PDHonline Course S261 (4 PDH)

Vehicle Barrier Analysis

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Vehicle Barrier Analysis

1. INTRODUCTION
Vehicle barriers fall into one of three general categories, depending upon their primary function. They are:

1.1 Non-Impact Barriers
The function of these barriers is to delineate safe traffic lanes and are not designed to stop vehicles. Their primary function is protection of the vehicle occupants from either a dangerous situation, such as an approaching train, prevent access to private property, or, temporarily redirect traffic in a safe manner. This last process is called ‘channelization’. The devices in this category include tubular markers, cones, signage barricades, active traffic control devices for grade crossings, as well as valleys or berms separating high speed highway lanes.

1.2 Low Impact Barriers
The function of first type of these barriers is also related to vehicle occupant safety while driving on a roadway and are designed to deflect, rather than completely stop, vehicle travel on a roadway. Jersey barriers and highway guard rails meet this need. A second type of low impact barrier is that used to protect vehicles and/or property during a parking maneuver, such as bollards or barrier cable systems in parking garages. These are, in general, designed to the prevailing building code, typically to resist a static force of 6000 pounds applied either 18 or 27 inches above the driving surface.

1.3 High Impact Barriers
The function of these types is to stop a vehicle traveling at speed into a restricted or dangerous area. Full vehicle tests at speed or analyses directly following from these tests are using in the design and construction of these barriers. Classifications within each vehicle category include approach speeds, and vehicle weights. Another classification relates to penetration distance, the distance between initial contact and final stop. Kinetic energies in high impact barriers exceed 1000000 foot-pounds for some classes.
A further distinction is made between active (can be lowered and raised manually or automatically) versus passive (permanently in place) barriers.

2. NONIMPACT BARRIERS [1]

2.1 Channelizing Devices
Reference [1] gives the following diagrams of channelizing devices on page 605:

Traffic cones were invented in 1914 and originally made of concrete. Modern cones (pylons) are either thermoplastic or rubber.
The cones are typically fluorescent “safety” orange, range in height from 12 inches to 36 inches, and weigh no more than 10 pounds. Some cones are constructed with bases that can be filled with ballast to withstand wind. Sandbag rings that can be dropped over cones are sometimes used for added stability.

2.2 Grade Crossing Traffic Control Device [1]

The drawing below is from page 770 of reference [1].

The warning system must provide not less than 20 second warning for normal operation of through trains.
The gate arms must be in the horizontal position at least 5 seconds before arrival of the train at the crossing.
The detection of the train is accomplished by circuits sensing a shunting resistance of 0.06 ohm or less across the track rails. Maintenance is required to prevent sand, rust, dirt, grease, or other foreign matter from preventing proper operation of the shunting surface because of their higher resistance. When grade crossings are located near a signaled intersection, the adjacent signal must be automatically overridden by the approaching train.

3. LOW IMPACT BARRIERS

3.1 Jersey barriers [2,3]

These are modular concrete barriers primarily used to separate lanes of traffic, and/or prevent vehicles from traveling into a dangerous situation, such as a roadside cut.
They derive their name from their initial development at the Stevens Institute of Technology in Hoboken, New Jersey, under the direction of the New Jersey State Highway Department. Their first major use is to prevent head on collisions between vehicles traveling in opposite directions and separated by a narrow median.
A second major use is called a shoulder barrier. These barriers are modular, and may or may not be anchored, and may or may not be coupled to an adjacent barrier by a pin and loop connection.
A typical median barrier shown on Oregon Standard Drawing RD500 [2] is reproduced on the next page.
As shown above in the end views, the cross-section is designed to deflect shallow angle hits, by allowing the vehicle tires to ride up on the lower face slope, and then, hopefully, to be deflected back into the same direction with minimum damage.

The barrier shown weighs approximately 6180 pounds.
Note that the scupper cutouts can also be used for fork clearance in lifting.

EXAMPLE 1.
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Consider the use of a typical Jersey barrier, such as the one shown above, without either anchor rods or attachment to other barriers, to meet the requirements of the 2009 IBC (International Building Code),[3]. The vehicle barrier system section(1607.7.3) states that the barrier must withstand a force of 6000 pounds applied either 1′-6″ or 2′-3″ above the supporting surface, whichever causes the most stress.

