6 in. to a diameter of 8 in. The density of the fluid in the pipe is constant at 60.8 lb/ft\(^3\). If the flow velocity is 22.4 ft/s in the 6 in. section, what is the flow velocity in the 8 in. section?

\[ m \text{ (inlet)} = m \text{ (outlet)} = \]

\[ (p_1 \times (R_1)^2 \times v_1) \text{ inlet} = (p_2 \times (R_2)^2 \times v_2) \text{ outlet} = \]

\[ v_2 \text{ (outlet)} = v_1 \times \frac{p_1 \times (R_1)^2}{p_2 \times (R_2)^2} \]

\[ p = p_1 = p_2 \]

\[ v_2 \text{ (outlet)} = v_1 \times \frac{p_1 \times (R_1)^2}{p_2 \times (R_2)^2} = \frac{22.4 \text{ ft/s} \times 60.8 \text{ lb/ft}^3 \times (3)^2}{60.8 \text{ lb/ft}^3 \times (4)^2} \]

\[ v_2 \text{ (outlet)} = 12.6 \text{ ft/s} \] (decrease in flow velocity in the 8 in. section).

Example 9:

The inlet diameter of the centrifugal pump, shown in figure below, is 28 in. and the outlet flow through the pump is 9200 lb/s. The density of the water is 49 lb/ft\(^3\). What is the velocity at the pump inlet?

\[ A = \pi r^2 = \pi \times (14 / 12)^2 = 4.28 \text{ ft}^2 \]

\[ m = \rho \times A \times v = 9200 \text{ lb/s} \]

\[ v = \frac{9200 \text{ lb/s}}{A \cdot \rho} = \frac{9200 \text{ lb/s}}{4.28 \text{ ft}^2 \times 49 \text{ lb/ft}^3} = \]

\[ v = 43.9 \text{ ft/s} \]
7. Reynolds Number:

The Reynolds Number, based on studies of Osborn Reynolds, is a dimensionless number comprised of the physical characteristics of the flow. The flow regime, called commonly laminar or turbulent, is determined by evaluating the Reynolds Number of the flow.

If the Reynolds number is less than 2000, the flow is laminar. Reynolds numbers between 2000 and 3500 are sometimes referred to as transitional flows. If it is greater than 3500, the flow is turbulent. Most fluid systems in plant facilities operate with turbulent flow. The equation used to calculate the Reynolds Number for fluid flow is:

\[
Re = \frac{\rho v D}{\mu} \quad \text{or} \quad Re = \frac{\rho v D}{\mu} = \frac{\rho g_c c}{\mu}
\]

Where:

- \(Re\) = Reynolds Number (unitless)
- \(v\) = Velocity (ft/sec)
- \(D\) = Diameter of pipe (ft)
- \(\mu\) = Absolute Viscosity of fluid (lbf.s/ft²)
- \(\rho\) = Fluid Density (lb/ft³)
- \(g_c\) = Gravitational constant (32.17 ft-lbm/lbf-s²)

Reynolds numbers can also be conveniently determined using a Moody Chart.

8. Simplified Bernoulli Equation:

Bernoulli’s equation, results from the application of the first Law of Thermodynamics to a flow system in which no work is done by the fluid, no heat is transferred to or from the fluid, and no temperature change occurs in the internal energy. So, the general energy equation is simplified to equation below:

\[
mz_1 + mv_1^2 + P_1 = mz_2 + mv_2^2 + P_2
\]

Where:

- \(m\) = Mass of the fluid (lbm)
- \(z\) = Height above reference (ft)
- \(v\) = Velocity (ft/s)
- \(g\) = Acceleration due gravity (32.17 ft/s²)
- \(g_c\) = Gravitational constant, (32.17 ft-lbm/lbf-s²)

Note: The factor \(g_c\) is only required when the English System of measurement is used and mass is measured in pound mass. It is essentially a conversion factor needed to allow the units to come out directly.

No factor is necessary if mass is measured in slugs or if the metric system of measurement is used. Multiplying all terms of the above equation, by the factor \(g_c / m.g\), the form of Bernoulli’s equation is:

\[
z_1 + \frac{v_1^2}{2g} + P_1 = z_2 + \frac{v_2^2}{2g} + P_2
\]
9. Head:

The term head is used in reference to pressure. It is a reference to the height, typically in feet, of a column of water that a given pressure will support. The pressure head represents the flow energy of a column of fluid whose weight is equivalent to the pressure of the fluid.

The sum of the elevation head, velocity head, and pressure head of a fluid is called the total head. Thus, Bernoulli’s equation states that the total head of the fluid is constant.

Example 10:

Assume frictionless flow in a long, horizontal, conical pipe. The diameter is 2.0 ft at one end and 4.0 ft at the other. The pressure head at the smaller end is 16.0 ft of water. If water flows through this cone at a rate of 125.6 ft³/s, find the velocities at the two ends and the pressure head at the larger end.

\[ v_1 = \frac{Q_1}{A_1} \quad v_2 = \frac{Q_2}{A_2} \]

\[ v_1 = \frac{125.6}{\pi (1)^2} \quad v_2 = \frac{125.6}{\pi (2)^2} \]

\[ v_1 = 40 \text{ ft/s} \quad v_2 = 10 \text{ ft/s} \]

\[ z_1 + \frac{v_1^2}{2g} + P_1 + \frac{v_1 g_c}{g} = z_2 + \frac{v_2^2}{2g} + P_2 + \frac{v_2 g_c}{g} \]

\[ P_2 v_2 g_c = P_1 v_1 g_c + (z_1 - z_2) + \frac{v_1^2 - v_2^2}{2g} \]

Considering that, \( P_1 v_1 g_c = P_{h1} = 16 \text{ ft} \); \( P_2 v_2 g_c = P_{h2} \); and \( (z_1 - z_2) = 0 \)

\[ P_{h2} = 16 \text{ ft} + 0 + \frac{(40 \text{ ft/s})^2 - (10 \text{ ft/s})^2}{2(32.17 \text{ ft-lbm/lbf-s}^2)} \]

\[ P_{h2} = 16 \text{ ft} + 0 + \frac{1600 - 100}{64.34} = 39.3 \text{ ft} \]

10. Extended Bernoulli Equation:

The Bernoulli equation can be modified to take into account gains and losses of head. The head loss due to fluid friction (Hf) represents the energy used in overcoming friction caused by the walls of the pipe.

Then, the Extended Bernoulli equation is very useful in solving most fluid flow problems as shown below:

\[ z_1 + \frac{v_1^2}{2g} + P_1 v_1 g_c + H_p = z_2 + \frac{v_2^2}{2g} + P_2 v_2 g_c + H_f \]

Where:
z = Height above reference level (ft)
v = Velocity of fluid (ft/s)
P = Pressure of fluid (lbf/ft²)
n = Volume of fluid (ft³/lbm)
Hp = Head added by pump (ft)
Hf = Head loss due to fluid friction (ft)
g = Acceleration due to gravity (ft/s²)

**Example 11:**

Water is pumped from a large reservoir to a point 65 ft higher. How many feet of head must be added by the pump, if 8000 lb/h flows through a 6 inch pipe and the frictional head loss (Hf) is 2.0 ft? The density of the fluid is 62.4 lb/ft³, and the cross-sectional area of the pipe is 0.2006 ft².

\[ m = \rho \cdot A \cdot v \]
\[ v = \frac{m}{\rho \cdot A} \]
\[ v = \frac{8000 \text{ lb/h}}{(62.4 \text{ lb/ft}^3) (0.2006 \text{ ft}^2)} \]
\[ v = 639 \text{ ft/h} = 0.178 \text{ ft/s} \]

Using the Extended Bernoulli equation to determine the required pump head:

\[ z_1 + \frac{v_1^2}{2g} + P_1 \cdot \frac{g}{2} + H_p = z_2 + \frac{v_2^2}{2g} + P_2 \cdot \frac{g}{2} + H_f + H_t \]

\[ H_p = (z_2 - z_1) + \frac{v_1^2 - v_2^2}{2g} + (P_2 - P_1) \cdot \frac{g}{2} + H_f \]

Considering that, \( (z_2 - z_1) = 65 \text{ ft} \); \( (P_2 - P_1) \cdot \frac{g}{2} = 0 \); \( v_1 = 0.178 \text{ ft/s} \); and \( H_f = 2.0 \text{ ft} \)

\[ H_p = 65 \text{ ft} + \frac{(0.178 \text{ ft/s})^2}{2 \cdot (32.17 \text{ ft-lbm/lbf-s}^2)} + 0 + 2 \text{ ft} = 67 \text{ ft} \]

11. Head Loss, Darcy – Weisbach & Moody Chart:

Head loss is a measure of the reduction in the total head (sum of elevation head, velocity head and pressure head) of the fluid as it moves through a fluid system. The head loss is directly proportional to the length of pipe, the square of the velocity, and a term for fluid friction called the friction factor.

Darcy-Weisbach Head Loss, \( H_f = \frac{f \cdot L \cdot v^2}{D \cdot 2g} \)

Where:

\( f \) = Friction Factor (see Moody Chart)
\( L \) = Length of pipe, ft
v = Velocity of fluid, ft/s  
D = Diameter of pipe, ft  
g = Acceleration due gravity (ft/s²)

### 12. Friction Factor, Moody Chart:

The **Moody Chart** can be used to determine the friction factor based on the **Reynolds Number** and the **relative roughness**, which is equals the average height of surface irregularities (ε) divided by the pipe diameter (D) – see specific table.

**Moody Chart:**

![Moody Diagram](image)

**Example 14:**

Determine the friction factor (f) for fluid flow in a pipe that has a **Reynolds number of 40,000** and a relative roughness of 0.01.

Using the **Moody Chart**, a Reynolds number of 40,000 intersects the curve corresponding to a relative roughness of 0.01 at a friction factor of 0.038 and indicates a **transition zone** (see top of graphic).
As a rule of thumb, for transition flow with Reynolds numbers between 4,000 and 100,000, SI friction factors will be of the order suggested by equation 1, whilst Imperial friction factors will be of the order suggested by equation 2. Consider the equations below only for an estimating calculation.

\[
f = \sim \frac{0.55}{\text{Re}^{0.25}} \quad f = \sim \frac{0.3}{\text{Re}^{0.25}} \quad \text{Example 14 (above): } f = \sim 0.55 = 0.039
\]

13. Darcy-Weisbach Equations:

The Darcy-Weisbach equation can be calculated using a relationship known as frictional head loss. The calculation takes two distinct forms. The first form is associated with the piping length and the second form is associated with the piping fittings and accessories, with a coefficient “K”.

\[H_f = f \times L \times \frac{v^2}{D \times 2g}\]

Where:

- \( f \) = Friction factor (unitless)
- \( L \) = Length of pipe (ft)
- \( D \) = Diameter of pipe (ft)
- \( v \) = Velocity of fluid (ft/s)
- \( g \) = Acceleration due gravity (ft/s²)

Example 15:

A pipe 100 ft long and 20 inches in diameter contains water at 200°F flowing at a mass flow rate of 700 lb/s. The water has a density of 60 lb/ft³ and a viscosity of \( 1.978 \times 10^{-7} \) lbf·s/ft². The relative roughness of the pipe is 0.00008. Calculate the head loss for the pipe.

\[
m = \rho \times A \times v = \]

\[
v = \frac{m}{\rho \times A} = \frac{700 \text{ lb/s}}{\frac{(60 \text{ lb/ft}^3) \pi \times (10 \text{ in})^2}{144}} \]

\[v = 5.35 \text{ ft/s}\]

The Reynolds Number is:

\[
R_n = \frac{\rho \times v \times D}{\mu \times g_c}
\]

\[
R_n = \frac{60 \times 5.35 \times (20)}{12} \times \frac{12}{(1.978 \times 10^{-7})(32.17)} = 8.4 \times 10^7
\]

The Moody Chart for a Reynolds Number of \( 8.4 \times 10^7 \) and a relative roughness of 0.00008, \( f = 0.012 \).
\[ H_f = \frac{f}{D} \frac{L v^2}{2g} \]

\[ \begin{align*}
H_f &= (0.012) \frac{100}{20} \left( \frac{(5.35)^2}{2 \times 32.17} \right) \\
H_f &= 0.32 \text{ ft}
\end{align*} \]

\section*{b) Darcy – Weisbach minor losses for fittings and accessories with a coefficient “K”:}

