**PDHonline Course M476 (3 PDH)**

# The Reynolds Number - Units in a Dimensionless Number

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# The Reynolds Number - Units in a Dimensionless Number

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## Introduction

So you've been tasked with finding the pressure drop in a simple piping system. You know the pipe diameter, the fluid density, and even the flow rate. You find a formula for the Reynolds Number in your reference material; $R_e = \rho vD/\mu g_c$. You know you need the Reynolds Number to determine the friction factor which will be used to determine the pressure loss. You look up the relative roughness of the pipe you are using, determine the K-factors for the fittings and find a table that gives the kinematic viscosity for a variety of fluids in a variety of units. Recalling $12^{th}$ grade physics, you know units can be your downfall when solving equations in science and engineering.
Finding the velocity is simple enough to calculate; $v = 0.4085GPM/d^2$ (….but where did the 0.4085 constant come from??). You also find a different table with absolute viscosity in units of lbm/ft-sec. This value will work in the formula for Reynolds Number you just found, but you realize that the EXCEL program you want to use requires kinematic viscosity in units of centistokes. Where to begin? We'll discuss the various forms of the Reynolds Number first, and then look at viscosity and the various units used for that physical property and finally discuss the origin of $g_c$. Using this knowledge, we'll also solve a simple fluid flow problem using EXCEL.

## The Reynolds Number

You may have been introduced to the Reynolds Number in the Dimensional Analysis chapter in an introduction to Fluid Mechanics course. This dimensionless number is the ratio of the inertial forces to the viscous forces in a fluid flow. Inspecting the basic formula $R_e = \rho vD/\mu g_c$ , and separating the equation in terms of numerator and denominator, we have $\rho vD/g_c$ and $\mu$. Using primary dimensions of M (Mass) = lbm, L (Length) = ft, F (Force) = lbf and T (Time) = sec, we can substitute the primary dimension values into the numerator and denominator of the Reynolds Number.

Recall $g_c$= 32.2 lbm-ft/lbf-sec$^2$ (or in primary dimensions M-L/F-T$^2$) - more on $g_c$ later.

Then the numerator $\rho vD/g_c$ becomes M/L$^3$ * L/T * L * (F-T$^2$/M-L) which reduces to F-T/L$^2$, and the denominator μ (in English units absolute velocity can be expressed as lbf-sec/ft$^2$) becomes F-T/L$^2$. Both are the same and therefore cancel. The result is a dimensionless number.

The Reynolds Number appears in Crane (Technical Paper No. 410 Flow of Fluids) in fifteen different forms. We'll focus on just a few of these forms and determine how they were formulated and how they are related to each other. (Note: Nomenclature will be consistent with Crane, with a few exceptions)

$$R_e = \frac{\rho v D}{\mu g_c} \qquad \text{(Eq. 1)}$$

Where ρ = density in units of lbm/ft$^3$
v = velocity in units of ft/sec
D = pipe ID in units of ft
μ = absolute viscosity in units of lbf-sec/ft$^2$; ($\mu'_e$ in Crane)
$g_c$ = 32.2 lbm-ft/lbf-sec$^2$
(The term $\mu g_c$ is given as $\mu_e$ in Crane and has units of lbm/ft-sec)

This form of the Reynolds Number is easy to solve if you have viscosity in terms of $\mu g_c$. Chemical Rubber Co. CRC Handbook of Tables for Applied Engineering Science (2$^{nd}$ Edition, 1973) provides tables with absolute viscosity in units of lbm/ft-sec.
As an example, Benzene flowing through a 12 in Sch 80 pipe:

Let ρ = 54.7 lbm/ft$^3$
v = 12.06 ft/sec
D = 0.9478 ft
$\mu g_c$ = 4.04 x 10$^{-4}$ lbm/ft-sec

$R_e$ = (54.7 lbm/ft$^3$) (12.06 ft/sec)(0.94788 ft)/4.04 x 10$^{-4}$ lbm/ft-sec

$R_e$ = 1.55 x 10$^6$

(A general rule of thumb is that a flow with a Reynolds Number below 2000 is laminar, above 4000 fully turbulent. Flows in between are in transition)

One of the other versions of Reynolds Number uses flow rate in GPM and kinematic viscosity in centistokes

$$\mathbf{R_e = \frac{3165Q}{d\nu}} \qquad \text{(Eq. 2)}$$

Where Q = flow rate in gallons/minute (GPM)
d = pipe ID in units of inches
ν = kinematic viscosity in units of centistokes (1 centistoke = $0.000001\ m^2/sec$)

But how did $R_e = \rho vD/\mu g_c$ become $R_e = 3165Q/d\nu$??

The conversion is a fairly straight forward bookkeeping exercise with a few conversion factors sprinkled in.

$R_e = \rho vD/\mu g_c$ ⇒ Pipe ID will be expressed in inches; convert inches to feet by multiplying by 1/12
$R_e = (1ft/12in)\rho vd/\mu g_c$ ⇒ recall kinematic viscosity $\nu = \mu g_c/\rho$; pipe ID is now expressed in units of inches "d"
$R_e = (1ft/12in)vd/\nu$ ⇒ velocity v can be expressed as q/A; where $q = ft^3/sec$ and $A= (\pi d^2(1/144)/4)ft^2$ ; $q = Q * 0.00223\ ft^3/sec/GPM$
$R_e = (1ft/12in)d(Q*0.00223(\pi d^2(1/144)/4))/\nu$ ⇒ cancelling and combining terms
$R_e = 48Q*0.00223/\pi d\nu$ ⇒ ν will be in units of centistokes (cSt), not $ft^2/sec$
$R_e = 48Q*0.00223/(\pi d\nu*1.07639 \times 10^{-5}\ ft^2/sec/cSt)$; ⇒ the result is…..

$R_e = 3165Q/d\nu$, which is Equation 2.

Using data from the example above, $q = Av = \pi/4(0.9478\ ft)^2*(12.06\ ft/sec) = 8.51\ ft^3/sec$. Therefore,
$Q = (8.51\ ft^3/sec) / (0.00223\ ft^3/sec/GPM) = 3816$ GPM, and
d = 0.9478 ft x 12 = 11.3736 inches

This equation requires kinematic viscosity in centistokes (cSt). Curiously, cSt is a metric unit used in a formula that otherwise has English units. Crane promotes the use of the centipoise (also a metric unit) as the standard unit of absolute viscosity. 1 centipoise = 0.01 gram/cm-sec or multiply centipoise by 1/density (g/cm$^3$) to get centistokes. More viscosity conversions are provided later in this text.