SOLUTION
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The barrier must be checked for factors of safety against sliding and overturning.

Factor of safety against sliding:
Restraining force = coefficient of friction times weight
Restraining force = 0.4 x 6180 = 2472 pounds
Factor of safety = restraining force/applied force
Factor of safety = 2472/6000 = .412 < 1.75, n.g.

Factor of safety against overturning:
Overturning moment = force x 27 ″ = 162000 lbf-in.
Righting moment = weight x 12″ = 74160 lbf-in.
Factor of safety = 74160/162000 = .458 < 2.0, n.g.

Thus it is seen that if a single typical Jersey barrier is used as an IBC vehicle barrier system, it must be anchored to the supporting surface.

3.2 Cable Barriers [4 5]

Cable barriers are designed for two general applications, namely as median barriers and vehicle restraint at open edges of ramps in parking garages. Only the second use is discussed here, generally designed to IBC 2009 Sections 1013 (guards), 1607.7.3 (equivalent static loading), and 2207 (steel cable structures [3].
The barriers are of two types, low tension and high tension. Both work to restrain the vehicle by
absorbing its kinetic energy and dissipating it in elastic lengthening of the cables. The vehicle moves a short distance to absorb this energy. This method reduces the forces on the vehicle and its occupants, but the distance must be accounted for in the design. The restraint forces on the vehicle and its occupants are greatly increased by a stiff barrier, such as a wall. This short distance is limited by the distance from the undeflected cable line to ramp edge, wall, or other properly parked vehicle. Low tension cables typically use steel cables used in non-prestressed work. Material types are either ASTM A586 (structural steel strand) or ASTM A603 (structural wire rope). High tension cables use ASTM A416, “Seven-Wire Strand for Prestressed Concrete”, with tensile strength to 270 ksi. Single cables are mounted on vertical centers. The first section below considers sag, which applies to both low and high tension cables. A general analysis is then given to find the force and deflection for any cable, using the IBC criteria (static loading) and also vehicle kinetic energy. The section concludes with a discussion of temperature effects conclude the section.

3.2.1 Sag

Sag in a cable is caused by the low flexural resistance of steel wire. It is best shown by an example.

EXAMPLE 2.
----------
Consider a steel cable, \( \frac{1}{2} '' \) dia., 27’ long, \( E=28500 \) ksi. Find vertical deflection at center caused by sag.

SOLUTION
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The equation for deflection of a beam at midspan is:
\[ \Delta = \frac{5}{384} \times \frac{wL^4}{E*I} \text{ inches} \]
where \( w = \) distributed load, kip/in.
\( L = \) length, inches
\( E = \) Young’s Modulus, ksi
\( I = \) moment of inertia, in.\(^4\)
\( w = \) cable density x cross-sectional area
\[ w = .283 \ lbf/in.^3 \times 1 \ kip/1000 \ lbf \times \pi \times (1/2)^2/4 \]
\[ w = 5.557 \times 10^{(-5)} \ kip/in. \]
\[ I = \Pi d^4/64 = 3.068 \times 10^{-3} \text{ in.}^4 \]

Solving, \[ \Delta = 5 \times 5.557 \times 1.1100 \times 10^5 / \]
\[ 3.84 \times 2.85 \times 3.068 \times 10^3 \]
\[ \Delta = 22.80 \text{ in.}, \text{unsuitable} \]

The deflection problem is solved by applying an equal and opposite force (prestress) outward on each end of the cable. Deflection due to flexural action is usually neglected. The resulting form is called a catenary, whose shape is similar, but not equal to, that of a parabola. The defining equation of a catenary is:
\[ \Delta = a(\cosh(x/a)-1) \]

where:
- \[ a = P0/w \text{ in inches} \]
- \[ x = \text{distance from center of cable, ½ span in inches} \]
- \[ P0 = \text{cable tension (prestress) in kip} \]
- \[ w = \text{cable weight in kip/in.} \]

If the span, \( P0 \), and \( w \) are known, the deflection can be found directly.