Darcy-Weisbach equation minor losses for piping fittings and accessories is the second form, expressed in terms of the equivalent length of pipe, considering a resistance coefficient “K”, to be used according to table below:

\[ H_f = K \left( \frac{v^2}{2g} \right) = \]

\begin{center}
\begin{tabular}{|l|c|c|c|c|c|c|c|c|c|c|}
\hline
Fitting & LD & \(\frac{1}{2}\) & \(\frac{3}{4}\) & 1 & 1\(\frac{1}{4}\) & 1\(\frac{1}{2}\) & 2 & 2\(\frac{1}{2}\)-3 & 4 & 6 & 8-10 & 12-16 & 18-24 \\
\hline
Angle Valve & 55 & 1.48 & 1.38 & 1.27 & 1.21 & 1.16 & 1.05 & 0.99 & 0.94 & 0.83 & 0.77 & 0.72 & 0.66 \\
Angle Valve & 150 & 4.05 & 3.75 & 3.45 & 3.30 & 3.15 & 2.85 & 2.70 & 2.55 & 2.25 & 2.10 & 1.95 & 1.80 \\
Ball Valve & 3 & 0.08 & 0.08 & 0.07 & 0.07 & 0.06 & 0.06 & 0.05 & 0.05 & 0.04 & 0.04 & 0.04 & 0.04 \\
Butterfly Valve & 8 & 0.22 & 0.20 & 0.18 & 0.18 & 0.15 & 0.15 & 0.14 & 0.14 & 0.12 & 0.11 & 0.10 & 0.10 \\
Gate Valve & 340 & 9.2 & 8.5 & 7.8 & 7.5 & 7.1 & 6.5 & 6.1 & 5.8 & 5.1 & 4.8 & 4.4 & 4.1 \\
Globe Valve & 340 & 9.2 & 8.5 & 7.8 & 7.5 & 7.1 & 6.5 & 6.1 & 5.8 & 5.1 & 4.8 & 4.4 & 4.1 \\
Plug Valve Branch Flow & 90 & 2.43 & 2.25 & 2.07 & 1.98 & 1.89 & 1.71 & 1.62 & 1.53 & 1.35 & 1.26 & 1.17 & 1.08 \\
Plug Valve Straightway & 18 & 0.48 & 0.45 & 0.41 & 0.40 & 0.38 & 0.34 & 0.32 & 0.31 & 0.27 & 0.25 & 0.23 & 0.22 \\
Plug Valve 3-Way Thru-Flow & 30 & 0.81 & 0.75 & 0.69 & 0.66 & 0.63 & 0.57 & 0.54 & 0.51 & 0.45 & 0.42 & 0.39 & 0.36 \\
Standard Elbow & & & & & & & & & & & & & \\
\text{90°} & 30 & 0.81 & 0.75 & 0.69 & 0.66 & 0.63 & 0.57 & 0.54 & 0.51 & 0.45 & 0.42 & 0.39 & 0.36 \\
\text{45°} & 16 & 0.43 & 0.40 & 0.37 & 0.35 & 0.34 & 0.30 & 0.29 & 0.27 & 0.24 & 0.22 & 0.21 & 0.19 \\
\text{long radius 90°} & 16 & 0.43 & 0.40 & 0.37 & 0.35 & 0.34 & 0.30 & 0.29 & 0.27 & 0.24 & 0.22 & 0.21 & 0.19 \\
\hline
\end{tabular}
\end{center}

\section*{Example 16:}

Calculate the frictional head loss (in ft) for a flow rate of 0.60 ft³/sec of water at 50°F, through a length of 100 ft with 6 inch diameter galvanized iron pipe. Use the Moody Chart to find “f”.

At 50°F the properties of water are:

\[
\begin{align*}
\text{Density} &= \rho = 1.94 \text{ slugs/ft}^3, \\
\text{Viscosity} &= \mu = 2.73 \times 10^{-5} \text{ lb-s/ft}^2
\end{align*}
\]

Water velocity = \[ V = \frac{Q}{\pi(D/2)^2} = \frac{0.60/(\pi(6/12)^2)}{4} = 3.1 \text{ ft/sec} \]

Reynolds Number = \[ Re = \frac{D \times V \times \rho}{\mu} = (0.5)(3.1)(1.94) / (2.73 \times 10^{-5}) = 1.08 \times 10^5 \]

From the pipe roughness table (page 16), for Galvanized Iron: \[ \varepsilon = 0.0005 \text{ ft} \]

Pipe roughness ratio = \[ \frac{\varepsilon}{D} = 0.0005/0.5 = 0.001 \]

From the Moody diagram, the point for \[ Re = 1.08 \times 10^5 \] and \[ \varepsilon/D = 0.001 \], then \[ f = \sim 0.02 \]
The frictional head loss becomes: $H_f = (0.02) (100) (3.1)^2 = 0.58 (0.5) x 2 (32.17)$

### 14. Hazen-Williams Equation:

Since the approach does not require so efficient trial and error, an alternative empirical piping head loss calculation, like Hazen-Williams equation, may be preferred, as indicated below:

$$H_f = \frac{0.2083 \times (100 / C)_1^{1.85} \times Q_1^{1.85}}{D_1^{4.8655}} = \text{(in feet)}; \quad H_f = \frac{10.64 \times Q_1^{1.85}}{C_1^{1.85} D_1^{4.8655}} = \text{(in meters)}$$

Where:

- $f$ = Friction head loss in feet of water (per 100 ft of pipe)
- $C$ = Hazen-Williams roughness constant (see table below)
- $Q$ = Volume flow (gpm)
- $D$ = Inside pipe diameter (inches)
- $L$ = Length of pipe, (in. or m)

### 14.1 Hazen Williams Calculation Table:

<table>
<thead>
<tr>
<th>SIZE</th>
<th>4&quot; PIPE</th>
<th>6&quot; PIPE</th>
<th>8&quot; PIPE</th>
<th>10&quot; PIPE</th>
<th>12&quot; PIPE</th>
<th>14&quot; PIPE</th>
<th>16&quot; PIPE</th>
<th>18&quot; PIPE</th>
<th>20&quot; PIPE</th>
</tr>
</thead>
<tbody>
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<td>VEL. IN FT PER MIN.</td>
<td>VEL. IN FT PER MIN.</td>
<td>VEL. IN FT PER MIN.</td>
<td>VEL. IN FT PER MIN.</td>
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<td>2.45</td>
<td>2.45</td>
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<td>2.45</td>
</tr>
</tbody>
</table>

**Velocity** calculated using the formula...

$$V = \frac{Q}{\pi \times D^2}$$

**Head loss** calculated using Hazen-Williams formula with $C=100$...

$$F = \frac{0.2083 \times (100 / C)_1^{1.85} \times Q_1^{1.85}}{D_1^{4.8655}}$$

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14.2. L/D Method for Equivalent Piping Length:

L/D Method is another calculation way that may be used to find the equivalent piping length for fittings and accessories and can be determined by multiplying the value of L/D of that component by the diameter of the pipe. Friction factors (f), friction minors and approximate values of L/D for common piping components, using water flow, are listed in table below:

### Friction Loss of Water in Pipe Fittings in Terms of Equivalent Length - Feet of Straight Pipe

<table>
<thead>
<tr>
<th>Nominal pipe size</th>
<th>Actual inside diameter inches</th>
<th>Friction factor f</th>
<th>Gate valve - full open</th>
<th>90° elbow</th>
<th>Long radius 90° or 45° standard elbow</th>
<th>Std tee - through flow</th>
<th>Std tee - branch flow</th>
<th>Close return bend</th>
<th>Swing check valve - full open</th>
<th>Angle valve - full open</th>
<th>Globe valve - full open</th>
<th>Butterfly valve</th>
<th>90° Welding elbow</th>
<th>Mitre bend</th>
</tr>
</thead>
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<td>½</td>
<td>.622</td>
<td>.027</td>
<td>.41</td>
<td>.83</td>
<td>1.55</td>
<td>.31</td>
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<td></td>
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<tr>
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<td>46.7</td>
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<td>53</td>
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<td>982</td>
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<td>56.7</td>
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<td>1249</td>
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<td>66.7</td>
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<td>192</td>
<td>767</td>
<td>1534</td>
<td>78.0</td>
<td>76.7</td>
<td>46</td>
<td>58</td>
</tr>
</tbody>
</table>

**Example 17:**

A fully-open Gate Valve is installed in a pipe with a diameter of 10 inches. What L/D equivalent length of pipe would cause the same head loss?

From the table above, we find that the value of L/D for a full open Gate Valve is 10.

\[ L_e = \frac{(L/D) D}{L_e} = \frac{10 \times (10 \text{ inches})}{100 \text{ inches}} = 1 \]

14.3. Hazen-Williams Coefficients “C” Table:

The usual coefficients for friction loss calculation for some common materials can be found in the table below:
Pipe or Duct Material | Hazen-Williams Coefficient - C -
---|---
Aluminum | 130 - 150
Fiber Glass Pipe - FRP | 150
Cast Iron, Wrought Plain | 120
Polyethylene, PE, PEH | 140
Galvanized Steel, Standard Steel Pipe | 100

15. Pipe Roughness Ratio:

The relative piping roughness is the ratio of the surface roughness (ε – see table below), divided by the diameter (D) of the pipe or duct, as a result of equation ε / D.

<table>
<thead>
<tr>
<th>Pipe or Duct Material</th>
<th>Surface Roughness, ε</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Feet</td>
</tr>
<tr>
<td>PVC, Plastic or Glass</td>
<td>0.0</td>
</tr>
<tr>
<td>Commercial Steel or Wrought Iron</td>
<td>0.00015</td>
</tr>
<tr>
<td>Galvanized Iron</td>
<td>0.0005</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>0.00085</td>
</tr>
</tbody>
</table>

16. Simplified Pressure Drop:

The equation for calculating the simplified pressure drop is:

\[ \Delta p = \rho \times g \times H_f \]

Where:

\( \rho \) = Density of fluid, in slugs/ft\(^2\);
\( g \) = Acceleration due gravity, 32.17 ft/s\(^2\);
\( H_f \) = Frictional head loss.

**Example 18:**

Using the same example in problem 16 (page 13), calculate the simplified pressure drop (in psi), knowing that the frictional head loss is \( H_f = 0.58 \) and fluid density is 1.94 slugs/ft\(^2\).

The simplified pressure drop is:

\[ \Delta p = \rho \times g \times H_f = \]

\[ \Delta p = 1.94 \times 32.17 \times 0.58 = 36 \text{ lb/ft}^2 \]

\[ \Delta p = 36/144 \text{ psi} = 0.25 \text{ psi} \]
17. Hydraulic Diameter:

The hydraulic diameter uses the perimeter and the area of the conduit to provide the diameter of a pipe which has proportions such that conservation of momentum is maintained.

The hydraulic diameter of a Circular Tube or Duct can be expressed as:

\[ D_h = 2r \]

Where:

- \( r \) = Pipe or Duct radius (ft)

The hydraulic diameter of a Circular Tube with an inside Circular Tube can be expressed as:

\[ D_h = 2(R - r) \]

Where:

- \( r \) = Inside radius of the outside tube (ft)
- \( R \) = Outside radius of the inside tube (ft)

The hydraulic diameter of Rectangular Tubes or Ducts can be expressed as:

\[ D_h = \frac{2bc}{b + c} \]

Where:

- \( b \) = width/height of the duct (ft)
- \( c \) = height/width of the duct (ft)

18. Converting Head to Pressure:

Converting head in feet to pressure, in psi:

\[ p = 0.433 \times h \times SG \]

Where:

- \( p \) = Pressure (psi)
- \( h \) = Head (ft)
- \( SG \) = Specific Gravity

Converting head in meter to pressure, in bar:

\[ p = 0.0981 \times h \times SG \]

Where:

- \( h \) = Head (m)
- \( p \) = Pressure (bar)

Converting pressure in psi to head, in feet:
h = \frac{p \times 2.31}{SG}

Where:

h = Head (ft)
p = Pressure (psi)

Converting pressure in bar to head, in meter:

h = \frac{p \times 10.197}{SG}

Where:

h = Head (m)
p = Pressure (bar)

**Example 18:**

The pressure - psi - of a water pump operating with head 120 ft can be expressed as:

p = \frac{(120 \text{ ft}) \times 1.0}{2.31} =

p = 52 \text{ psi}

19. Viscosity and Density - Metric and Imperial System:

a) Metric or SI System: In this system of units the kilogram (kg) is the standard unit of mass, a cubic meter is the standard unit of volume and the second is the standard unit of time.

Density $\rho$: The density of a fluid is obtained by dividing the mass of the fluid by the volume of the fluid, normally expressed as kg / cubic meter.

$\rho = \frac{\text{kg}}{\text{m}^3}$

Water at a temperature of 20°C has a density of 998 kg/m³.

Sometimes the term “Relative Density” is used to describe the density of a fluid. Relative density is the fluid density divided by 1000 kg/m³. Water at a temperature of 20°C has a Relative density of 0.998.