From the first example, $\mu g_c = 4.04 \times 10^{-4}$ lbm/ft-sec
Multiply this value by 1488.0 to convert to centipoise; this equals 0.601152 centipoise
$\rho = 54.7$ lbm/ft$^3$ = 0.8765 g/cm$^3$ (from web search)
centipoise times 1/density (g/cm$^3$) = centistokes, therefore,
0.601152 centipoise x 1/0.8765 g/cm$^3$ = 0.685855 cSt

Plugging these values into $R_e = 3165Q/d\nu$ (Equation 2),

$R_e$ = 3165*3816 GPM/(11.3736in)*( 0.685855 cSt)
$R_e = 1.55 \times 10^6$

This is the same value obtained using Eq. 1.

Another version of the Reynolds Number uses velocity, pipe ID in inches and kinematic viscosity. The derivation of this equation is somewhat simpler than equation 2.

$$\mathbf{R_e = \frac{7741dv}{\nu}} \qquad \text{(Eq. 3)}$$

Where v = velocity of fluid in ft/sec
d = pipe ID in units of inches
ν = kinematic viscosity in units of centistokes

$R_e = \rho vD/\mu g_c$ ⇒ Pipe ID will be expressed in inches; convert inches to feet by multiplying by 1/12
$R_e = (1ft/12in)\rho vd/\mu g_c$ ⇒ recall kinematic viscosity $\nu = \mu g_c/\rho$; pipe ID "d" is now expressed in units of inches
$R_e = (1ft/12in)vd/\nu$ ⇒ velocity v is expressed in ft/sec; converting cSt to ft$^2$/sec; the version $R_e = (1/12)vd/\nu$ is used in the spreadsheet that accompanies this course. ν is in

units of $ft^2$/sec.

$R_e = (1ft/12in)vd/\nu\ (cSt \times 1.07639 \times 10^{-5}\ ft^2/sec/cSt)$ ⇒ the result is…..

$R_e = 7741dv/\nu$

Using the same example with Benzene flowing through a 12 in diameter pipe,
Let d = 11.3736 inches
v = 12.06 ft/sec
ν = 0.685855 cSt

Plugging these values into equation 3,

$R_e = 7741(11.3736\ in)(12.06\ ft/sec)/(\ 0.685855\ cSt)$
$R_e = 1.55 \times 10^6$

Again, the result is the same as when using both equations 1 and 2, as it should be.

Looking at some of the other version of Reynolds Number in Crane,

$$\mathbf{R_e = \frac{Dv\rho}{\mu_e}} \qquad \text{(Eq. 4)}$$

$\mu_e$ is in units of lbm/ft-sec and D is pipe ID in feet (ft). $\mu_e$ also equals $32.2\mu'_e$, therefore this equation can be rewritten as

$$\mathbf{R_e = \frac{Dv\rho}{32.2\mu'_e}} \qquad \text{(Eq. 5)}$$

$\mu'_e$ is expressed in units of lbf-sec/$ft^2$

(The key point to remember here is that when you see viscosity expressed in units of lbm/ft-sec, the value listed is already multiplied by $g_c$.)

If equation 4 is to expressed with pipe ID in inches (d) and viscosity expressed in units of centipoise (μ), inches needs to be converted to feet by multiplying by 1/12 and viscosity has to be converted to lbm/ft-sec by multiplying by 0.000672 lbm/ft-sec per centipoise. The result is…..

$$\mathbf{R_e} = \frac{\mathbf{124dv\rho}}{\boldsymbol{\mu}} \qquad \text{(Eq. 6)}$$

Equation 6 can be expressed in flow rate (q in units of $ft^3$/sec) by expressing the velocity (v) as q/A. The area (A) is represented by ($\pi d^2$(1/144)/4)/ ($\pi$d/ 12)ft. This reduces to d/48. Factoring this into Equation 6, the result is…..

$$\mathbf{R_e} = \frac{\mathbf{22734q\rho}}{\mathbf{d\mu}} \qquad \text{(Eq. 7)}$$

Equation 7 can be manipulated to use the hydraulic radius ($R_H$) in place of d. The hydraulic radius is defined as the Area/Wetted Perimeter. For a circular pipe completely full, $R_H = ((\pi/4*d^2)/(\pi d))$ft. This reduces to d/4. Converted to feet, the result is d/48. Factoring this into Equation 7, the result is…..

$$\mathbf{R_e} = \frac{\mathbf{473q\rho}}{\boldsymbol{R_H}\mathbf{\mu}} \qquad \text{(Eq. 8)}$$

Equation 7 can be easily expressed in GPM. GPM multiplied by 0.00223 $ft^3$/sec/GPM provides the conversion back to $ft^3$/sec to reconcile the units. The result is…..

$$\mathbf{R_e} = \frac{\mathbf{50.6Q\rho}}{\mathbf{d\mu}} \qquad \text{(Eq. 9)}$$

Equation 7 can also be expressed in lbm/hr (W). Since the conversion back to $ft^3$/sec is already factored into the constant, simply convert seconds to hours by dividing by $60^2$. The result is……….

$$\mathbf{R_e} = \frac{\mathbf{6.31W}}{\mathbf{d\mu}} \qquad \text{(Eq. 10)}$$

There are several other forms of the Reynolds Number provided in Crane using units such as barrels per hour (B), and the specific volume of fluid in units of $ft^3$/lb. The student is encouraged to review the other formulas to become familiar with their structure.

## Viscosity

Viscosity is the measurement of the resistance a fluid has to shear. For Newtonian fluids such as water, oil, and air, the following formula applies:

$$\tau = \mu\left(\frac{dv}{dy}\right) \qquad \text{(Eq. 11)}$$

Where τ is the shear stress in units of $lbf/ft^2$ or $N/m^2$ (or in Primary Dimensions $F/L^2$) and (dv/dy) is the slope of the velocity distribution as described below. μ is the absolute viscosity and is considered the constant of proportionality between the shear stress and the velocity distribution.