**EXAMPLE 3.**

Find the sag of the cable in Example 2 for a prestress of 5 kip.

**SOLUTION**

\[ a = 5/1.389 \times 10^{+5} = 359926 \text{ in.} \]
\[ x/a = 162/359926 = 4.501 \times 10^{-4} \]
\( (x/a \text{ is dimensionless}) \)
\[ \cosh(x/a) = (e^{x/a}+e^{-x/a})/2 \]
\[ \cosh(x/a) = 1.000000101 \]
\[ \Delta = 359926 \times (1.000000101-1) = .036 \text{ in.} \]

It is seen that this problem formulation is “ill-conditioned”, that is it requires the subtraction of two nearly equal numbers, differing by a very small amount.

This problem may be solved by expressing the hyperbolic cosine as a Taylor series, and using only the first three terms.

\[ \cosh(u) = 1 + x^2/2! + x^4/4! + x^6/6! + \ldots \]

Taking the first three terms, \( \Delta \) may be written as:
\[ \Delta = a*(\cosh(x/a)-1) = a*(1+(x/a)^2/2+(x/a)^4/24 -1) \]
\[ \Delta = a*(12*(x/a)^2 + (x/a)^4)/24 \]
For this example,

\[ \Delta = \frac{359926}{24} \times (2.4311 \times 10^{-6} + 4.1043 \times 10^{-14}) \]
\[ \Delta = .036 \text{ in.} \]

It is seen, that for usual conditions of prestress, span, and self-weight that only the first two terms of the Taylor series are required.

Since \( a = P_0/w \) and \( x = \frac{1}{2} \text{ span} \),

\[ \Delta = \frac{w \times \text{span}^2}{8 \times P_0} \]

EXAMPLE 4.

Find the prestress required to keep sag at \( \frac{1}{2} \) “ for a cable weighing 12.0 lbf per foot, with a span of 10 feet.

SOLUTION

Consistent units used here are lbf and inches.

The equation for sag above may be rewritten to show prestress as:

\[ P_0 = \frac{w \times \text{span}^2}{8 \times \Delta} \]

\[ P_0 = \frac{(12.0/12) \times (10 \times 12)^2}{8 \times (.5)} = 3600 \text{ lbf} \]
3.2.2 Cable Calculations Given Impacting Force [4]

The following diagram and glossary give the defining equations for cable calculations for prestress, total cable tension and deflection when an impacting force is given, such as in the 2009 IBC.

\[ A = \text{Cross-section area, 1 cable, in.}^2 \]
\[ E = \text{Young's modulus, ksi} \]
\[ f_y = \text{Yield stress, ksi} \]
\[ F_0 = \text{Applied force, 1 cable, kip} \]
\[ F_1 = \text{Local post reaction, 1 cable, kip} \]
\[ L = \text{Original length, in.} \]
\[ N_{UTS} = \text{Min. ultimate tensile strength, kip} \]
\[ N_A = \text{Total no. of cables} \]
\[ N_A = \text{No. active cables} \]
\[ P_0 = \text{Prestress, each cable, kip} \]
\[ T = \text{Total tension, 1 cable, kip} \]
\[ x_i = \left( \frac{L}{2} \text{ post post spacing} \right) - \text{Vehicle width, in.} \]
\[ y_0 = \text{Cable deflection, 1 cable line, in.} \]
\[ \Delta = \text{Cable deflection in length, in.} \]
\[ \theta_0 = \tan^{-1}\left( \frac{y_0}{x_1} \right) \]

1. \[ \Delta = \frac{T - P_0}{AE} L \quad \text{Cable elongation} \]
2. \[ F_0 = 2T \sin \theta_0 \quad \text{Static equilibrium} \]
3. \[ \cos \theta_0 = \frac{x_1}{x_1 + \frac{1}{2} \Delta} \quad \text{Geometry} \]
In problems of this type, it is usually required to find the horizontal deflection, $y_0$, and the fixed terminal reactions.

Given parameters are $A$, $E$, $F_0$, $L$, $N_0$, $N_1$, post-post spacing, yield force for single cable, and $y_0$.