Dynamic Viscosity $\mu$: Viscosity describes a fluid's resistance to flow. Dynamic Viscosity (sometimes referred to as Absolute Viscosity) is obtained by dividing the shear stress by the rate of shear strain.

The units of Dynamic Viscosity are: Force / area x time. This unit can be combined with time (sec) to define Dynamic Viscosity. Centipoise (cP) is commonly used to describe Dynamic Viscosity.

The Pascal unit (Pa) is used to describe pressure = force / area.

$\mu = \text{Pa.s}$

1.00 Pa.s = 10 Poise = 1000 Centipoise.

**Obs.**: Water temperature of 20°C has a viscosity of 1.002 cP must be converted to $1.002 \times 10^{-3} \text{ Pa.s}$. 
**Kinematic Viscosity** $\nu$: Kinematic Viscosity is measured by timing the flow of a known volume of fluid from a viscosity measuring cup, whose value is in Centistokes (cSt).

The formula of the Kinematic Viscosity is $\nu = \frac{\text{area}}{\text{time}}$, then:

$$
\nu = \frac{m^2}{s} \\
1.0 \, m^2/s = 10,000 \text{ Stokes} = 1,000,000 \text{ Centistokes}.
$$

Water at a temperature of 20°C has a viscosity of $1.004 \times 10^{-6} \, m^2/s$ or 1.004000 Centistokes. This value must be converted back to $1.004 \times 10^{-6} \, m^2/s$ for use in calculations.

**Note:** The kinematic viscosity can also be determined by dividing the Dynamic Viscosity by the fluid density.

$$
\text{Centistokes} = \frac{\text{Centipoise}}{\text{Density}} = \frac{\nu}{\rho}
$$

To understand the metric units involved in this relationship it will be necessary to use an example:

Dynamic viscosity $\mu = \text{Pa.s}$
Substitute for $\text{Pa} = N/m^2$ and $N = \text{kg.m/s}^2$

Therefore $\mu = \text{Pa.s} = \text{kg} / (\text{m} \cdot \text{s})$

$$
\text{Density} \ \rho = \text{kg/m}^3
$$

Kinematic Viscosity $= \nu = \frac{\mu}{\rho} = \frac{(\text{kg/(m} \cdot \text{s}) \times 10^{-3})}{(\text{kg/m}^3)} = \frac{\text{m}^2/s \times 10^{-6}}{}
$$

b) Imperial or US Units: In this system of units the **pound (lb)** is the standard unit of weight, a *cubic foot* is the standard unit of volume and the **second** is the standard unit of time.

The **standard unit of mass** is the slug.

This is the mass that will **accelerate by 1 ft/s** when a force of **one pound (lbf)** is applied to the mass. The **acceleration due to gravity (g)** is 32.17 ft per second per second.

To obtain the **mass of a fluid the weight (lb)** must be **divided by 32.17**.

**Density $\rho$:** Density is normally expressed as mass (slugs) per cubic foot. The weight of a fluid can be expressed as pounds per cubic foot.

$$
\rho = \text{slugs/ft}^3
$$

Water at a temperature of 70°F **has a density of 1.936 slug/ft}^3 = (62.286 \text{ lb/ft}^3)

**Dynamic Viscosity $\mu$:** The units of dynamic viscosity are: Force / area x time, $\mu = \text{lb.s/ft}^2$. Water at a temperature of 70°F **has a viscosity of 2.04 \times 10^{-5} \text{ lb.s/ft}^2**.

$$
1.0 \, \text{lb}\cdot\text{s/ft}^2 = 47880.26 \text{ Centipoise}
$$

**Kinematic Viscosity** $\nu$: The formula of Kinematic Viscosity is $\nu = \frac{\text{area}}{\text{time}}$, then:

$$
\nu = \frac{\text{ft}^2}{\text{s}} \\
1.00 \, \text{ft}^2/s = 929.034116 \text{ Stokes} = 92903.4116 \text{ Centistokes}
$$
Obs.: Water at a temperature of 70°F has a viscosity of $10.5900 \times 10^{-6} \text{ ft}^2/\text{s}$ (0.98384713 Centistokes).

**Kinematic Viscosity** = Dynamic Viscosity / Density:

$$\nu = \frac{\mu}{\rho}$$

**Note:** The Imperial unit of Kinematic Viscosity is ft²/s. To understand the Imperial units involved in this relationship it will be necessary to use an example:

Dynamic viscosity $\mu = \text{lb}\cdot\text{s}/\text{ft}^2$

Density $\rho = \text{slug}/\text{ft}^3$

Substitute for slug = lb/32.17 ft•s²

Density $\rho = (\text{lb}/32.17 \text{ ft}^3)/\text{ft}^3 = (\text{lb}/32.17 \text{ s}^2)/\text{ft}^4$

Obs: slugs/ft³ can be expressed in terms of lb.s²/ ft. Kinematic Viscosity $\nu = (\text{lb.s}/\text{ft}^2)/(\text{slug}/\text{ft}^3)$, substitute lb.s²/ ft for slug/ft³ =

Kinematic Viscosity $\nu = (\text{lb.s}/\text{ft}^2) / (\text{lb.s}^2/ \text{ft}^4) = \text{ft}^2/\text{s}$

**Conversions:** It is possible to convert between the Imperial system and the Metric system by substituting the equivalent of each dimension with the appropriate value.

1 slug/ft³ = 515.36 kg/m³. The density of water is 1.94 slug/ft³ or 1000 kg/m³ (1 gr/cm³).

**Table of Water Properties**

<table>
<thead>
<tr>
<th>Fluid</th>
<th>T (°F)</th>
<th>Density (slug/ft³)</th>
<th>$\nu$ (ft²/s)</th>
<th>T (°C)</th>
<th>Density (kg/m³)</th>
<th>$\nu$ (m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>70</td>
<td>1.936</td>
<td>$1.05 \times 10^5$</td>
<td>20</td>
<td>998.2</td>
<td>$1.00 \times 10^6$</td>
</tr>
<tr>
<td>Water</td>
<td>40</td>
<td>1.94</td>
<td>$1.66 \times 10^5$</td>
<td>5</td>
<td>1000</td>
<td>$1.52 \times 10^6$</td>
</tr>
<tr>
<td>Seawater</td>
<td>60</td>
<td>1.99</td>
<td>$1.26 \times 10^5$</td>
<td>16</td>
<td>1030</td>
<td>$1.17 \times 10^6$</td>
</tr>
</tbody>
</table>

**20. Moody Friction Factor, Re & $\varepsilon$/D Relationship:**

There are equations available that give the relationships between Moody friction factor, Re and $\varepsilon$/D for four different flow regions of the Moody diagram. The four regions of the Moody diagram are:

- **a)** Laminar flow - $\text{Re} < 2100$ - the straight line at the left side of the Moody diagram;

- **b)** Smooth pipe turbulent flow – $\text{Re} > 4000$ - the dark curve labeled “smooth pipe” in the Moody diagram – “$f$” is a function of Re only in this region;

- **c)** Complete turbulent flow - the portion of the diagram above and to the right of the dashed line labeled “complete turbulence” – “$f$” is a function of $\varepsilon$/D only in this region);

- **d)** Transition region - $\text{Re} > 2100 < 4000$ - the diagram between the “smooth pipe” solid line and the “complete turbulence” dashed line – “$f$” is a function of both Re and $\varepsilon$/D in this region.

The equations to find the friction factor “$f$” for these four regions are shown in the box below:
Example 19:

Calculate the value of the Moody friction factor “f” for a 6” pipe, 100 ft long, 270 GPM, \( \varepsilon/D = 0.005 \), assuming completely a turbulent flow - \([f = 1.14 + 2 \log_{10}(D/e)]^2\).

Solution:

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe Diameter, ( D = )</td>
<td>6 in</td>
</tr>
<tr>
<td>Pipe Diameter, ( D = )</td>
<td>0.5000 ft</td>
</tr>
<tr>
<td>Pipe Roughness, ( e = )</td>
<td>0.005 ft</td>
</tr>
<tr>
<td>Friction Factor, ( f = )</td>
<td>0.03785</td>
</tr>
<tr>
<td>Pipe Length, ( L = )</td>
<td>100 ft</td>
</tr>
<tr>
<td>Cross- Sect. Area, ( A = )</td>
<td>0.1963 ft²</td>
</tr>
<tr>
<td>Pipe Flow Rate, ( Q = )</td>
<td>0.602 ft³/sec</td>
</tr>
<tr>
<td>Velocity, ( V = )</td>
<td>3.1 ft/sec</td>
</tr>
<tr>
<td>Fluid Density, ( r = )</td>
<td>1.94 slugs/ft³</td>
</tr>
<tr>
<td>Reynolds number, ( Re = )</td>
<td>110.147</td>
</tr>
<tr>
<td>Fluid Viscosity, ( m = )</td>
<td>0.000027 lb-sec/ft²</td>
</tr>
</tbody>
</table>

**Note:** The Atmospheric pressure at sea level is 14.7 pounds per square inch (psi). This pressure with perfect vacuum, will maintain a line 29.9 inches of mercury or a column of water, 33.9 feet high.

### III. PUMPS CALCULATION PRINCIPLES:

1. **Head:**

Head is a measurement of the height of a liquid column which the pump could create resulting from the kinetic energy the pump gives to the liquid. The basic principle is a pump shooting a jet of water straight up into the air, the height of the water goes up would be the head.

The **head** is measured in units of feet while **pressure** is measured in pounds per square inch (psi), and is independent of pressure or liquid density. To convert head to pressure (psi) the following formula applies:

\[
\text{Head (ft)} = \frac{\text{Pressure (psi)}}{2.31} \times \frac{1}{\text{Specific Gravity (SG)}}
\]

For water considering atmospheric pressure at sea level it is: \( \text{Head} = 14.7 \times 2.31 / 1.0 = 33.9 \text{ ft} \)

Thus, **33.9 feet** is the theoretical **maximum suction lift** for a pump at sea level.
1.1. Types of Head:

a) **Static Head**: Is the *vertical distance from the water level* at the source to the *highest point* where the water must be delivered. It is the sum of static lift and static discharge.

b) **Static Suction Head**: Or static lift is the *vertical distance between the center line of the pump* and the *height of the water source* when the pump is not operating.

**Note**: The *Static Suction Head* (h) is *positive* when liquid line is *above pump centerline* and *negative* when liquid line is *below pump centerline*, as can be seen at the sketch below:

![Fluid Diagram](image)

1. c) **Static Discharge Head**: The static discharge head is a measure of the *elevation difference* between the center line of the pump and the *final point* of use.

   ✓ **Pressure Head**: Refers to the *pressure on the liquid in the reservoir* feeding a pump operating in a pressurized tank. If the fluid is under *vacuum* we can convert to the absolute pressure to head instead of atmospheric pressure. *Vacuum* is often read in inches of mercury, then a formula to convert it to head is:

   \[
   \text{Feet of liquid} = 1.133 \times \text{inches of mercury} = \frac{\text{Specific gravity}}{}
   \]

   ✓ **Total Dynamic Head**: Is the vertical distance from source water level to point of discharge when pumping at required capacity, includes Velocity Head, friction, inlets and exit losses.

   ✓ **Total Dynamic Discharge Head**: Is the Total Dynamic Head minus Dynamic Suction Lift or plus Dynamic Suction Head.

   ✓ **Dynamic Suction Head**: Is the vertical distance from source water level to centerline of pump, minus Velocity Head, entrance, friction, but not minus internal pump losses.
Velocity Head: Velocity head also known as dynamic head is a measure of a fluid’s kinetic energy. In most installations velocity head is negligible in comparison to other components of the total head (usually less than one foot). Velocity head is calculated using the following equation:

\[ Vh = \frac{v^2}{2g} \]

Where:

- \( Vh \) = Velocity head, ft.
- \( v \) = Velocity of water, ft/s
- \( g \) = Acceleration of gravity 32.17 ft/s².

The velocity head varies at different points in the cross section of a flow. A Pitometer may be used to take a number of readings at different points in piping, as can be seen in the table below:

<table>
<thead>
<tr>
<th>Velocity ft/s</th>
<th>Velocity Head ft.</th>
<th>Velocity Head ft.</th>
<th>Velocity ft/s</th>
<th>Velocity Head ft.</th>
<th>Velocity ft/s</th>
<th>Velocity Head ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.02</td>
<td>0.56</td>
<td>1.4</td>
<td>12.0</td>
<td>2.24</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>0.06</td>
<td>0.76</td>
<td>1.55</td>
<td>13.0</td>
<td>2.62</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>0.14</td>
<td>1.0</td>
<td>1.7</td>
<td>14.0</td>
<td>3.05</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>0.25</td>
<td>1.12</td>
<td>1.87</td>
<td>15.0</td>
<td>3.50</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>0.39</td>
<td>1.25</td>
<td>2.05</td>
<td>20.0</td>
<td>6.20</td>
<td></td>
</tr>
</tbody>
</table>

Imperial and Metric Relations:

1 foot of head = 0.433 psi = ~0.030 kg/cm²
1.0 psi = 0.0703 kg/cm² = 2.31 feet

Note:

In the Imperial system of units, the unit used for mass is the slug and not the lbm.