The classic example involves a plate of area "A" being pulled on top of a layer of viscous fluid of thickness "y" by a force of "$F_1$". Another plate at the bottom of the fluid layer is stationary. As the top plate moves, a linear fluid velocity profile develops starting at zero (0) velocity at the bottom stationary plate and reaching a maximum velocity of $v_1$ (this is also equal to the velocity of the moving plate) at the fluid surface which is in contact with the moving plate. This velocity profile or velocity distribution is the slope and is represented by (dv/dy); see Figure 1. The shear stress associated with $F_1$ is designated as $\tau_1$ and in this case is defined by the pulling force $F_1$ divided by the area of the moving plate A. Therefore, $\tau_1 = F_1/A$ (again, units are $lbf/ft^2$ or $N/m^2$).

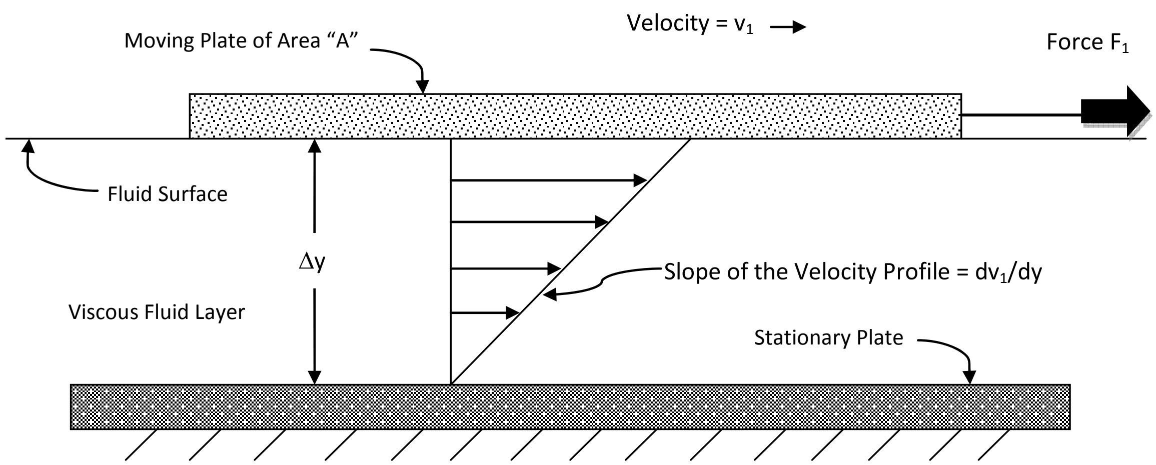


**Figure 1**

If another force, $F_2$, was applied to the same configuration of plates, a different shear stress, $\tau_2$, would develop in addition to a different velocity, $v_2$.

If a series of different forces were plotted as $\tau_N$ versus $dv_N/dy$, the slope of that line is the absolute viscosity "μ", i.e.

$$\mu = \frac{\Delta\tau}{\Delta\left(\frac{dv}{dy}\right)}$$

Stated another way, the shear stress τ (F/A or in Primary Dimensions $F/L^2$) is proportional to the fluid velocity (v) and inversely proportional to the distance between the moving and stationary plates (y), with the absolute viscosity (μ) representing the constant of proportionality, resulting in the equation…….

$\tau = \mu\,(dv/dy)$, which is Equation 11. See Figure 2.

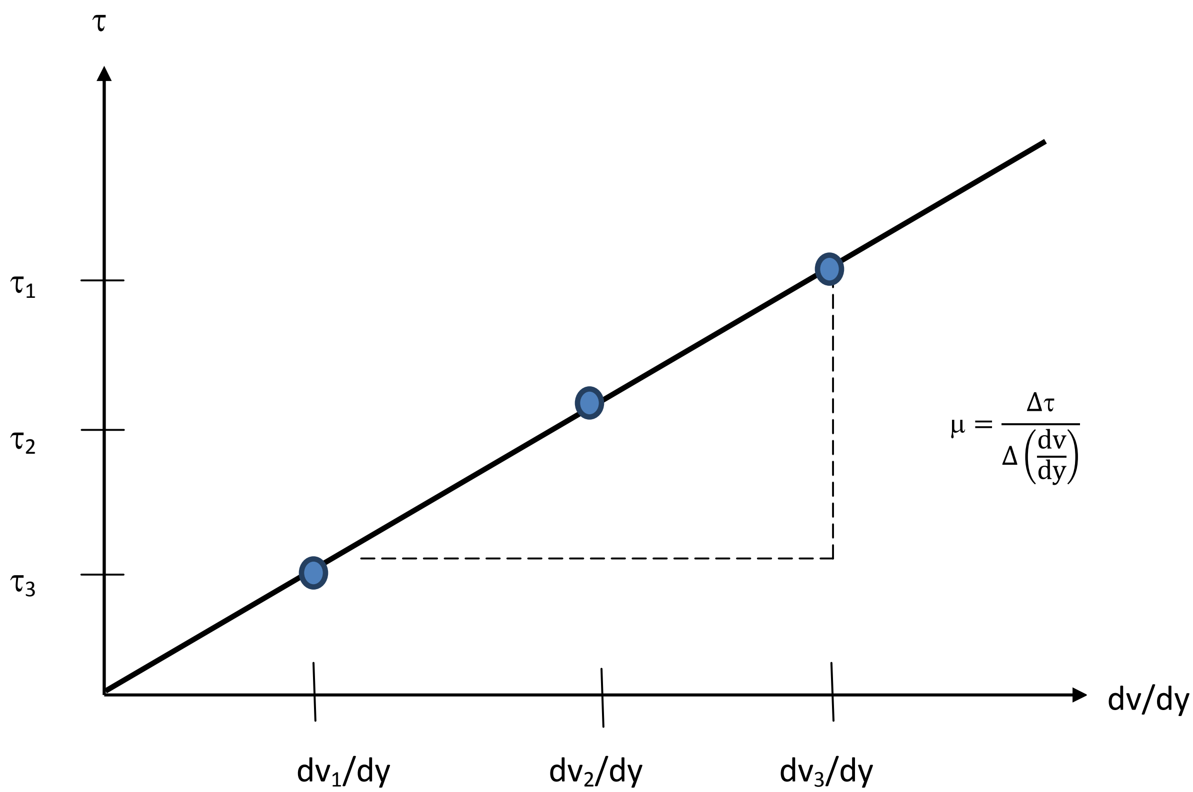


**Figure 2**

To determine the units for this constant of proportionality, let's use Primary Dimensions in Equation 11:

$F/L^2 = \mu\ (L/T)/L$
$F/L^2 = \mu\ (1/T)$
Solving for μ
$\mu = FT/L^2$; all units of absolute viscosity ultimately reduce to this form.
(Note – Dynamic Viscosity is another name for Absolute Viscosity; Absolute is the preferred term.)