The following procedure may be used:

(a) Find $\theta_0$, given $x_1 = \frac{1}{2}$ post-post spacing, and $y_0$.  
    $\theta_0 = \tan^{-1}(y_0/x_1)$

(b) Use equation (3) above, reworked, to find $\Delta$, the cable increase in length.  
    $\Delta = \text{post-post spacing}(1/\cos(\theta_0) - 1)$

(c) Reconfigure equation (2) above, to find the total tension in one cable.  
    $T = F_0/(2*\sin(\theta_0))$

(d) Substitute results for $\Delta$ and $T$ into (1), again reworked, to find the required prestress force, $P_0$.  
    $P_0 = T - A*E*\Delta/L$, where $L$ is the total length of cable.

(e) Check total force in cable versus allowable force to find factor of safety.  
    $F.S. = \text{Tallowable}/T$

(f) Find end post reaction = $N_1*T + (N_0-N_1)*P_0$

Example 5.  
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Find prestress force, total cable force, and end post reaction, given the following parameters.

$A = 0.140 \text{ in.}^2$
$E = 26000000 \text{ lbf/in.}^2$
$F_0 = 10000 \text{ lbf (evenly distributed over 5 cables)}$
$L = 8 \text{ continuous local spans of 27 ft each}$
$N_0 = 7$, total number of cables
$N_1 = 5$, number of active cables in this case
$Y_0 = 1.5 \text{ ft, specified maximum deflection under load } F_0$
Yield force of single cable = 23350 lbf

Solution  
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Use lbf and inch consistent units.

(a) $\theta_0 = \tan^{-1}(18/(324/2)) = 6.3042^\circ$
(b) $\Delta = 324*(1/\cos(6.3042^\circ) - 1) = 1.9939 \text{ in.}$
(c) $T = (10000/5)/(2*\sin(6.3042^\circ)) = 9055.4 \text{ lbf}$
(d) $P_0 = 9055.4 - A*E*\Delta/L = 6255.4 \text{ lbf}$
(e) $F.S. = 23350/9055.4 = 2.579$
(f) Total force on end posts = $5*9055.4 + 2*6255.4$
    $= 57788 \text{ lbf}$!
3.2.3 Cable Calculations Given Vehicle Velocity

Additional Terms:
- $E_0$ = initial cable strain energy, kip-in.
- $E_1$ = final cable strain energy, kip-in.
- $g$ = gravitational constant
- $g = 386.1$ in./sec$^2$
- $KE$ = initial vehicle kinetic energy, kip-in.
- $KE = m*v^2/2$
- $m$ = vehicle mass, kip-sec$^2$/in. ($m = \text{wgt}/g$)
- $v$ = initial vehicle velocity, in./sec
- wgt = weight of vehicle, kip
- $x_0$ = width of vehicle, in.
- $x_1$ = (post-post spacing $- x_0)/2$

Consistent units here are kip, inches, and seconds.

$E_0 = (P_0^2)*L*N_1/(2*E*A)$, cable strain energy before impact

$E_1 = (T^2)*L*N_1/(2*E*A)$, cable strain energy after impact

The basis of the solution method is the assumption that the vehicle kinetic energy is equal to the change in cable strain energy, $KE = E_1 - E_0$. 
The three basic equations derived from this assumption, cable deflection $\Delta$, and geometry are:

1. $KE = \frac{L*N1*(T^2-P0^2)}{(2*E*A)}$
2. $\Delta = \frac{(T-P0)*L}{(A*E)}$
3. $(x1)^2 + (y0)^2 = (x1+\Delta/2)^2$

This is three equations in four unknowns, namely $T$, $P0$, $\Delta$, and $y0$. One of these factors must be given to obtain a solution. In these problems, $y0$ is usually given, so as to obtain a certain minimum displacement of the cables after impact.

Equation 3 may be reworked to obtain $\Delta$ in terms of $y0$:

4. $\Delta = -2*x1+2*sqrt((x1)^2+(y0)^2)$

Now (2) may be solved to obtain $P0$ and substituted into (1) to obtain $T$. After a fairly amount of algebra:

5. $T = \frac{KE}{(\Delta*n1)} + \frac{A*E*\Delta}{(2*L)}$

Finally, an expression for $P0$ may be gotten from (2)

6. $P0 = T - \frac{A*E*\Delta}{L}$

Example 6.