1 slug = 32.174 lbm.

International System: 1 Newton (N) = 1 kg m/s²  Imperial - 1 lbf = 1 slug ft/s²

Obs:

Water: \( \rho_w = 62.4 \text{ lb/ft}^3 \) - do not use this value - instead, use \( \rho_w = 1.94 \text{ slug/ft}^3 \).

Manometry: \( \rho \cdot g \cdot h = (\text{kg/m}^3) \cdot (\text{m/s}^2) \cdot \text{m} = \text{kg m/s}^2 / \text{m}^2 = \text{N/m}^2 \)

\( \text{slug/ft}^3 \) \( \times \text{ft/s}^2 \) \( \times \text{ft} = (\text{slug ft/s}^2) / \text{ft}^2 = \text{lbf/ft}^2 \)

2. Altitude and Atmospheric Pressure:

Atmospheric pressure is often measured with a mercury barometer, and a height of approximately 760 millimeters (30 in) of mercury is often used to measure the atmospheric pressure. At sea level, the weight of the air presses on us with a pressure of approximately 14.7 lbs/in².

1 atmosphere = 100 kPa or 14.7 psi is the pressure that can lift water approximately 10.3 m (33.9 ft).
Thus, a diver underwater 10.3 m (33.9 ft) experiences a pressure of about 2 atmospheres (1 atm of air plus 1 atm of water). This is the suction maximum height to which a column of water can be drawn up. At higher altitudes, less air means less weight and less pressure, then, pressure and density of air decreases with increasing elevation. Altitude and atmospheric pressure are according to tables below:

### ALTITUDE AND ATMOSPHERIC PRESSURE

<table>
<thead>
<tr>
<th>Feet</th>
<th>Meters</th>
<th>Psia</th>
<th>Kg/cm² abs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>14.69</td>
<td>1.033</td>
</tr>
<tr>
<td>500.0</td>
<td>153.0</td>
<td>14.43</td>
<td>1.015</td>
</tr>
<tr>
<td>1000.0</td>
<td>305.0</td>
<td>14.16</td>
<td>0.956</td>
</tr>
<tr>
<td>1500.0</td>
<td>458.0</td>
<td>13.91</td>
<td>0.978</td>
</tr>
<tr>
<td>2000.0</td>
<td>610.0</td>
<td>13.66</td>
<td>0.960</td>
</tr>
<tr>
<td>2500.0</td>
<td>763.0</td>
<td>13.41</td>
<td>0.943</td>
</tr>
<tr>
<td>3000.0</td>
<td>915.0</td>
<td>13.17</td>
<td>0.926</td>
</tr>
<tr>
<td>3500.0</td>
<td>1068.0</td>
<td>12.93</td>
<td>0.909</td>
</tr>
<tr>
<td>4000.0</td>
<td>1220.0</td>
<td>12.69</td>
<td>0.892</td>
</tr>
<tr>
<td>4500.0</td>
<td>1373.0</td>
<td>12.46</td>
<td>0.876</td>
</tr>
<tr>
<td>5000.0</td>
<td>1526.0</td>
<td>12.23</td>
<td>0.860</td>
</tr>
<tr>
<td>6000.0</td>
<td>1831.0</td>
<td>11.78</td>
<td>0.828</td>
</tr>
<tr>
<td>7000.0</td>
<td>2136.0</td>
<td>11.34</td>
<td>0.797</td>
</tr>
<tr>
<td>8000.0</td>
<td>2441.0</td>
<td>10.91</td>
<td>0.767</td>
</tr>
<tr>
<td>9000.0</td>
<td>2746.0</td>
<td>10.50</td>
<td>0.738</td>
</tr>
<tr>
<td>10000.0</td>
<td>3050.0</td>
<td>10.10</td>
<td>0.710</td>
</tr>
<tr>
<td>15000.0</td>
<td>4577.0</td>
<td>8.29</td>
<td>0.583</td>
</tr>
</tbody>
</table>

### PRACTICAL SUCTION LIFTS AT VARIOUS ELEVATIONS ABOVE SEA LEVEL

<table>
<thead>
<tr>
<th>ELEVATION</th>
<th>Barometer Reading (lb/sq. in.)</th>
<th>Theoretical Suction Lift (feet)</th>
<th>Practical Suction Lift (feet)</th>
<th>Vacuum Gauge (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>At sea level</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>¼ mile – 1320 ft – above sea level</td>
<td>14.0</td>
<td>32.4</td>
<td>21</td>
<td>18.6</td>
</tr>
<tr>
<td>½ mile – 2640 ft – above sea level</td>
<td>13.3</td>
<td>30.8</td>
<td>20</td>
<td>17.7</td>
</tr>
<tr>
<td>¾ mile – 3960 ft – above sea level</td>
<td>12.7</td>
<td>29.2</td>
<td>18</td>
<td>15.9</td>
</tr>
<tr>
<td>1 mile – 5280 ft – above sea level</td>
<td>12.0</td>
<td>27.8</td>
<td>17</td>
<td>15.0</td>
</tr>
<tr>
<td>1 ¼ mile – 6600 ft – above sea level</td>
<td>11.4</td>
<td>26.4</td>
<td>16</td>
<td>14.2</td>
</tr>
<tr>
<td>11/4 mile – 7920 ft – above sea level</td>
<td>10.9</td>
<td>25.1</td>
<td>15</td>
<td>13.3</td>
</tr>
<tr>
<td>2 miles – 10560 ft – above sea level</td>
<td>9.9</td>
<td>22.8</td>
<td>14</td>
<td>12.4</td>
</tr>
</tbody>
</table>

Obs: Multiply barometer in inches by 0.491 to obtain lbs. per sq. in (psi).

### 3. Density Alternatives and Pressure Relationships:

\[ \gamma = \rho \times g, \] where, \( \gamma \) - specific weight = weight per unit volume (N/m³, lbf/ft³).

Water: \( \gamma = 9790 \text{ N/m}^3 \approx 1000 \text{ Kg/m}^3 = 62.4 \text{ lbf/ft}^3 = 1.94 \text{ slug/ft}^3 \)

Air: \( \gamma = 11.8 \text{ N/m}^3 \approx 1.2 \text{ Kg/m}^3 = 0.0752 \text{ lbf/ft}^3 = 0.00237 \text{ slug/ft}^3 \)
Density $\rho$ is usually at 4°C, but some references will use $\rho$ at 20°C, thus, **Specific Gravity** is:

**Water** ($\rho$) = at 1 atm, 4°C = 1000 kg/m$^3$ - $SG = 1000 / 1000 = 1.0$

**Air** ($\rho$) = at 1 atm, 4°C = 1.205 kg/m$^3$ - $SG = 1.205 / 1000 = \sim 0.0012$

**PRESSURE AND EQUIVALENT FEET HEAD OF WATER**

<table>
<thead>
<tr>
<th>lb /sq. in. (psi)</th>
<th>Feet Head</th>
<th>lb /sq. in. (psi)</th>
<th>Feet Head</th>
<th>lb /sq. in. (psi)</th>
<th>Feet Head</th>
<th>lb /sq. in. (psi)</th>
<th>Feet Head</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>2.31</td>
<td>20.0</td>
<td>46.28</td>
<td>120.0</td>
<td>277.07</td>
<td>225.0</td>
<td>519.51</td>
</tr>
<tr>
<td>2.0</td>
<td>4.62</td>
<td>25.0</td>
<td>57.72</td>
<td>125.0</td>
<td>288.62</td>
<td>250.0</td>
<td>577.24</td>
</tr>
<tr>
<td>3.0</td>
<td>6.93</td>
<td>30.0</td>
<td>69.27</td>
<td>130.0</td>
<td>300.16</td>
<td>275.0</td>
<td>643.03</td>
</tr>
<tr>
<td>4.0</td>
<td>9.24</td>
<td>40.0</td>
<td>92.36</td>
<td>140.0</td>
<td>323.25</td>
<td>300.0</td>
<td>692.69</td>
</tr>
<tr>
<td>5.0</td>
<td>11.54</td>
<td>50.0</td>
<td>115.45</td>
<td>150.0</td>
<td>346.34</td>
<td>325.0</td>
<td>750.41</td>
</tr>
<tr>
<td>6.0</td>
<td>13.85</td>
<td>60.0</td>
<td>138.54</td>
<td>160.0</td>
<td>369.43</td>
<td>350.0</td>
<td>808.13</td>
</tr>
<tr>
<td>7.0</td>
<td>16.16</td>
<td>70.0</td>
<td>161.63</td>
<td>170.0</td>
<td>392.52</td>
<td>375.0</td>
<td>865.89</td>
</tr>
<tr>
<td>8.0</td>
<td>18.47</td>
<td>80.0</td>
<td>184.72</td>
<td>180.0</td>
<td>415.61</td>
<td>400.0</td>
<td>922.58</td>
</tr>
<tr>
<td>9.0</td>
<td>20.78</td>
<td>90.0</td>
<td>207.81</td>
<td>190.0</td>
<td>438.90</td>
<td>500.0</td>
<td>1154.48</td>
</tr>
<tr>
<td>10.0</td>
<td>23.09</td>
<td>100.0</td>
<td>230.90</td>
<td>200.0</td>
<td>461.78</td>
<td>1000.0</td>
<td>2310.00</td>
</tr>
<tr>
<td>15.0</td>
<td>34.63</td>
<td>110.0</td>
<td>253.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example 20:**

Determine the static pressure: 18 cm (0.59 ft) column of fluid with a **Specific Gravity** of 0.85.

$\Delta P = \rho g h = SG \gamma h = 0.85 \times 9790 \text{ N/m}^3 \times 0.18 \text{ m} = 1498 \text{ N/m}^2 = 1.5 \text{ kPa} = 0.015 \text{ bar}$

$\Delta P = \rho g h = SG \gamma h = 0.85 \times 62.4 \text{ lb/ft}^3 \times 0.59 \text{ ft} = 31.3 \text{ lb/ft}^2 = 31.3 \text{ lb/144} = 0.217 \text{ psi}$

4. **Centrifugal Force Theory:**

The equation that describes the relationship of velocity, height and gravity applied to a falling body is:

$v^2 = 2 gh$

Where:

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v = Velocity of the body, ft/s;
g = Acceleration due gravity, 32.2 ft/s²;
h = Distance the body falls, ft.

The peripheral velocity or, the outside travelling point of a rotating body in one second is:

\[ v = \pi \frac{D \cdot n}{60} = \]

Where:

D = Diameter of rotating body or impeller, inches
n = Rotation of the rotating body or impeller in minutes, RPM

Example 21:

What is the velocity of a stone thrown from a building window 100 ft high?

\[ v^2 = 2 \cdot g \cdot h = \]
\[ v^2 = 2 \times 32.2 \times 100 = 6440 \text{ ft}^2/\text{s}^2 = \]
\[ v = 80.3 \text{ ft/s} \]

The same equation applies when pumping water with a centrifugal pump. If we rearrange the falling body equation we get the velocity head - known as dynamic head as a measure of a fluid’s kinetic energy:

\[ h = \frac{v^2}{2g} = \]

This relationship is one of fundamental laws of centrifugal pumps. Applying this theory with a practical application, take the example below:

Example 22:

Installing an 1800 RPM centrifugal pump, what will be the necessary diameter of the impeller to develop a head of 200 ft?

\[ v^2 = 2 \cdot g \cdot h = \]
\[ v^2 = 2 \times 32.2 \times 200 = 12880 \text{ ft}^2/\text{s}^2 = 113 \text{ ft/s} \]

The peripheral velocity is:

\[ v = \pi \frac{d \cdot n}{60}, \text{ then:} \]
\[ d = 60 \frac{v}{\pi \cdot n} = 60 \times 113 / \pi \times 1800 = 1.2 \text{ ft (14.4 inches)} \]

5. Static Head:

The static head, sometimes referred to as the pressure head, is a term primarily used in hydraulics to denote the static pressure in a pipe, channel, or duct flow. It has the physical dimensions of length (hence the term "head") and represents the flow-work per unit weight of fluid. Static head is the difference in height between the source and destination of the pumped liquid (see figure below):
The static head at a certain pressure depends on the weight of the liquid and can be calculated with this equation:

\[
\text{Head (in feet)} = \text{Pressure (psi)} \times 2.31 = \frac{\text{Specific Gravity}}{6}
\]

6. Vapor Pressure:

A fluid’s vapor pressure is the force per unit area that a fluid exerts as an effort to change phase from a liquid to a vapor, and depends on the fluid’s chemical and physical properties. At 60°F, the vapor pressure of water is approximately 0.25 psia; at 212°F (boiling point of water) the vapor pressure is 14.7 psia (atmospheric pressure).