In Crane μ is in units of centipoise. 1 centipoise = 0.001 Pascal-sec = 0.001 N-sec/$m^2$
$\mu_e$ has units of lbm/ft-sec. This quantity, when divided by $g_c$ results in units of lbf-sec/$ft^2$; this is $\mu'_e$.
As you can see, the units are consistent with the form as expressed in Primary

Dimensions.

Viscosity can be expressed in a multitude of different units. Some of the more common units have already been discussed. Other systems used to measure viscosity include Seconds Saybolt Universal (SSU), Seconds Saybolt Furol (SSF), Seconds Redwood 1 Standard, Seconds Redwood 2 Admiralty, Degrees Engler and Degrees Barbey. Crane provides conversion tables for kinematic viscosity in centistokes to SSU and SSF. Conversion tables for some of the other systems noted above can be found on the web.

Some conversion factors are provided below:

**Kinematic Viscosity**

| **To Convert** | **Into** | **Multiply By** |
|---|---|---|
| $ft^2$/sec | cSt | 92903.04 |
| $ft^2$/sec | $m^2$/sec | 0.092903 |
| $m^2$/sec | $ft^2$/sec | 10.7639 |
| $m^2$/sec | cSt | $1.0 \times 10^6$ |
| cSt | $m^2$/sec | $1.0 \times 10^{-6}$ |
| cSt | $ft^2$/sec | $1.07639 \times 10^{-5}$ |

**Absolute Viscosity**

| **To Convert** | **Into** | **Multiply By** |
|---|---|---|
| lbf-sec/$ft^2$ | centipoise | 47889.26 |
| lbf-sec/$ft^2$ | Pascal-sec | 47.8803 |
| centipoise | kg-sec/$m^2$ | 0.000102 |
| kg-sec/$m^2$ | centipoise | 9810.0 |
| centipoise | lbf-sec/$ft^2$ | $2.0885 \times 10^{-5}$ |
| Pascal-sec | lbf-sec/$ft^2$ | 0.0208854 |
| Pascal-sec | centipoise | 1000 |
| centipoise | Pascal-sec | 0.001 (1 Pascal-sec = 1N-sec/$m^2$) |
| centipoise | lbm/ft-sec | 0.000672 |
| lbm/ft-sec | centipoise | 1488.0 |

## Absolute to Kinematic Viscosity

| **To Convert** | **Into** | **Multiply By** |
|---|---|---|
| centipoise | centistokes | 1/density ($g/cm^3$) |
| centipoise | $ft^2/sec$ | 0.00067197/density ($lbm/ft^3$) |
| $lbf\text{-}sec/ft^2$ | $ft^2/sec$ | $g_c$/density ($lbm/ft^3$); where $g_c = 32.2\ lbm\text{-}ft/lbf\text{-}sec^2$ |
| $kg\text{-}sec/m^2$ | $m^2/sec$ | 9.8/density ($kg/m^3$) |
| Pascal-sec | centistokes | 1000/density ($g/cm^3$) |
| Pascal-sec | $m^2/sec$ | 0.001/density ($g/cm^3$) |

## Kinematic to Absolute Viscosity

| **To Convert** | **Into** | **Multiply By** |
|---|---|---|
| centistokes | centipoise | density ($g/cm^3$) |
| $m^2/sec$ | $kg\text{-}sec/m^2$ | 0.10197 x density ($kg/m^3$) |
| $ft^2/sec$ | $lbf\text{-}sec/ft^2$ | 0.03108 x density ($lbm/ft^3$) |
| $ft^2/sec$ | centipoise | 1488.16 x density ($lbm/ft^3$) |
| centistokes | Pascal-sec | 0.001 x density ($g/cm^3$) |
| $m^2/sec$ | Pascal-sec | 1000 x density ($g/cm^3$) |

## Equations of Flow

Now that we've discussed various formulas used to calculate the Reynolds Number and some of the units used for viscosity, we can apply that knowledge to solving a simple fluid flow problem. The friction factor will be determined using some of the forms of the Reynolds Number already discussed. This information can then be used to calculate the pressure loss in a piping system due to friction.

Starting with the Bernoulli equation, which is based on the principle of conservation of energy, you can see that the equation has terms for pressure energy ($p/\rho$), kinetic energy ($v^2/2g_c$), potential energy ($gz/g_c$), as well as terms for hydraulic machines ($h_A$ = added energy, i.e. a pump; $h_E$ = extracted energy; i.e. a turbine) and friction loss ($h_f$). All the terms in this equation are in units of ft-lbf/lbm or "specific energy".

$$\frac{p_1}{\rho} + \frac{v_1^2}{2g_c} + \frac{gz_1}{g_c} + h_A = \frac{p_2}{\rho} + \frac{v_2^2}{2g_c} + \frac{gz_2}{g_c} + h_E + h_f \qquad \text{(Eq. 12)}$$

Many texts are not clear on how to convert from units of specific energy (ft-lbf/lbm) to units of feet (ft). Some texts even mix terms, compounding the confusion. Lindeburg, 12th Edition states that "Foot and ft-lbf/lbm may be thought of as one and the same" while acknowledging that "lbf" and "lbm" do not "really" cancel out to give feet. Further, it is stated that sometimes the term $g/g_c$ is omitted, giving dimensionally inconsistent equations. A caveat is offered stating that the resulting equation will also be "numerically incorrect" in a "non-standard gravitational field".

So the question is how to legitimately convert from specific energy in units of ft-lbf/lbm to head in units of feet (ft). The solution is quite simple; multiply by $g_c/g$. Performing this operation will reconcile the units, resulting in feet (ft) of head and compensate for a non-standard gravity field if that situation is encountered.
When the terms of Equation 12 are multiplied by $g_c/g$, the result is…..

$$\frac{p_1 g_c}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_A = \frac{p_2 g_c}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_E + h_f \qquad \text{(Eq. 13)}$$

All units are in feet (ft).