Assume, as in Example 5., a cable length of 8x27 feet, 5 active cables, 7 total cables, vehicle weight = 4000 lbf, vehicle initial velocity = 10 miles per hour, vehicle width = 9 feet, and maximum cable deflection = 18 inches. $E = 26000$ ksi and $A = .140$ in.$^2$

Solution

(a) $x2 = 12*(27-9)/2 = 108”$
(b) From (4), $\Delta = -216+2*sqrt(108^2+18^2) =2.9795$ in.
(c) mass = 4.000/386.1, where 386.1 is gravitational acceleration in in./sec$^2$, and weight in kips.
   mass = 0.01036 kip-sec$^2$/in.
(d) $v = 10$ miles/hour x 5280 feet/mile x 1 hr/3600sec
   X 12 inches/1 foot = 176 inches/second
   Note this conversion technique is called “unit fractions” where each of the factors represents a
physical quantity divided by the same physical quantity, expressed differently.

(e) KE = (.01036)*176^2/2 = 160.456 kip-in.

(f) Substitute values for KE, velocity, N1, L, and Δ into (5) to obtain total single active cable tensile force.

\[ T = 10.770 + 2.0921 = 12.863 \text{ kip} \]

(g) Use (6) to find \( P0 = 8.6788 \text{ kip} \)

(h) Total force on end posts = 5*T + 2*P0 = 81.672 kip!

3.2.4 Temperature Effects on Cables

Temperature strain is given as \( \delta L / L = \varepsilon = \alpha (\delta T) \) where:

- \( L \) = original cable length, in.
- \( \alpha \) = temperature coefficient of expansion, in./in./°F
- \( \delta T \) = change in temperature, °F
- \( \varepsilon \) = strain caused by temperature if cable is not restrained in length

Now if the cable is restrained from moving, a stress \( \sigma \) is induced, \( \sigma = E*\varepsilon \), tensile for a temperature drop, and compressive for a temperature rise.

Example 7.
----------

Find the change in tension for a .140 in.^2 cable, \( E = 26000 \text{ ksi} \), installed at 80°F, when the temperature drops to -20°F. Let \( \alpha = 0.0000065 \text{ in./in./}°\text{F} \)

Solution
--------

\[ \varepsilon = 0.0000065*100 = 0.00065 \text{ in./in.} \]
\[ \sigma = 26000(0.00065) = 16.90 \text{ ksi} \]

increase in cable tension = \( \sigma*A = 2.366 \text{ kip} \)

Note that this increase is not a function of length.

A commercially available cable system by Dywidag [5] is shown on the next page.
3.3 Bollards [6,7,8]

A bollard is a large post, typically a round hollow structural section (HSS), embedded in a concrete foundation. It is usually filled with concrete which both increases its strength and prevents interior corrosion, and is anchored to a foundation. Two purposes are served by a bollard, namely to define the end of an open driving space, as in store fronts, and to provide a resistance to an impacting vehicle traveling towards a protected area. It may be used singly, or in groups, where they are mounted close enough to block ordinary cars or trucks, but let pedestrians and special smaller vehicles through.

The analysis here discusses a single bollard with a circular concrete foundation (pier), and is divided into the following sections:

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3.1</td>
<td>Soil Assumptions</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Force impact, no restraint at base</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Kinetic energy impact, no restraint at base</td>
</tr>
<tr>
<td>3.3.4</td>
<td>Force and kinetic energy impact, restraint at base</td>
</tr>
<tr>
<td>3.3.5</td>
<td>Round HSS and pier design</td>
</tr>
</tbody>
</table>

3.3.1 Soil Assumptions [6]

(1) The spring rate of the soil = n*y*x(y), where n = estimated coefficient of lateral soil reaction, units of psi/in./in. = lbf/in.^4

Reference [6] gives the following table:

<table>
<thead>
<tr>
<th>Class of Materials</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy gravel and/or gravel (GW,GP)</td>
<td>1.92901</td>
</tr>
<tr>
<td>Sand, silty sand, clayey gravel (SW,SP,SM,SC,GM,GC)</td>
<td>0.48225</td>
</tr>
<tr>
<td>Clay, sandy clay, silty clay, clayey silt (CL,ML,MH,CH)</td>
<td>0.24113</td>
</tr>
<tr>
<td></td>
<td>0.04823</td>
</tr>
</tbody>
</table>
The alphabetic characters above refer to classifications of soils given by the “Unified Soil Classification System”, ASTM D2487.