Water Vapor Pressure - Suction Head:

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Abs. Water Vapor Pressure</th>
<th>Max. Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>C°</td>
<td>psi/psia</td>
<td>(m)</td>
</tr>
<tr>
<td>F°</td>
<td>bar</td>
<td>(ft)</td>
</tr>
<tr>
<td>0</td>
<td>0.0886</td>
<td>0.062</td>
</tr>
<tr>
<td>5</td>
<td>0.1217</td>
<td>0.085</td>
</tr>
<tr>
<td>10</td>
<td>0.1781</td>
<td>0.125</td>
</tr>
<tr>
<td>15</td>
<td>0.2563</td>
<td>0.180</td>
</tr>
<tr>
<td>21</td>
<td>0.3631</td>
<td>0.255</td>
</tr>
<tr>
<td>25</td>
<td>0.4593</td>
<td>0.322</td>
</tr>
<tr>
<td>30</td>
<td>0.6152</td>
<td>0.432</td>
</tr>
<tr>
<td>35</td>
<td>0.8153</td>
<td>0.573</td>
</tr>
<tr>
<td>40</td>
<td>1.069</td>
<td>0.751</td>
</tr>
<tr>
<td>45</td>
<td>1.389</td>
<td>0.976</td>
</tr>
<tr>
<td>50</td>
<td>1.789</td>
<td>1.258</td>
</tr>
</tbody>
</table>

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The centrifugal pumps are generally the most economical followed by the rotary and reciprocating pumps. Although, positive displacement pumps are generally more efficient than centrifugal pumps, the benefit of higher efficiency tends to be offset by increased maintenance costs.
Note: As can be seen from the above laws, doubling the rotating speed of the centrifugal pump will increase the power consumption by 8 times. This forms the basis for energy conservation in centrifugal pumps with varying flow requirements.

9. Pump Performance Curve:

The rate of flow at a certain head is called the duty point. The pump performance curve is made up of many duty points. The pump operating point is determined by the intersection of the system curve and the pump curve as shown below:

```
Example 23:

A centrifugal pump, at 1750 RPM, has the following performance, \( Q = 1000 \) GPM; \( h = 150 \) ft.; \( N = 45 \) HP. What will the performance of this pump at 2900 RPM?

a) \( Q = 1000 \times (2900 / 1750) = 1660 \) GPM
b) \( h = 150 \times (2900 / 1750)^2 = 411 \) ft
c) \( N = 45 \times (2900 / 1750)^3 = 205 \) HP
```
10. Specific Speed:

The specific speeds of centrifugal pumps range from **500 to 20,000** depending upon the design. Pumps of the same specific speed (Ns), but with different sizes are considered to be geometrically similar, one pump being a size-factor of the other, as indicated in table below:

\[
Ns = \frac{N \times Q^{0.5}}{H^{0.75}}
\]

**Ns** = Specific speed, dimensionless;

**Q** = Flow capacity at best efficiency point at maximum impeller diameter, GPM;

**H** = Head at maximum impeller diameter, ft;

**N** = Pumps speed, RPM.

### Specific Speeds for Centrifugal Pumps

<table>
<thead>
<tr>
<th>Pump Type</th>
<th>Application</th>
<th>Specific Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial Vane</td>
<td>Low capacity/high head</td>
<td>500 - 1000</td>
</tr>
<tr>
<td>Francis - Screw Type</td>
<td>Medium capacity/Medium head</td>
<td>1000 - 4000</td>
</tr>
<tr>
<td>Mixed - Flow Type</td>
<td>Medium to high capacity, low to medium head</td>
<td>4000 - 7000</td>
</tr>
<tr>
<td>Axial - Flow Type</td>
<td>High capacity/low head</td>
<td>7000 - 20,000</td>
</tr>
</tbody>
</table>

**Example 24:**

Given a centrifugal pump at **3570 RPM**, flow capacity **2000 GPM** and head of **500 ft**, the specific speed is calculated as:

\[
Ns = \frac{3570 \times 2000^{0.5}}{500^{0.75}} = 1510
\]

10. Pump Pressure:

Pump manufacturers supply in feet (or meters) of head. In the final analysis, they are the same, just expressed from two different points of view. The pressure rises as flow progresses from the suction to discharge. Pressure is expressed in **“psi”**, but also can be expressed in **feet of water, water gauge, head or static head**:

**Head, h, ft. water, water gauge, = psi x 2.31 / SG, where SG is the Specific Gravity.**

11. Total Dynamic Head:

In fluid dynamics, **Total Dynamic Head** (TDH) is the total equivalent height that a fluid is to be pumped, taking into account friction losses in the pipe. **TDH = Static Height + Static Lift + Friction Loss.**
Static Height is the maximum height reached by the pipe after the pump (also known as the “discharge head”). When a pump is installed, the developed pressure as explained above, is also commonly called discharge head at the exit side of the pump and suction head on the inlet side of the pump.

**Figure # 1:**

Total Dynamic Head (TDH) is the total dynamic discharge head minus the total dynamic suction head when installed with a suction head.

The suction head is positive because the liquid level is above the centerline of the pump:

\[
TDH = \text{discharge head} - \text{suction head}
\]

\[
TDH = H_d - H_s \quad \text{(with a suction head)}
\]

**Figure # 2:**

Total Dynamic Head (TDH) is the total dynamic discharge head plus the total dynamic suction head when installed with a suction lift.

The suction head is negative because the liquid level is below the centerline of the pump:

\[
TDH = \text{discharge head} + \text{suction head}
\]

\[
TDH = H_d + H_s \quad \text{(with a suction lift)}
\]

The formulae are:

The total suction head (Hs) consists of three separate heads:

\[
H_s = h_{ss} + h_{ps} - h_{fs}
\]

hss = Suction static head;

hps = Suction surface pressure head;

hfs = Suction friction head.

The total discharge head (Hd) is also made from three separate heads:

\[
H_d = h_{sd} + h_{pd} + h_{fd}
\]

hsd = Discharge static head;

hpd = Discharge surface pressure head;

hfd = Discharge friction head.
Pumping System: Basic installation with negative suction head and main components:

12. Pump Standards:

Centrifugal pumps can be segmented into groups based on design, application, models and service type. Pumps can belong to several different groups depending on their construction and application. The following examples demonstrate various segments:

Industry standards:

HI - Hydraulic Institute Standards
ANSI Pump - ASME B73.1 Specifications (chemical industry)
API Pump - API 610 Specifications (oil & gas industry)
DIN Pump - DIN 24256 Specifications (European standard)
ISO Pump - ISO 2858, 5199 Specifications (European standard)
Nuclear Pump - ASME Specifications
UL/FM Fire Pump - NFPA Specifications
Example 25:

Calculate the Total Dynamic Head (TDH) according to Figure #2, below:

![Figure #2](image)

a) The total suction head \((H_s)\) calculations are:

1. The suction head is negative because the liquid level is below the centerline of the pump:
   \[ h_{ss} = -6 \text{ feet} \]

2. The suction surface pressure: the tank is open, so pressure equals atmospheric pressure:
   \[ h_{ps} = 0 \text{ feet, gauge} \]

3. Assume the suction friction head as:
   \[ h_{fs} = 4 \text{ feet} \]

4. The total suction head is:
   \[
   H_s = h_{ss} + h_{ps} - h_{fs} =
   -6 + 0 - 4 = -10 \text{ feet}
   \]

b) The total discharge head \((H_d)\) calculations are:

1. The static discharge head is:
   \[ h_{sd} = 125 \text{ feet} \]

2. The discharge surface pressure: the discharge tank is also open to atmospheric pressure, thus:
   \[ h_{pd} = 0 \text{ feet, gauge} \]

3. Assume the discharge friction head as:
   \[ h_{fd} = 25 \text{ feet} \]

4. The total discharge head is:
\[ Hd = hsd + hpd + hfd = \]
\[ Hd = 125 + 0 + 25 = 150 \text{ feet} \]

The Total Dynamic Head calculation is:

\[ \text{TDH} = \text{Hd} - \text{Hs} = \]
\[ \text{TDH} = 150 - (-10) = 160 \text{ feet} \]

**Example 26:**

Take the following data:

1. Transferring 1000 GPM weak acid from the vacuum receiver to the storage tank;
2. Specific Gravity – SG = 0.98;
3. Viscosity - equal to water;
4. Piping – suction and discharge piping - all 6'' Schedule 40 steel pipes;
5. Discharge piping rises 40 feet vertically, plus 400 feet horizontally. Only one 90° flanged elbow.
6. Suction piping has a square edge inlet, 4 feet long, one gate valve and one 90° flanged elbow;
7. The minimum level in the vacuum receiver is 5 feet above the pump centerline.
8. The pressure on top of the liquid in the vacuum receiver is 20 inches of mercury, vacuum.

![Diagram of pump system](image)

a) The total suction head (Hs) calculation is:

1. The suction static head is 5 feet above suction centerline. The suction pipe is 4 ft long.
   \[ hss = 5 \text{ feet} \]
2. To calculate the suction surface pressure use one of the following formulae:
   - Feet of Liquid = Inches of mercury x 1.133 / Specific Gravity
   - Feet of Liquid = Pounds per square inch x 2.31 / Specific Gravity
   - Feet of Liquid = Millimeters of mercury / (22.4 x Specific Gravity)

   Then, using the first formula, the suction surface pressure is:
   \[ hps = -20 \text{ Hg} \times 1.133 / 0.98 = -23.12 \text{ feet water} \]
3. The suction friction head (hfs) equals the sum of all the friction losses in the suction line. Friction loss in 6" pipe, at a flow rate 1000 GPM, considering the Hazen-Williams equation is 12.26 feet per 100 feet of pipe.

The friction loss for a 6"diameter x 4 ft long pipe is = 4/100 x 12.26 = 0.49 feet.

The friction loss coefficients (K factors) for the inlet, elbow and valve can be added together and multiplied by the velocity head. There is no K factor for the square inlet, assume K = 0.45.

<table>
<thead>
<tr>
<th>Fittings</th>
<th>K</th>
<th>From Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>6&quot; – Square edge inlet</td>
<td>0.45</td>
<td>Page 14</td>
</tr>
<tr>
<td>6&quot; - 90º flanged elbow</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>6&quot; - Gate valve</td>
<td>0.12</td>
<td></td>
</tr>
</tbody>
</table>

Total coefficient, K = 1.02

The total friction loss (hfs) on the suction side is:

\[ hfs = 0.49 + 1.02 = 1.51 \text{ feet} \]

4. The total suction head (Hs) then becomes:

\[ Hs = hss + hps - hfs = \]

\[ Hs = 5 + (-23.12) - 1.51 = -19.6 \text{ feet} \]

b) The total discharge head (Hd) calculation is:

1. Static discharge head = hsd = 40 feet
2. Discharge surface pressure = hpd = 0 feet gauge
3. Discharge friction head = hfd = sum of the following losses:

The friction loss for a 6" pipe at 1000 GPM from table indicated above is: 6.17 feet / 100 feet of pipe.

Considering the 440 feet of pipe, the friction loss = 440/100 x 6.17 = 27.2 feet

The friction loss for a 6" elbow, K = 0.45

Q = 1000 GPM = 2.3 ft³/s, and pipe radius = 3" /12 = 0.25 ft
The flow velocity, \( v = \frac{Q}{A} = 2.3 \text{ ft}^3/\text{s} / \pi \times 0.25^2 = 11.36 \text{ ft/s} \)
From equation, \( Vh = v^2 / 2g = 11.36^2 / 64.34 = 2.0 \text{ ft} \)

Friction loss = K x Vh = 0.45 x 2.0 = 0.9 feet

The friction loss in the sudden enlargement at the end of the discharge line is called the exit loss. Then the velocity in discharge tank friction loss at exit is:

\[ Vh = v^2 / 2g = 2.0 \text{ feet} \]

The discharge friction head (hfd) is the sum of the above losses, that is:

\[ hfd = 27.2 + 0.9 + 2.0 = 30.1 \text{ feet} \]
4. The total discharge head (Hd) becomes:

\[Hd = hsd + hpd + hfd = 40 + 0 + 30.1 = 70.1 \text{ feet}\]

a) The Total Dynamic Head (TDH) calculation:

\[TDH = Hd - Hs = 70.1 - (-19.6) = \]

\[TDH = 89.7 \text{ feet}\]

13. Pump System Power:

The Brake Horsepower (BHP) is the actual horsepower delivered to the pump shaft, defined as follows:

\[BHP = Q \times H \times SG / 3960 \times \eta\]

Where:

- **Q** = Capacity in gallons per minute;
- **H** = Total Differential Head in absolute feet;
- **SG** = Specific Gravity of the liquid;
- **\(\eta\)** = Pump efficiency as a percentage.