The Darcy formula can be used to calculate head loss (in units of ft) resulting from friction:

$$h_f = \frac{fLv^2}{2Dg} \qquad \text{(Eq. 14)}$$

The Darcy formula is part of the Bernoulli Equation that accounts for the energy loss due to friction. In its "energy" form it is written as:

$$h_f = \frac{fLv^2}{2Dg_c} \qquad \text{(Eq. 15)}$$

Notice the only difference is the "g" has now been replaced with "$g_c$". As noted earlier, by multiplying Equation 15 (in units of ft-lbf/lbm) by $g_c/g$, the result is Equation 14 in units of feet (ft). This general rule should also be applied to the terms for hydraulic machines, $h_A$ and $h_E$.

**The Friction Factor "f"**

The friction factor "f" can be found from the Moody diagram (Figure 3) or by using a curve fit equation such as the Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon}{3.7065D} + \frac{2.5226}{R_e\sqrt{f}}\right) \qquad \text{(Eq. 16)}$$

Equation 16 is valid for completely full conduits and Reynolds Numbers greater than 4000. Since the friction factor "f" appears on both sides of the equation, iterative techniques are required to solve the equation. A simple macro can be embedded into an EXCEL spreadsheet to solve for "f" using this equation. See Appendix A.

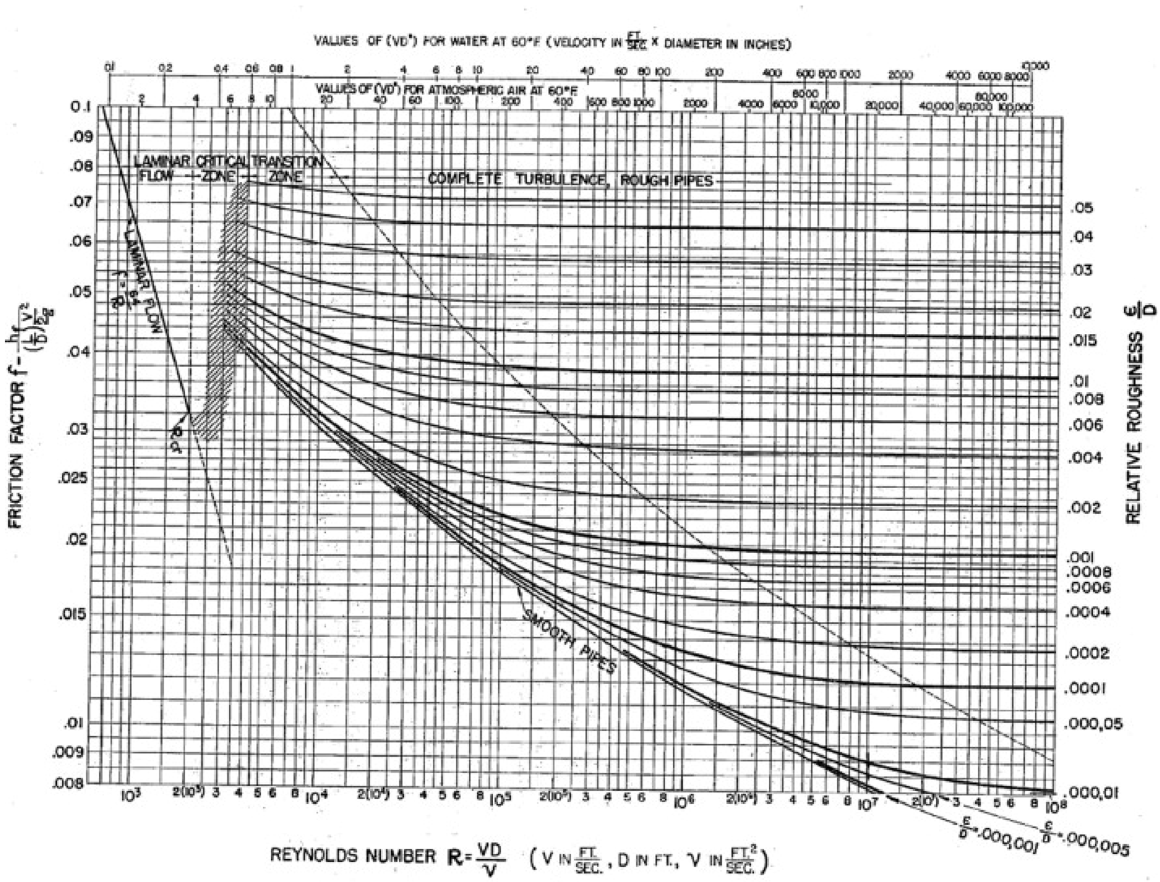


Fig. 1

Figure 3

**Absolute Roughness "ε"**

Another factor used to determine the friction factor is the Absolute Roughness "ε". This factor represents the wall roughness of the interior of a pipe and is in units of feet (ft). The Reynolds Number and the ratio ε/D (known as the Relative Roughness; this ratio is dimensionless) are plotted on the Moody Diagram to determine the friction factor "f" (For Reynolds Numbers less than 2000, i.e. laminar flow, f = 64/Re). ε/D is also used in the Colebrook Equation and other curve fit equations for the Moody Diagram. The Absolute Roughness "ε" for a variety of pipe materials can easily be found on the web. As an example, new cast iron pipe has an Absolute Roughness of 0.00085 ft.

**Minor Losses**
Minor losses due to valves, fittings, etc., must also be considered.
The minor losses can be determined using two methods - The Method of Loss Coefficients and The Method of Equivalent Lengths.

Using the **Method of Loss Coefficients**, a loss coefficient "K" is substituted into the Darcy formula (Equation 14) for fL/D resulting in the formula

$$\mathbf{h_f = \frac{Kv^2}{2g}} \qquad \text{(Eq. 17)}$$

For example, a regular threaded 45 deg elbow will have a K value of 0.35 (this coefficient is dimensionless); a long radius flanged 45 deg elbow has a K value of 0.17.

If there are several fittings in a system, the "K" values are simply added up resulting in a total "K" for the system.