(2) Rigid post and foundation

(3) Constant circular cross-section pier footing

The diagram below describes the geometry of the two general solutions of earth pressure, namely no restraint at base (Sections 3.3.2 and 3.3.3) and restraint at base, typically by a slab on grade (Section 3.3.4)
3.3.2 Force Impact, No Restraint at Grade

With no restraint at grade, there is a shifting of the post at grade, illustrated by the parameter \( x_0 \).

The pressure at depth \( y \) = spring rate*\( x(y) \)
\[ \text{Note that positive } x \text{ gives negative pressure and vice versa.} \]

The force on a differential element of footing = \( dF \)
\[ \text{Therefore, } dF = -W*n*y*(x_0-(x_0/y_0)*y)dy \]

Force balance of forces, \( F_0 + F_2 = F_1 \)
\[ \begin{bmatrix} y_0 \\ y_0 \end{bmatrix} \]
\[ \begin{bmatrix} F_0 + dF \\ 0 \end{bmatrix} \]

This integral equation can be solved to give :
(1) \( F_0 = W*n*x_0*(L^2/2-L^3/(3*y_0)) \)

Equating moments of \( F_1,F_2 \) and \( F_3 \) about \( y_0 \) gives :
\[ \begin{bmatrix} 0 \\ y_0 \end{bmatrix} = \]
\[ \begin{bmatrix} -W*n*(x_0-(x_0/y_0)*y)*y*(y_0-y)dy \\ -W*n*(X0-(x_0/y_0)*y)*y*(y-y_0)dy \end{bmatrix} \]
\[ \text{where CW moment = sum of CCW moments} \]

This integral equation may be solved to give :
(2) \( F_0*(H+y_0) = W*n*x_0*(y_0*L^2/2-2*L^3/3+L^4/(4*y_0)) \)

Now solving (1) and (2) simultaneously results in :
(3) \( y_0 = (4*H*L+3*L^2)/(6*H+4*L) \)

which is independent of \( n, W \), and \( x_0 \), that is, it
\( y_0 \) is fixed given \( H \) and \( L \), given any \( n, W \), or \( x_0 \).

To solve for footing moment, the equation is :
\[ \begin{bmatrix} y_f \\ 0 \end{bmatrix} = \]
\[ \begin{bmatrix} \int \end{bmatrix} -W*n*(x_0-(x_0/y_0))*y*(y_f-y)dy \]
where \( y_f \) is an arbitrary point between \( y=0 \) and \( y=L \). Solving and letting \( y_f = y \),

\[
M(y) = -F_0(H+y) + W*n*x_0*(y^3/6 - (y^4/(12*y_0)))
\]

The maximum of \( M(y) \) may be found by setting its derivative equal to zero. This results in:

\[
y^3 - 1.5*y_0*y^2 + 3*y_0*F_0/(W*n*x_0) = 0
\]

All cubic and quartic (fourth order) equations may be solved explicitly, but none of higher order. The method is fairly involved and only applies to the equation shown here.

A more general method of solution is the Newton-Raphson iteration, which applies to any function whose derivative is known or can be approximated. The algorithm is:

\[
x(n+1) = x(n) - f(x(n))/f'(x(n)), \text{n = iteration number}
\]

This method is very useful in equations containing transcendental functions, i.e., functions which cannot be expressed by any algebraic function involving only their variables and constants, such as trigonometric functions.

The method is perhaps best explained by an example:

Consider the equation \( \cos x + x - 1.3 = f(x) \), where we wish to find the value of \( x \) which satisfies \( f(x) = 0 \).