The actual or brake horsepower (BHP) of a pump will be greater than the WHP by the amount of losses incurred within the pump through friction, leakage and recirculation, defined as follows:

\[WHP = Q \times H \times SG / 3960\]

Where

- **Q** = Capacity in gallons per minute;
- **H** = Total Differential Head in absolute feet;
- **SG** = Specific Gravity of the liquid.

Obs.: The constant (3960) is the number of foot-pounds in one horsepower (33,000) divided by the weight of one gallon of water (8.33 pounds).

14. Recommended Flow Velocity:

In general - a rule of thumb - is to keep the suction fluid flow speed below the following values:

<table>
<thead>
<tr>
<th>Pipe Diameter</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>inches/mm</td>
<td>m/s</td>
</tr>
<tr>
<td>1 25</td>
<td>0.5</td>
</tr>
<tr>
<td>2 50</td>
<td>0.5</td>
</tr>
<tr>
<td>3 75</td>
<td>0.5</td>
</tr>
<tr>
<td>4 100</td>
<td>0.55</td>
</tr>
<tr>
<td>6 150</td>
<td>0.6</td>
</tr>
<tr>
<td>8 200</td>
<td>0.75</td>
</tr>
<tr>
<td>10 250</td>
<td>0.9</td>
</tr>
<tr>
<td>12 300</td>
<td>1.4</td>
</tr>
</tbody>
</table>
Note: Fluid velocity should not exceed 4 ft/s and, depending on the pipe sizes involved, always select the next larger pipe diameter, that will result in acceptable pipe velocities.

The velocity formulae may be:

\[ v = Q \times 0.4085 / d^2 \]  
(Imperial Units)

or,

\[ v = (Q \times 0.321) / A = \]

Where:

- \( v \) = Velocity (ft/s)
- \( Q \) = Volume flow (GPM)
- \( d \) = Pipe inside diameter (inches)
- **Constant** = 0.4085 and 0.321 (used to convert GPM into cubic feet and then, velocity in ft/s).

\[ v = 1.274 Q / d^2 \]  
(Metric Units)

\[ v = \text{Velocity (m/s)} \]
\[ Q = \text{Volume flow (m}^3/\text{s)} \]
\[ d = \text{Pipe inside diameter (m)} \]

A handy formula for the pump impeller speed is:

\[ V = N \times D = \frac{229}{V} \]

\( V \) = Peripheral impeller velocity, ft/s
\( N \) = Impeller rotation, RPM
\( D \) = Impeller diameter

**Example 27:**

What is the velocity of flow for a 1" polyethylene sewage pipe, 1.189" ID, with a flow rate of 8 GPM?

\[ v = 0.4085 \times 8 / (1.189)^2 \]
\[ v = 2.3 \text{ ft/s} \]

**Example 28:**

An inlet pressure gage is installed in a 2 inches pipe directly in front of a pump delivering 100 gpm oil with Specific Gravity \( \text{SG} = 0.9 \), reading 10 psig. Calculate Velocity Head and Total Suction Pressure.

Pipe net Area:

\[ A = 3.14 \times d^2 / 4 = 3.14 \times 2^2 / 4 = 3.14 \text{ in}^2 \]

Velocity:

\[ v = (Q \times 0.321) / A = (100 \times 0.321) / 3.14 = 10.2 \text{ ft/s} \]
The Velocity Head is:

\[ Vh = \frac{v^2}{2g} = \frac{10.2^2}{2 \times 32.2} = 1.6 \text{ ft.}, \text{ or,} \]
\[ Vh = 1.6 \times 0.9 / 2.31 = 0.6 \text{ psi}. \]

The **Total Suction Pressure** then is:

\[ Hs = 10 + 0.6 = 10.6 \text{ psi}, \text{ or,} \]
\[ Hs = 10.6 \times 2.31 / 0.9 = 27.2 \text{ feet of water} \]

**15. Capacity Relationship:**

As liquids are essentially incompressible, the capacity is directly related with the velocity of flow in the suction pipe. Flow rate Q is defined to be the volume of fluid passing by some location through an area during a period of time. Flow rate and velocity are related, but quite different, physical quantities. The GPM relationship is as follows:

\[ \text{GPM} = 449 \times v \times A \]

Where

\( v = \) Velocity of flow, feet per second (fps)
\( A = \) Area of pipe, ft²

**16. Pipe Diameter – Minimum Recommended:**

The recommended suction inlet size (D) may be:

\[ D = (0.0744 \ Q)^{0.5} \]

Where:

\( D = \) Pipe diameter, inches
\( Q = \) Flow rate in gallons per minute (GPM).

Clear fluids:

\[ d = \sqrt[0.73]{\frac{Q}{SG}} = \frac{Q}{\rho^{0.33}} \]

Corrosive fluids:

\[ d = \sqrt[1.03]{\frac{Q}{SG}} = \frac{Q}{\rho^{0.33}} \]

\( d = \) Pipe inner diameter, in
\( Q = \) Flow rate, GPM
\( SG = \) Specific Gravity,
\( \rho = \) Fluid density, lb/ft²
17. Calculating the NPSH:

The term NPSH means Net Positive Suction Head. The motive to calculate the NPSH of any pump is to avoid the cavitation or corrosion of the parts during the normal process.

The main concepts of NPSH to be understood are the NPSHr (required) and NPSHa (available).

The NPSHr can be found in a manufacturing catalog of pumps, a technician or an engineer is choosing to apply in a project or installation. The manufacturer always shows the graphic curves of all line pumps manufactured by the company, indicating the required NPSH for each product.

The NPSHa is the normal calculation the technician or the engineer has to perform to find which of pump, from that manufacturing catalog, will better fit in his project or installation. Then, to calculate the available NPSH of a pump is necessary to know the following concepts:

a) NPSH: NPSHa (available) > NPSHr (required).

b) Vapor Pressure: The vapor pressure units, commonly given in feet or meters, depend completely from the temperature and the altitude. At 212°F or 100°C (boiling point of water) the water vapor pressure is 33.9 feet (14.7 psia) or 10.33 m (1.033 kg/cm²). See the basic tables below:

<table>
<thead>
<tr>
<th>Temperature, °F / °C</th>
<th>32 0</th>
<th>40 5</th>
<th>50 10</th>
<th>60 15</th>
<th>70 21</th>
<th>122 50</th>
<th>149 65</th>
<th>167 75</th>
<th>212 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vapor Pressure, feet / meters</td>
<td>0.204</td>
<td>0.280</td>
<td>0.410</td>
<td>0.591</td>
<td>0.837</td>
<td>4.126</td>
<td>8.384</td>
<td>12.919</td>
<td>33.9</td>
</tr>
<tr>
<td>Vapor Pressure, psia / kg/cm²</td>
<td>0.062</td>
<td>0.085</td>
<td>0.125</td>
<td>0.180</td>
<td>0.255</td>
<td>1.258</td>
<td>2.555</td>
<td>3.938</td>
<td>10.33</td>
</tr>
<tr>
<td>Temperature, °F / °C</td>
<td>32 0</td>
<td>40 5</td>
<td>50 10</td>
<td>60 15</td>
<td>70 21</td>
<td>122 50</td>
<td>149 65</td>
<td>167 75</td>
<td>212 100</td>
</tr>
<tr>
<td>Pressure, psi / kg/cm²</td>
<td>0.088</td>
<td>0.122</td>
<td>0.178</td>
<td>0.256</td>
<td>0.363</td>
<td>1.789</td>
<td>3.635</td>
<td>5.601</td>
<td>14.7</td>
</tr>
<tr>
<td>Pressure, psi / kg/cm²</td>
<td>0.006</td>
<td>0.008</td>
<td>0.012</td>
<td>0.018</td>
<td>0.025</td>
<td>0.123</td>
<td>0.255</td>
<td>0.394</td>
<td>1.033</td>
</tr>
<tr>
<td>Altitude at Sea Level, Feet / Meters</td>
<td>0 0</td>
<td>500 153</td>
<td>1000 305</td>
<td>1500 458</td>
<td>2000 610</td>
<td>3000 915</td>
<td>5000 1526</td>
<td>7000 2136</td>
<td>10000 3050</td>
</tr>
<tr>
<td>Pressure, psi / kg/cm²</td>
<td>33.9</td>
<td>10.33</td>
<td>33.28</td>
<td>10.15</td>
<td>9.56</td>
<td>9.78</td>
<td>9.60</td>
<td>9.26</td>
<td>8.60</td>
</tr>
<tr>
<td>Pressure, psi / kg/cm²</td>
<td>14.7</td>
<td>1.033</td>
<td>14.43</td>
<td>1.015</td>
<td>1.015</td>
<td>0.956</td>
<td>0.978</td>
<td>0.960</td>
<td>0.926</td>
</tr>
</tbody>
</table>

c) Static Head: Is positive when liquid line is above pump centerline and negative when liquid line is below pump centerline.

Head, feet = \( \frac{\text{psi} \times 2.31}{\text{Sg}} \), or, Vapor Pressure (psi) \( \times 2.31 = \frac{\text{Sg}}{\text{Sg}} \)

d) Atmospheric Pressure: When the pump to be installed is according to altitude from sea level (see table above).

Pressure, psi = Head x Sg = \( \frac{\text{Head} \times \text{Sg}}{2.31} \)

e) Specific Gravity: is the substance density compared to water. The density of water at standard temperature is 1 g/cm³ = 1 g/liter. So, the Specific Gravity (Sg) of water is 1.0.

f) Friction Loss: is a measure of the reduction in the total head (sum of elevation head, velocity head and pressure head) of the fluid as it moves through a fluid system.
Head Loss, \( H_f = f \frac{L v^2}{D} \) \( 2g \)

The technician or engineer also needs to know the formulae that show how to convert vacuum readings to feet of head.

The main formulae to convert vacuum readings to feet of head are:

- Feet of Liquid = **Inches of mercury** x 1.133 / Specific Gravity
- Feet of Liquid = **Pounds per square inch** x 2.31 / Specific Gravity
- Feet of Liquid = **Millimeters of mercury** / (22.4 x Specific Gravity)

The side graphic shows the conditions of each item in a complete NPSH process:

### 18. Calculation of the NPSH Process:

As explained above the calculation is for the NPSHa.

\[ \text{NPSHa} = + H + \text{Pa} - \text{Pv} - H_f \]

- \( H = \) Static Suction Head (positive or negative), in feet
- \( \text{Pa} = \) Atmospheric pressure (psi x 2.31/Sg), in feet
- \( \text{Pv} = \) Vapor pressure (psi x 2.31/Sg), in feet.
- \( H_f = \) See tables indicating friction loss. Fittings friction loss is (K x v²/2g), in feet.

**Example 29:**

1) Find the NPSHa from below data:

**Steel Piping** = suction and discharge - 2 inch diameter, total length10 feet, plus 2 x 90° elbow;

**Cold water pumping,** \( Q =100 \text{ gpm} @ 68^\circ\text{F};\)

**Flow velocity,** \( v = 10 \text{ ft/s} \) (maximum);

**Specific gravity,** \( Sg = 1.0 \) (clean water).

\( H = \) Liquid level is above pump centerline = + 5 feet

\( \text{Pa} = \) Atmospheric pressure = 14.7 psi - the tank is at sea level

\( \text{Pv} = \) Water vapor pressure at 68°F = 0.339 psi.

According to pump manufacturer the NPSHr (required), as per the pump curve) = 24 feet.