Likewise if there are several different pipe sizes in a system being analyzed, the sum of each of the individual sections (fL/D) is accounted for in a modified Bernoulli equation as follows:

$$\mathbf{\frac{p_1 g_c}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_A = \frac{p_2 g_c}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_E + \sum \frac{fLv^2}{2Dg} + \sum \frac{Kv^2}{2g}} \qquad \text{(Eq. 18)}$$

The **Method of Equivalent Lengths** applies a value of an equivalent length of straight pipe ($L_e$) to each valve or fitting in a system. These values of equivalent length are available from a variety of sources; the Hydraulic Institute is one such source. For example, a 2" long radius 90 deg elbow has an equivalent length of 3.6 ft (Schedule 40 pipe). The individual values of each equivalent length are then added up and the sum is substituted into the Darcy equation (Equation 14) resulting in

$$\mathbf{h_f = \frac{f L_e v^2}{2Dg}} \qquad \text{(Eq. 19)}$$

This value for head loss in feet can then be used in the Bernoulli equation (Equation 13). Of course, $L_e$ should include the length of straight pipe in addition to the values for the fittings. If there are different pipe sizes in the system, an $h_f$ term should be developed for each section. Sometimes the Equivalent Length is given in terms of the ratio $L_e$ /D and is defined as the equivalent length in "pipe diameters". In this form, a single value will cover all sizes of particular type of fitting.

Please note that neither method can be considered an exact representation of the pressure loss through a valve or fitting. Both methods have advocates among practicing engineers and the merits of each can be the subject of spirited debates. See Appendix B for details on more refined methods related to calculating minor losses. The student is encouraged to engage in further study regarding these methods.

**Formulas for Pressure Loss**

Crane provides numerous formulas for pressure loss through pipe and fittings in units of feet (ft) of head and psi. These many formulas utilize a large assortment of different variables to find the pressure loss; we'll focus on just a few.

All the formulas for pressure loss start with one basic formula, the Darcy formula, Equation 14,

$$\mathbf{h_f = \frac{fLv^2}{2Dg}}$$

By considering the conversion from inches to feet, this formula can be expressed as....

$$\mathbf{h_f = \frac{0.1863fLv^2}{d}} \quad \text{(units are ft)} \qquad \text{(Eq. 20)}$$

Converting Equation 20 into "psi" requires conversion of "$ft^2$" to "$in^2$" and bringing the density term "ρ" to the other side of the equation, resulting in.....

$$\mathbf{\Delta p = \frac{0.001294fL\rho v^2}{d}} \qquad \text{(Eq. 21)}$$

A careful examination of this equation shows that it results in units of $lbm/in^2$.

To produce units of $lbf/in^2$, the equation has to be multiplied by $g/g_c$. Justification for this step is provided below.

Recall that the pressure head (in units of ft - let's call it $h_p$) is the $pg_c/\rho g$ term in the Bernoulli equation (Equation 13).

Let $h_p = pg_c/\rho g$ where the subscript "p" represents pressure, then solving for p results in…

$p = h_p\rho g/g_c$, therefore, reconciliation of the units requires multiplication by the ratio of $g/g_c$ (not $g_c/g$ as was done in the conversion of Equation 12 to Equation 13).

Another formula used to calculate pressure loss in "psi" incorporates the loss coefficient "K".

Starting with the equation…

$$\Delta p = \frac{fL\rho v^2}{144D2g} \qquad \text{(Eq. 22)}$$

(This is another variation of Equation 14, expressed in units of "psi".)

Substituting "K" for fL/D (D is in units of feet), into Equation 22 results in the following…

$$\Delta p = 0.0001078K\rho v^2 \qquad \text{(Eq. 23)}$$

The equation also requires multiplication by $g/g_c$ to reconcile the units.

Equations 21 and 23 will be used in the spreadsheet accompanying this course.

**EXCEL Spreadsheet "PSI-LOSS"**

A simple tabulated EXCEL spreadsheet can be useful for doing an analysis of a fluid flow problem. Unlike an internet web page which typically only gives you a single solution to an equation and no way to save results, a spreadsheet is useful when you want to compare a variety of scenarios.

The EXCEL spreadsheet "PSI-LOSS" uses both the "Method of Loss Coefficients" and the "Method of Equivalent Lengths" to account for minor losses, Equations 2 and 3 to solve for Reynolds Number, and Equations 21 and 23 to solve for pressure loss in "psi". A macro is embedded in the spreadsheet to calculate the friction factor "f" using the Colebrook equation.

Data is entered into the following columns in spreadsheet PSI-LOSS:

Column A: Type of fluid, info only
Column B: Temperature, info only
Column C: Misc data, info only
Column D: K for fittings using the Method of Loss Coefficients; Equivalent length for fittings using the Method of Equivalent Lengths
Column E: Pipe ID in inches
Column F: Length of pipe in feet
Column G: Kinematic viscosity in centistokes (cSt) using the Method of Loss Coefficients; in $ft^2/sec$ using the Method of Equivalent Lengths
Column H: Absolute roughness in feet
Column I: Density in $lbm/ft^3$
Column J: Flow rate in gallons per minute, GPM
Column Q: Notes, info only

The following columns have calculated values:

Column K: Velocity in ft/sec using the formula $v = 0.4085GPM/d^2$ (The course opened with a question regarding the formula for velocity, $v = 0.4085GPM/d^2$. This equation is derived from the continuity equation. Knowing $Q = Av$, you can solve for v, convert area in square inches to square feet and convert GPM to $ft^3/sec$, resulting in the equation above.
Column L: Reynolds Number using Equation 2 for the Method of Loss Coefficients; for the Method of Equivalent Lengths, Reynolds Number $= (1/12)vd/\nu$; $\nu$ is in units of $ft^2/sec$ - this is an intermediate form of Equation 3.
Column M: Friction factor "f" using embedded macro for the Colebrook equation
Column N: Total K ($fL/d + K$) for the Method of Loss Coefficients; Total equivalent length (Minor losses + pipe length) for the Method of Equivalent Lengths

Column O: Pressure loss in psi using Equation 23 for the Method of Loss Coefficients and Equation 21 for the Method of Equivalent Lengths

Column P: Reynolds Number using Equation 3

As a test case, let's use the same example used in the beginning of the text with some additional data needed solve for pressure loss.