Taking the derivative, \( f'(x) = -\sin x + 1 \)

Now choose a starting value, such as \( x = 0 \), and set up the table as:

<table>
<thead>
<tr>
<th>( x_n )</th>
<th>( f(x_n) )</th>
<th>( f'(x_n) )</th>
<th>( x_{n+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0.000000</td>
<td>-1.000000</td>
<td>+1.000000</td>
<td>+0.300000</td>
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<tr>
<td>+0.300000</td>
<td>-0.044664</td>
<td>+0.704480</td>
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<td>+0.363399</td>
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<td>+0.644546</td>
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<td>+0.641783</td>
<td>+0.366364</td>
</tr>
<tr>
<td>+0.366364</td>
<td>-0.000000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example 8.

Consider a single bollard impacted by 10000 lbf at 27 inches above ground surface. Assume firm silty sand soil, and a pier-type footing sixteen (16) inches in diameter. Let maximum displacement at impact be four inches.

Find maximum pipe moment at base, length of pier, and location and value of maximum pier moment.

Solution

(a) Moment at base = 10000 x 27 = 270000 lbf-in.
Shear at base = 10000 lbf

(b) Solve (1) and (3) above simultaneously to get:
\[ L^3 - \frac{18F_0}{3Wn*x_0}L + \frac{24F_0H}{Wn*x_0} = 0 \]

Using cuberoot.c, \( L = 90.315 \) in.

Use \( L = 90" \) (7’-6’)

As a check, first solve (3) for \( y_0 = 65.172 \) in.

Then reconfigure (1) to solve for \( x_0 \).

\[ x_0 = \frac{F_0}{Wn*(L^2/2-L^3/(3*y_0))} = 4.032 \text{ in.}, \text{ o.k.} \]

(c) Using (5) above, solve for \( y \) giving maximum \( M(y) \):

\[ y^3 - 97.759y^2 + 62845 = 0 \rightarrow y = 30.587 \text{ in.} \]

From (4) above, \( M_{\text{max}} = -462310 \text{ lbf-in.} \)
3.3.3 Bollard Characteristic with Vehicle KE Input, No Constraint at Grade

The development for this case assumes that the kinetic energy of the impacting vehicle = the work done by the bollard on the earth.

\[
\text{Work done} = \int_0^L W*n*(x_0-(x_0/y_0)*y)^2*ydy
\]

This is solved to yield:

\[
\text{Work done} = W*n*(x_0^2)*(L^2)*(1/2-2*L/(3*y_0)+L^2/(4*y_0^2))
\]

Equating this to the vehicle kinetic energy, either \(x_0\) or \(L\) may be solved, given the other.

Example 9.
---
Consider a 4000 lbf vehicle impacting the bollard at 10 miles per hour. Assume pier depth = 90 inches (7′-6”). Find deflection \(x_0\) at surface.

Solution
---
vehicle mass = 4000/386.1 = 10.360 lbf-sec^2/inch
velocity = 10*5280*12/3600 = 176 inches/second

\[\text{KE} = m*v^2/2 = 160456 \text{ lbf-inches}\]

Solving the equation above gives \(x_0^2 = 45.7448 \text{ in.}^2\), and \(x_0 = 6.7635 \text{ inches}\).
3.3.4 Bollard with Post Constrained at Base

In this case, the post is constrained for no lateral movement at the ground surface, typically by a slab on grade. Here the post rotates about a point on the surface, rather than below the surface, as is the case for an unconstrained post, as in 3.3.2 and 3.3.3 above. It is again assumed that the post is rigid, with axes as shown.

The equation of the post displacement is \( x = y \tan \theta \), where \( \theta \) = angle of rotation of post.

The governing equations of the earth restraint is:

\[
dF = Wnxydy = Wn\tan \theta y^2dy
\]

Solving for moment balance about the rotation point :

\[
\left\{ \begin{array}{l}
F0H = Wn y \tan \theta y^2dy = (1/4)Wn\tan \theta L^4
\\
0
\end{array} \right.
\]

Now we can solve for \( L \) given \( F0, H \), and \( \theta \) (design) or for \( \theta \) given \( F0, H \) and \( L \) (analysis).