Using the above formula:
\[ NPSHa = + H + Pa - Pv - Hf \]

\[ H - \text{ Static head} = +5 \text{ feet} \]

\[ Pa - \text{ Atmospheric pressure} = \text{ psi} \times 2.31/\text{Sg.} = 14.7 \times 2.31/1.0 = +34 \text{ feet absolute} \]

\[ Pv - \text{ Water vapor pressure at 68°F} = \text{ psi} \times 2.31/\text{Sg} = 0.339 \times 2.31/1.0 = 0.78 \text{ feet} \]

\[ Hf - 100 \text{ gpm} - \text{ through 2 inches pipe shows a loss of 36.1 feet for each 100 feet of pipe}, \text{ then:} \]

\[ \text{Piping friction loss} = Hf_1 = 10 \text{ ft} / 100 \times 36.1 = 3.61 \text{ feet} \]

\[ \text{Fittings friction loss} = Hf_2 = K \times v^2/2g = 0.57 \times 10^2 \times (x 2) = 1.77 \]

\[ 2 \times 32.17 \]

Total friction loss for piping and fittings = \( Hf = (Hf_1 + Hf_2) = 3.61 + 1.77 = 5.38 \text{ feet} \).

\[ NPSHa (\text{available}) = + H + Pa - Pv - Hf = \]

\[ NPSHa (\text{available}) = + 5 + 34 - 0.78 - 5.38 = \]

\[ NPSHa (\text{available}) = 32.34 \text{ feet (NPSHa)} > 24 \text{ feet (NPSHr)}, \text{ so, the system has plenty to spare.} \]

**Example 30:**

2) Using the same data above, find the NPSHa in metric numbers:

Steel Piping = suction and discharge - 2 inch diameter, total length 3.0 m, plus 2 x 90° screwed elbow; Cold water pumping – 100 gpm = 0.379 m³/min (22.7 m³/h) at 20º C (68º F);

Flow velocity for a 2 inches piping – 10 ft/s = ~3.0 m/s

\[ H = \text{ Liquid level above pump centerline} = +1.5 \text{ m} \]

\[ Pa = \text{ Atmospheric pressure} = 1.033 \text{ kg/cm}^2 = \text{ at sea level} \]

\[ Pv = \text{ Vapor pressure at 20º C} = 0.024 \text{ kg/cm}^2 \]

\[ Sg = \text{ Specific gravity} = 1.0 \text{ (1000 kg/m}^3\text{)} \]

According to pump manufacturer the NPSHr (required), as per the pump curve) = 7.32 m

1) Converting \( Pa =1.033 \text{ kg/cm}^2 \text{ in kg/m}^2 \text{ we have -} 1.033 \text{ kg/cm}^2 \times 10,000 = 10330 \text{ kg/m}^2 \)

\[ \text{Pa} = \text{ Water density} 1000 \text{ kg/m}^3, \text{ then} - \overline{10330} \text{ kg/m}^2 = 10.33 \text{ m of water column (WC)}; \]

\[
\frac{1000 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = \]

2) Converting \( Pv = 0.024 \text{ kg/cm}^2 \text{ in kg/m}^2 \text{ we have} - \overline{0.024} \text{ kg/cm}^2 \times 10,000 = 240 \text{ kg/m}^2 \)

\[ \text{Pv} = \text{ Water density} 1000 \text{ kg/m}^3, \text{ then} - \overline{240} \text{ kg/m}^2 = 0.24 \text{ m of water column (WC)}; \]

\[
\frac{1000 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = \]

3) Total Friction Loss, \( Hf \):

Piping 2 inches, total length = ……………………………………3.0 m

Equivalent length - 2 inches elbows = 1.1 m (x 2) = ………2.2 m

Total equivalent length = …………………………………………..5.2 m
a) According to the metric tables: for a flow rate 22.7 m³/h (100 gpm) using piping diameter 2 inches (0.05 m) and length of 100.0 m, the total friction loss is = \sim{25}\% \\

\[ H_f = 5.2 \times \frac{25}{100} = 1.30 \text{ m} \]

b) Using the Darcy - Weisbach formula:

\[ H_f = f \cdot \frac{L \cdot v^2}{D_i \cdot 2 \cdot g} \]

Where:

\[ f = \text{Friction} = 0.019 \text{ (see table in page 15)} \]
\[ D_i = \text{Pipe inside diameter} = 0.052 \text{ m}, \]
\[ L = \text{Piping length} = 5.2 \text{ m}, \]
\[ v = \text{Velocity rate} = 3.2 \text{ m/s}, \]
\[ g = \text{Velocity due gravity} = 9.8 \text{ m/s}^2 \]

\[ H_f = 0.019 \times 5.2 \times 3.2^2 = \sim{1.0} \]
\[ 0.052 \times 2 \times 9.8 \]

The calculated product, \( H_f = 1.0 \) will be used in this example:

Then:

\[ \text{NPSHa} = +H + Pa - P_v - hf \]

\[ \text{NPSHa} = +1.5 + 10.33 - 0.24 - 1.0 = 9.69 \text{ m (NPSHa)} > 7.32 \text{ m (NPSHr)}. \]

---

**Example 31:**

3) Compute the NPSHa, according to the following data below:

Water flow rate = 100 GPM 
Piping = 4 inches diameter 
Static Suction Lift Length= 15 ft + 2 ft (foot valve) + 1 elbow 90º + 1 elbow 45º; 
Water Vapor Pressure at 74º = 0.441 
Atmospheric pressure - corrected = 6 ft 
Consider a safety factor for atmospheric pressure = 2.0 ft 
Consider a friction loss correction = 0.71 

\[ \text{NPSHr} – \text{according to performance pump curves catalog} = 5.0 \text{ ft} \]
a) Calculate the Total Dynamic Suction Lift:

<table>
<thead>
<tr>
<th></th>
<th>STATIC SUCTION LIFT (Length =)</th>
<th>15 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Suction Piping, Friction:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) Pipe diameter, 4”</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pipe total length + Foot valve =</td>
<td>17 ft</td>
</tr>
<tr>
<td></td>
<td>b) One elbow 90°, diameter, 4” =</td>
<td>6 ft</td>
</tr>
<tr>
<td></td>
<td>c) One elbow 45°, diameter, 4” =</td>
<td>4 ft</td>
</tr>
<tr>
<td></td>
<td>Fittings total length =</td>
<td>10 ft</td>
</tr>
<tr>
<td></td>
<td>Total equivalent length =</td>
<td>27 ft</td>
</tr>
<tr>
<td></td>
<td>d) Pipe friction loss (see tables) = 4.43 ft</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Friction loss = 27’/100 x 4.43 = ~1.20 ft</td>
<td></td>
</tr>
<tr>
<td></td>
<td>f) Correction factor = 0.71</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Friction Loss = 1.20 x 0.71 = 0.85 ft</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Total Dynamic Suction Lift =</td>
<td>15.85 ft</td>
</tr>
</tbody>
</table>

The sketch below shows the calculated above data:
b) How to calculate the NPSHa:

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Atmospheric pressure at sea level</td>
<td>33.90’</td>
</tr>
<tr>
<td>E</td>
<td>Atmospheric pressure - corrected</td>
<td>- 6.00’</td>
</tr>
<tr>
<td>F</td>
<td>Atmospheric pressure available at job site</td>
<td>27.90’</td>
</tr>
<tr>
<td>G</td>
<td>Deductions from available atmospheric pressure:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1. Total dynamic suction lift</td>
<td>15.85’</td>
</tr>
<tr>
<td></td>
<td>2. Vapor pressure 74º (0.441 x 2.31 / 1.0)</td>
<td>1.00’</td>
</tr>
<tr>
<td></td>
<td>3. Safety factor (for atmospheric pressure)</td>
<td>2.00’</td>
</tr>
<tr>
<td>H</td>
<td>Net deductions from available atmospheric pressure</td>
<td>18.85’</td>
</tr>
<tr>
<td>I</td>
<td>NPSHa, F - H =</td>
<td>+ 9.05’</td>
</tr>
<tr>
<td>J</td>
<td>NPSHr - according to pump catalog =</td>
<td>- 5.00’</td>
</tr>
<tr>
<td>L</td>
<td>NPSH excess available, or excess atmospheric, I - J =</td>
<td>4.05’</td>
</tr>
</tbody>
</table>

19. Cavitation:

Cavitation is associated with head loss, as relationship between NPSHr and Total Dynamic Head. In 1920 the German engineer Dieter Thoma described a parameter known as the Thoma’s cavitation factor.

$$\sigma = \frac{(P_a - P_v - H_s)}{H} = \text{(Thoma's Formula)}$$

Where:

- \( \sigma \) = Thoma’s number
- \( P_a \) = Atmospheric pressure (at sea level = 33.90 ft)
- \( P_v \) = Vapor pressure (ft)
- \( H_s \) = Suction head (ft)
- \( H \) = Total dynamic head (ft)

Specific speed (Ns) and suction specific speed (S) are terms that are no longer limited to the interest of pump designers. The equation for specific speed is:

$$Ns = n \times \sqrt{\frac{Q}{H^{0.75}}}$$

The formula for suction specific speed is an indicator of impeller inlet geometry:

$$S = n \times \sqrt{\frac{Q}{\text{NPSHr}^{0.75}}}$$

In Imperial system, when the NPSHr from a pump manufacturer is not available, since experience has shown that \( s = 9000 \) is a reasonable value of suction specific speed; it can be estimated by the following equation:

$$9000 = n \times \sqrt{\frac{Q}{\text{NPSHr}^{0.75}}}$$

The common calculation to find the NPSHa should be 50% bigger than the NPSHr.
Example 32:

Given data: Pump flow 2,000 GPM; head 600 ft. What NPSHa will be required?

Considering that with a head of 600 ft., 3500 RPM operation will be required, then:

\[ 9000 = 3500 \times \sqrt[0.75]{2000} = NPSHr \]

\[ NPSHr^{0.75} = \frac{3500 \times \sqrt{2000}}{9000} = \]

\[ NPSHr = 17.4^{1.333} \]

\[ NPSHr = 45 \text{ ft} \]

Thus, the NPSHa will become:

\[ NPSHa = 45 \times 1.5 \text{ (factor)} = 67.5 \]

Example 33:

Calculate the specific speed (Ns) of a centrifugal pump with 1750 RPM, a flow 0.045 m³/s and total dynamic head of 45.61 m. Consider Pa = 9.5 m, Pv = 0.235 m, Hs = 2.40 m.

\[ Q = 0.045 \times 1000 \times 60s / 3.78 \text{ liters} = 714 \text{ GPM} \]

\[ H = \frac{45.61m}{0.305 m} = \sim 150 \text{ ft} \]

\[ Ns = n \times Q^{0.5} / H^{0.75} = \]

\[ Ns = 1750 \times 714^{0.5} / 150^{0.75} = 1090 \]

Thoma’s formula:

\[ \sigma = (Pa – Pv – Hs) / H = \]

\[ \sigma = (9.5 – 0.235 – 2.40) / 45.61 = 0.15 \]

Note: The Thoma’s number (0.15) and the specific speed Ns (1090) in figure below shows the calculation enters in a safe region. Then, there will be no cavitation.

In Metric system, when the NPSHr from a pump manufacturer is not available, it can be estimated by the following equation:

\[ NPSHr = \varphi \times n^{4/3} \times Q^{2/3} = \]

Where:

\( \varphi = 0.0011 \) for centrifugal pumps;
\( n = \) Impeller rotation (RPM);
\( H = \) Suction head, ft (m);
\( Q = \) Flow rate, CFS (m³/s).
Example 34:

Estimate the NPSHr: centrifugal pump flow rate = 50 m³/h (0.0139 m³/s); Impeller rotation = 3000 RPM

\[ NPSHr = 0.0011 \times 3000^{4/3} \times 0.0139^{2/3} = \]

\[ NPSHr = 0.0011 \times 43152 \times 0.058 = 2.75 \text{ m} \]

Example 35:

Given the data: pump with 1750 RPM e flow rate 0.045 m³/s. Estimate the NPSHr.

\[ NPSHr = 0.0012 \times n^{4/3} \times Q^{2/3} = \]

\[ NPSHr = 0.0012 \times 1750^{4/3} \times 0.045^{2/3} = 0.0012 \times 21088 \times 0.1265 = 3.2 \text{ m} \]

20. Elevation Equivalent Pressure Relationship:

- **Static Head** – The hydraulic pressure at a point in a fluid when the liquid is at rest.
- **Friction Head** – The loss in pressure or energy due to frictional losses in flow.
- **Velocity Head** – The energy in a fluid due to its velocity, expressed as a head unit.
- **Pressure Head** – A pressure measured in equivalent head units.
- **Discharge Head** – The outlet pressure of a pump in operation.
- **Total Head** – The total pressure difference between the inlet and outlet of a pump in operation.
- **Suction Head** – The inlet pressure of a pump when above atmospheric.
- **Suction Lift** – The inlet pressure of a pump when below the atmospheric.
These terms are sometimes used to express different conditions in a pumping system, and can be given dimensions of either pressure units (PSI) or head units (feet).

21. Centrifugal Pump Parts:

Example 36:

The sketch above is a clean water pumping system to be sized in order to feed a reservoir.

Given the data:
Static suction head is negative, hss = - 6 feet;
Suction piping length: 10 ft;
Static discharge head, hsd = 125 feet;
Discharge piping length: 9800 ft;
Flow rate: 480 GPM;
Galvanized steel piping, C = 100;
Elevation = 3000 ft (915 m) - atmospheric pressure, Pa = 13.17 psi; 30.37 ft (table page 39);
Water temperature = 77 °F (25 °C) – vapor pressure, Pv = 0.46 psi; 1.06 ft (table page 24).