Benzene flowing through 1000 feet of a 12 inch Schedule 80 Cast Iron pipe:

Let $\rho$ = 54.7 lbm/ft$^3$
d = 11.3736 inches
$\nu$ = 0.685855 cSt = 7.3825 x 10$^{-6}$ ft$^2$/sec
K for the fittings = 0.87 (two 45 deg regular elbows and one 45 deg long radius elbow)
$L_e$ for fittings = 48 ft (As noted earlier, there can be significant differences in results between the two methods)
Absolute Roughness "ε" = 0.00015 ft for wrought iron pipe
Flow rate = 3816 GPM

This input data results in a flow velocity of 12.05 ft/sec, a Reynolds Number of 1.548 x 10$^6$, a friction factor of 0.0139, a total K of 15.49 and a pressure loss of 13.26 psi (for the Method of Loss Coefficients)

Using the Method of Equivalent Lengths, the pressure loss equals 13.15 psi.

As another example, 180 deg F water is flowing through a flat heating coil made of 1" schedule 40 pipe at a rate of 15 GPM (this is example 4-11 in Crane). There are seven (7) 180 deg bends and two (2) 90 deg bends for a total K of the fittings equal to 4.51.
Other data entered:
The length of straight pipe = 18 ft
Pipe ID = 1.049 in
Kinematic viscosity = 0.34 cSt
Absolute roughness = 0.00015 ft
Density = 60.57 lbm/ft$^3$

Calculated results:
Reynolds Number = 1.33 x $10^5$
Friction factor = 0.0239
Total K = 9.44
Pressure loss = 1.91 psi

Using an approximation for the equivalent lengths of the fittings from the equation $K = f_T L_e/D$ (Note that f has been replaced with $f_T$) , $L_e$ = 17.1 (Remember, data regarding the equivalent length of fittings varies widely. Therefore, considerable differences in the resulting pressure loss can result.), and the resulting pressure loss is 1.95 psi. The factor "$f_T$" is defined as friction factor for flow in the zone of complete turbulence (on the Moody diagram). It is different than the friction factor used to calculate the pressure loss in a straight run of pipe (f) and as discussed in Appendix B is used to determine the resistance coefficient "K" for various valves and fittings (the "Crane K" method). In this example, other values of equivalent length for 90 degree and 180 degree bends may result in a much higher pressure loss.

Refer to line 6 of the EXCEL spreadsheet "PSI-LOSS".
Hydraulic oil at 120 deg F is flowing through 2-1/2" schedule 80 pipe (ID=2.32") at 200 GPM; the Reynolds Number is 6821. Using the "What-If Analysis" feature of "Goal Seek" in EXCEL we can determine the required flow rate for a Reynolds Number of 4000 (this is where flow is generally considered to be fully turbulent). Setting $R_e$ (cell L6) equal to 4000 by changing cell J6, the flow rate becomes 117.28 GPM. This type of analysis can be used with any of the variables in the spreadsheet.

**"g sub c"**

The factor $g_c$ is used extensively in the English system as a conversion factor to make equations dimensionally correct. To understand the origin of $g_c$, simply refer back to Newton's second law, F = ma. If this relationship is expressed in primary dimensions, it becomes……

$$\mathbf{F = M\frac{L}{T^2}} \qquad \text{(Eq. 24)}$$

In English units this becomes…

$$lbf = lbm \cdot (ft/sec^2)$$

 

Introducing the constant “32.2”, we will define 1lbf as the force required to accelerate 1 lbm at an acceleration of 32.2 ft/sec$^2$, or……

$$\mathbf{1\ lbf = 32.2\ lbm\left(ft/sec^2\right)} \qquad \text{(Eq. 25)}$$

The constant 32.2 is also the acceleration due to gravity, g = 32.2 ft/sec$^2$. This conveniently allows “lbm” to equal “lbf” (in Earth’s gravity) as follows…

If W = mg (Weight = mass times gravity), then for a mass of 100 lbm,

W = (100 lbm)*(32.2 ft/sec$^2$) = 3220 lbm-ft/sec$^2$

But this has to be converted to units of lbf…..

Rearranging Equation 25, the result is……

$$\mathbf{\frac{1\ lbf}{32.2\ lbm(ft/sec^2)} = 1} \qquad \text{(Eq. 26)}$$

Let’s call this relationship “k”.

The reciprocal of “k” is known as “$g_c$”….

$$\mathbf{\frac{1}{k} = g_c = \frac{32.2\ lbm\left(ft/sec^2\right)}{lbf}}$$, or in its more familiar form…..

$$\mathbf{g_c = 32.2\,\frac{lbm{\cdot}ft}{lbf{\cdot}sec^2}} \qquad \text{(Eq. 27)}$$

So in essence, when you multiply or divide by $g_c$, you are multiplying or dividing by unity or “1”; when performing this operation you change the form, not the value and reconcile the units.

Going back to the example W = (100 lbm)*(32.2 ft/sec$^2$), if we divide by $g_c$ the equation becomes…..

W = mg/$g_c$ or………

W = (100 lbm)*(32.2 ft/sec$^2$)/(32.2 lbm-ft/lbf-sec$^2$) = 100 lbf

Again, $g_c$ is the factor used to reconcile the units when calculating the weight in this example; it is also used in numerous other instances to reconcile units as noted earlier in this course.

## Conclusion

This course started with a review of the various forms of the Reynolds Number, but it quickly became apparent that one needs to focus on the units of the various factors used to calculate this dimensionless number, especially viscosity. The details surrounding the derivations of the various forms of Reynolds Number are not as important as correctly applying the equations, however understanding some details of how the formulas are derived will aid in keeping track of units within the equations.

Equations of flow were shown in units of feet and were also expressed in "specific energy" units of ft-lbf/lbm. Remember that not all reference materials correctly account for these units and occasionally interchange them. While this may not necessarily lead to incorrect numerical values, the equation will not be dimensionally correct.

"g sub c" ($g_c$) is the factor that, in the English system, ties equations together, providing for correct dimensions. Knowing how to derive $g_c$ is as easy remembering Newton's second law, F = ma.

It cannot be stressed enough that units require careful accounting. A review of units will typically be the first indication that the solution to an equation may have an error. If there is any doubt that units are important, read the details surrounding the loss of the Mars Climate Orbiter (MCO). This $125 million dollar spacecraft was lost due to an English to Metric conversion oversight. There are numerous other examples available if a web search of "Unit Mishaps" is done. These range from "expensive rice" to "airline disasters". Remember the lesson offered by your 12th grade physics teacher - "don't forget about the units". This tidbit of information will serve you well.

# Appendix A

## The Colebrook Equation

The Colebrook equation can be used to determine the friction factor and is a curve fit equation for the Moody diagram.