1) Minimum recommended suction diameter calculation:

\[
D = (0.0744 \times Q)^{0.5} = (0.0744 \times 480)^{0.5} = 6 \text{ inches} - \text{ID 6.07 inches} - \text{Sch. 40 steel piping.}
\]

2) Flow rate velocity evaluation:

\[
v = \frac{Q \times 0.4085}{d^2}
\]

\[
v = \frac{480 \times 0.4085}{6.07^2} = 5.3 \text{ ft/s}
\]

Suction fluid velocity should not exceed 4 ft/s then, the next larger pipe diameter that will result in acceptable pipe velocities, thus:

3) Suction and discharge piping diameter & velocity:

D = 8 inches – ID 7.98 inches – 0.665 ft - Sch. 40 steel piping
\[
v = \frac{480 \times 0.4085}{7.98^2} = 3.0 \text{ ft/s} - \text{this flow velocity is adequate.}
\]

a) Suction piping friction loss:

✓ Darcy-Weisbach equation associated with piping length:

\[
H_f = f \times \frac{L \times v^2}{D \times 2g} = 0.014 \times \frac{10 \times 3.0^2}{0.66 \times 64.34} = 0.029 \text{ ft}
\]

Where:

\[f = \text{Friction factor (8 inches pipe)} = 0.014 \text{ (Table page 15)}
\]

\[L = \text{Suction piping length} = 10 \text{ ft}
\]

\[D = \text{Internal diameter of pipe} = 7.98 \text{ in} = 7.98 / 12 = 0.66 \text{ ft}
\]

\[v = \text{Velocity of fluid} = 3.0 \text{ ft/s}
\]

\[g = \text{Acceleration due gravity} = 32.17 \text{ ft/s}^2
\]

b) The coefficient “K” of fittings to be used according to tables:

\[
H_f = \frac{v^2}{2g} = 3.0^2 = 0.14
\]

\[
\frac{1}{2g} = 64.34
\]

1 foot valve 8” with Strainer Hinged Disc - (K = 1.10) = 1.10 \times 0.14 = 0.154 \text{ ft}
1 elbow 8”, 90° - (K = 0.42) = 0.42 \times 0.14 = 0.056 \text{ ft}
1 reduction 8” x 6” – (K = 0.34) = 0.34 \times 0.14 = 0.047 \text{ ft}
\[ H_f (suction) = 0.029 + 0.154 + 0.056 + 0.047 = 0.29 \text{ ft} \]

c) Discharge piping friction loss:

- **Darcy-Weisbach** with piping length:

\[
H_f = f. \frac{L \cdot v^2}{D \cdot 2g} = 0.014 \times 9800 \times 3.0^2 = 29.0 \text{ ft}
\]

\[
D = 64.34
\]

\d) The coefficient “K” according to tables:

\[ H_f = \frac{v^2}{2g} = \frac{3.0^2}{64.34} = 0.14 \]

1 reduction 8” x 6” – (K = 0.34) = 0.34 x 0.14 = 0.047 ft
1 swing check valve 8” – (K = 1.40) = 1.40 x 0.14 = 0.196 ft
1 gate valve 8” – (K = 0.11) = 0.11 x 0.14 = 0.015 ft
1 elbow 8”, 90° – (K = 0.42) = 0.42 x 0.14 = 0.056 ft

\[ H_f (discharge) = 29 + 0.047 + 0.196 + 0.015 + 0.056 = 29.32 \text{ ft} \]

**Obs.:** When the Hazen-Williams are preferred, the equation is as indicated below:

\[ H_f = 0.2083 \left( \frac{100}{C} \right)^{1.85} \times Q^{1.85} \]

Where:

- \( f \) = Friction head loss in feet of water (per 100 ft of pipe);
- \( C \) = Hazen-Williams roughness constant;
- \( Q \) = Volume flow (gpm);
- \( D \) = Inside pipe diameter (inches);
- \( L \) = Length of pipe, (in. or m).

Using the Hazen – William equations as indicated below:

<table>
<thead>
<tr>
<th>Specification – Suction Piping Friction Loss:</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>L = length of pipe (ft)</td>
<td>10</td>
</tr>
<tr>
<td>C = Hazen-Williams roughness constant</td>
<td>100</td>
</tr>
<tr>
<td>Q = volume flow (gal/min)</td>
<td>480</td>
</tr>
<tr>
<td>Dh = inside or hydraulic diameter (inches)</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculated Pressure Loss</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head loss - ft of water</td>
<td>0.08</td>
</tr>
<tr>
<td>Head loss - psi</td>
<td>0.03</td>
</tr>
</tbody>
</table>

| Calculated Flow Velocity | 3.07    |

<table>
<thead>
<tr>
<th>Specification - Discharge Piping Friction Loss</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>L = length of pipe (ft)</td>
<td>9800</td>
</tr>
<tr>
<td>C = Hazen-Williams roughness constant</td>
<td>100</td>
</tr>
<tr>
<td>Q = volume flow (gal/min)</td>
<td>480</td>
</tr>
<tr>
<td>Dh = inside or hydraulic diameter (inches)</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

©2012 Jurandir Primo
Calculated Pressure Loss:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Head loss - ft of water</td>
<td>76.14</td>
</tr>
<tr>
<td>Head loss - psi</td>
<td>32.74</td>
</tr>
</tbody>
</table>

Calculated Flow Velocity

v = flow velocity (ft/s) 3.07

1. The Darcy-Weisbach calculation resulted in **0.029 ft for suction piping** and **29.0 ft for discharge piping** respectively. The above online friction loss resulted in **0.04 ft and 37.36 ft** respectively.

2. However, as can be seen in the figures above, the online calculation uses the parameters of the Moody Chart. Thus, surely is much more efficient.

3. As can be noticed the piping head loss calculation is empirical and many times, trial and error. Using the acceptable results of the link “light my pump” (http://www.pumpfundamentals.com) the piping friction loss becomes:

   \[ H_f (suction) = 0.04 + 0.154 + 0.056 + 0.047 = -0.3 \text{ ft} \]

   \[ H_f (discharge) = 37.36 + 0.047 + 0.196 + 0.015 + 0.056 = -37.7 \text{ ft} \]

2) Total Dynamic Head (TDH) calculation:

   a) Suction head (Hs):

   1. The suction head is negative, \( hss = -6 \) feet;
   2. The suction surface pressure, \( hps = 0 \) feet, gauge (tank is open, equals atmospheric pressure);
   3. The suction friction head is, \( hfs = 0.3 \) feet;
   4. The total suction head is, \( (Hs = hss + hps - hfs) \) then, \( Hs = -6 + 0 - 0.30 = -6.3 \) feet

   b) Discharge head (Hd):

   1. The static discharge head is, \( hsd = 125 \) feet
   2. The suction surface pressure, \( hpd = 0 \) feet, gauge (tank is open, equals atmospheric pressure);
   3. The discharge friction head is, \( hfd = 37.7 \) feet
   4. The total discharge head is, \( (Hd = hsd + hpd + hfd) \) then, \( Hd = 125 + 0 + 37.7 = 162.7 \) feet

   The Total Dynamic Head (TDH) is, \( (TDH = Hd - Hs) \) then, \( TDH = 162.7 - (-6.3) = 169.0 \) feet

   c) Brake Horsepower (BHP) calculation:

   \[ BHP = \frac{Q \times H \times SG}{3960 \times \eta} = \frac{480 \times 169 \times 1.0}{3960 \times 0.75} = 27 \text{ HP} \]

   Consider, \( BHP = 30 \text{ HP} \)
Q = Capacity, 480 GPM
H = Total Differential Head, 169 ft
SG = Specific gravity, 1.0
Ρη = Pump efficiency, assume 75%.

3) NPSHa calculation:

\[ H - \text{Static head} = -6 \text{ feet}; \]
\[ \text{Pa} - \text{Elevation} = 3000 \text{ ft (915 m)} - \text{atmospheric pressure, Pa} = 13.17 \text{ psi; 30.37 ft (table page 39)}; \]
\[ \text{Pv} - \text{Water temperature} = 77^\circ \text{F (25}^\circ \text{C)} - \text{vapor pressure, Pv} = 0.46 \text{ psi; 1.06 ft (table page 24)}; \]
\[ H_f (\text{suction}) = 0.3 \text{ ft} \]

NPSHa (available) = \( H - \text{Pa} - \text{Pv} - H_f \)

NPSHa (available) = -6 + 30.37 - 1.06 - 0.3 = 23.0 ft

a) The NPSHr is not known, it can be estimate:

\[ 9000 = n \times \sqrt{\frac{Q}{\text{NPSHr}^{0.75}}} = \frac{3500 \times \sqrt{480}}{\text{NPSHr}^{0.75}} \]

\[ \text{NPSHr}^{0.75} = \frac{3500 \times \sqrt{480}}{9000} \]

\[ \text{NPSHr} = 8.5^{1.333} \approx 17.0 \text{ ft} \]

NPSHa (available) = 23.0 feet (NPSHa) > 17 feet (NPSHr). The system is acceptable.

4) Check about cavitation. Assume a centrifugal pump with 1750 RPM:

\[ \text{Ns} = n \times \frac{Q^{0.5}}{H^{0.75}} = \]

\[ \text{Ns} = 1750 \times 480^{0.5} / 169^{0.75} = -820 \]

Thoma’s formula:

\[ \sigma = \frac{\text{Pa} - \text{Pv} - \text{Hs}}{H} = \]

\[ \sigma = 30.37 - 1.06 - (-6.3) / 169 = -0.20 \]

The Thoma’s number (0.20) and the specific speed Ns (820) in the graphic (page 45) shows the calculation enters in a safe region. Then, there will be no cavitation.

Notes:

a) The Hazen-Williams formula gives accurate head loss due to friction for fluids with kinematic viscosity of approximately 1.1 cSt and cold water at 60 °F (15.6 °C).

b) The Hazen Williams method is valid for water flowing at ordinary temperatures between 40 to 75 °F and the Darcy Weisbach method should be used for other liquids or gases.
### Hazen-Williams Equation for Pressure Loss in Pipes:

#### Imperial or US Units:

<table>
<thead>
<tr>
<th>Specified Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( l ) = length of pipe (ft)</td>
<td>200</td>
</tr>
<tr>
<td>( c ) = Hazen-Williams roughness constant</td>
<td>140</td>
</tr>
<tr>
<td>( q ) = volume flow (gal/min)</td>
<td>200</td>
</tr>
<tr>
<td>( dh ) = inside or hydraulic diameter (inches)</td>
<td>3</td>
</tr>
</tbody>
</table>

#### Calculated Pressure Loss

- \( f \) = friction head loss in feet of water per 100 feet of pipe (ft H₂O per 100 ft pipe) 9.73
- \( f \) = friction head loss in psi of water per 100 feet of pipe (psi per 100 ft pipe) 4.18

| Head loss (ft H₂O) | 19.46 |
| Head loss (psi) | 8.37 |

#### Calculated Flow Velocity

- \( v \) = flow velocity (ft/s) 9.08

#### SI Units:

<table>
<thead>
<tr>
<th>Specified Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( l ) = length of pipe (m)</td>
<td>30</td>
</tr>
<tr>
<td>( c ) = Hazen-Williams roughness constant</td>
<td>140</td>
</tr>
<tr>
<td>( q ) = volume flow (liter/sec)</td>
<td>10</td>
</tr>
<tr>
<td>( dh ) = inside or hydraulic diameter (mm)</td>
<td>76</td>
</tr>
</tbody>
</table>

#### Calculated Pressure Loss

- \( f \) = friction head loss in mm of water per 100 m of pipe (mm H₂O per 100 m pipe) 6406.62
- \( f \) = friction head loss in kPa per 100 m of pipe (kPa per 100 m pipe) 62.85

| Head loss (mm H₂O) | 1921.99 |
| Head loss (kPa) | 18.85 |

#### Calculated Flow Velocity

- \( v \) = flow velocity (m/s) 2.20

### Centrifugal Pumps Standards:

- **ASME B73.2 - 2003**: Specifications for Vertical In-Line Centrifugal Pumps for Chemical Process.
- **BS 5257 – 1975**: Specification for Horizontal End-Suction Centrifugal Pumps (16 bar).
LINKS AND REFERENCES:

✓ References:

Centrifugal Pumps - University of Sao Paulo, Engineering Lab

✓ Links:

http://www.tasonline.co.za/toolbox/pipe/veldyn.htm
http://www.lightmypump.com
http://www.mcnallyinstitute.com/