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon}{3.7065D} + \frac{2.5226}{R_e\sqrt{f}}\right)$$

Since the term "f" appears on both sides of the equation, iterative techniques are required to solve for "f". A simple macro embedded into an EXCEL spreadsheet performs this operation; for this case the function is named "Friction".

Note that for Reynolds Numbers less than 2000, the friction factor is simply equal to 64/(Reynolds Number).

Referring back to the cells in spreadsheet PSI-LOSS
H=Absolute Roughness, ε
E=Pipe ID in inches
L=Reynolds Number

The algorithm is provided below:

```
Function Friction(H2, E2, L2)
If L2 < 2000 Then
Friction = 64 / L2
Else
    FT = 0.001
    LS = (12 * H2) / (3.7065 * E2)
    RS = 2.5226 / (L2 * (FT ^ 0.5))
    FF = ((-2) * (0.43429) * Log(LS + RS)) ^ (-2)
    Do While Abs(FF - FT) > 0.00001
    FT = FF
    RS = 2.5226 / (L2 * (FT ^ 0.5))
    FF = ((-2) * (0.43429) * Log(LS + RS)) ^ (-2)
    Loop
    Friction = FF
End If
End Function
```

# Appendix B

## Methods for Determining Minor Losses through Valves and Fittings

The accompanying course covered two of the methods available to determine minor losses due to valves or fittings in a piping system - The Method of Loss Coefficients and The Method of Equivalent Lengths. As will be discussed in this appendix, there are other methods which can be considered more refined ways to calculate minor losses due to valves and fittings and it is important that the student be aware of these methods.

The Method of Loss Coefficients has a few variations as discussed below. "K" values for a wide variety of valves and fittings can be found in textbooks and on the web. Using these "K" values directly in Equation 17 may not produce the precision of other versions.

Another variation is discussed in Chapter 2 of Crane Technical Paper No. 410 (Flow of Fluids) and goes into great detail regarding the formulation of the factor "$f_T$" which is defined as the friction factor for flow in the zone of complete turbulence and is used to determine "K" factors in the Method of Loss Coefficients (called the "Crane K" method). The factor "$f_T$" takes into account the phenomena that there is usually a significantly higher amount of turbulence in a fitting than in the pipe (connected to that fitting) at a specific Reynolds Number.

Crane explains that pressure losses result primarily from four characteristics:
1. Loss due to pipe friction in a valve or fitting, in the inlet and outlet straight area of the component.
2. Changes in the direction of the flow path.
3. Obstructions in the flow path.
4. Sudden or gradual changes in the cross section and shape of the flow path.

The losses attributed to item 1 above can be considered small compared to the other three items, therefore K, as originally developed in Crane, was considered to be independent of both the friction factor and the Reynolds Number and can be treated as a constant for all flow conditions and will vary only with the size of the valve or fitting (to paraphrase Crane).

It is important to remember that this more refined application of the Method of Loss Coefficients involves solving for the pressure loss in the valves or fittings (using the "$f_T$" factor) separately from pressure loss in the pipe (using the "f" factor) and adding the two pressure losses together. "f" and "$f_T$" are not the same; as noted earlier, "$f_T$" is the friction factor for flow in the zone of complete turbulence and "f" is the friction factor in the pipe – and flow may be laminar at low Reynolds Numbers while the flow in the valve or fitting may have some turbulence.

Two other methods that further refine the derivation of the "K" factor are the "2K Method" developed by William Hooper in 1981 and the "3K Method" developed by Ron Darby in 1999. The development of the two methods acknowledges the fact that the pressure loss through a valve or fitting has some dependence on the Reynolds Number.
The "2K Method" uses two separate K's, as the name implies, to solve for an overall "K"; $K_1$ (the resistance coefficient at $R_e = 1$) and $K_\infty$ (the resistance coefficient for large fittings, $R_e = \infty$).

$$\mathbf{K = \frac{K_1}{R_e} + K_\infty\left(1 + \frac{1}{d}\right)}$$ d is in inches

The equation therefore takes into account both the effects of the Reynolds Number and the size of the fittings, and is valid for a wide range of Reynolds Numbers.

Although the "2K Method" was an improvement because it was valid over a wider range of Reynolds Numbers, it was shown that it was not as accurate for larger sizes of fittings and valves. Because of this shortfall, the "3K Method" was developed and is expressed by the following formula…

$$\mathbf{K = \frac{K_1}{R_e} + K_i\left(1 + \frac{k_d}{d^{0.3}}\right)}$$

Tabulated values for the coefficients $K_1$, $K_i$, and $K_d$ can be found in textbooks such as "Chemical Engineering Fluid Mechanics" by Darby. The equations listed above are also from this source.

The Method of Equivalent Lengths as described in this course is a less precise method for determining the pressure loss through a valve or fitting since it uses the same friction factor for the valves and fittings as is used for the straight pipe (i.e. "$L_e$" is multiplied by the friction factor of the pipe "f"). Because of this, some error is introduced.

A more precise result can be obtained by multiplying "$L_e$" by the friction factor in the zone of complete turbulence "$f_T$" and solving separately for the pressure loss through the fittings and then solving for the pressure loss through the pipe (using "f"), then adding up the results of each. The equivalent length "$L_e$" can even be converted to "K" (also using the "$f_T$" factor) and the pressure loss can be solved in this manner. Again, the important point to remember is to also use "$f_T$" for this method, as was done to determine "K", to generate a more precise result.

The Valve Flow Coefficient "$C_v$" is another method to determine minor losses and is generally used for control valves. "$C_v$" by definition is the quantity of US gallons per minute of water (at 60 deg F) that will pass through a flow restriction resulting in a pressure drop of 1 psi.

Two equations that apply are…..

$$\mathbf{Q = C_v\sqrt{\frac{\Delta p}{SG}}}$$

Where $C_v$ is the Valve Flow Coefficient, Δp is in psi, and SG is the specific gravity. Q is in gallons per minute.

$$\mathbf{C_v = \frac{29.9d^2}{\sqrt{K}}}$$

Where d is in inches and K is the Loss Coefficient discussed above.

**Conclusion**

This appendix by no means claims to be a comprehensive review of the various methods used to calculate minor losses in valves and fittings; it is just an overview and is intended as an introduction only. There is an abundance of information available in textbooks and on the web covering these concepts in greater detail. The student is encouraged to pursue further study in these areas.