The reaction at the surface restraint, \( F1 \), is given by the balance of forces equation :

\[
F1 = F0 + Wn\tan \theta (y^2)dy = F0 + (1/3)Wn(L^3)\tan \theta
\]

Solving the above two equations by eliminating \( \tan \theta \), we have

\( F1 = F0(1+4F0H/(3L)) \), the force on the surface restraint.

Maximum shear on pier = \( F1-F0 \)

Maximum moment at grade.

The governing equation for work done, in the kinetic energy approach is:

\[
\left\{ \begin{array}{l}
\text{Work done} = Wn\tan \theta (y^2)y\tan \theta dy = \text{kinetic energy}
\\
0
\end{array} \right.
\]

\[
\text{Work done} = (1/4)Wn(\tan \theta)^2L^4 = \text{kinetic energy}
\]
3.3.5 Post and Pier Design [7]

The design loads for bollards and piers have been described in the previous three sections. It remains to design the post and the pier. The system usually consists of a concrete-filled tube (CFT) enclosed within a cylindrical concrete pier below grade. The following procedure consists of four parts, namely:

1. Find moments and shears throughout length.
2. Find the moment capacity of the CFT.
3. Calculate load transfer from the CFT to the concrete and, hence, shear transfer.
4. Design of the concrete pier.

(1) Find Moments and Shears
-----------------------------
Using techniques in sections above, find moments and shears depending on restraint at grade and impact via force or kinetic energy.

(2) Concrete-Filled Tube (CFT) Design
---------------------------------
The formulas used are those shown on page I-6 and I-7 of reference [7], AISC Design Examples, Version 14.0, corresponding to the 2010 AISC code. The examples, as well as the code itself, are free downloads from the website shown in the Reference section.

A program for calculating the nominal moment capacity is given in Appendix 1.

The AISC code, Reference [8], gives a $\phi$ value of 0.9 for flexure, and, with a load factor of 1.6, the moment value given by the equations above must be reduced by $0.9/1.6 = 0.5625$ to get the maximum allowed working moment. The ultimate moment of the pipe alone is given by:

$$M = F_y*(d_o^3-d_i^3)/6,$$

for comparison with the CFT ultimate moment.

(3) Load Transfer from the CFT to the Pier
---------------------------------------
One of the methods of shear transfer from the CFT to the pier is by use of steel headed steel anchors.
As shown in Reference [8], the shear strength of each anchor is:

\[ Q_{nv} = F_u A_{sa}, \]

where:

- \( Q_{nv} \) = nominal shear strength of steel headed stud, kips
- \( A_{sa} \) = cross-section area of steel headed stud anchor, in.\(^2\)
- \( F_u \) = specified minimum tensile strength of a steel headed stud anchor, ksi, with a minimum height/diameter ratio of five (5), and with a \( \Phi \) value of 0.65.

The anchors shall have at least one (1) inch of lateral clear concrete cover, with a minimum center-center spacing of four (4) diameters in any direction and a maximum center-center spacing of 32 times the shank diameter.

(4) Pier Design
---------------

The pier is predominately a flexural, as opposed to compression, member, and should be designed for moment and shear, following ACI 318 rules as:

- Minimum tensile steel strain = 0.004
- Minimum tensile steel area:
  \[ As,\text{min} = 3\sqrt{f_{c'}} A_{gross}/f_y \]
  \[ As,\text{min} = 200 A_{gross}/f_y \]

where the last two are interpreted from ACI 318 Section 10.5.1, which apply to rectangular cross-sections.

- \( V_c \) = nominal shear capacity of concrete alone
  \[ V_c = 2 \sqrt{f_{c'} A_c}, \]
  \( A_c \) = concrete cross-sectional area

interpreted from ACI 318 Section 11.3.1.1, which also applies to rectangular cross-sections.

If stirrups required, max. spacing = \( \frac{1}{2} d \).

Shear reinforcement needed if \( V_u > 0.5 \Phi V_c \)

\( \Phi = 0.75 \) for shear

- \( V_s \) = shear strength by reinforcement
  \[ V_s = A_v f_y t d/s \]
  \( A_v \) = shear reinforcement area, in.\(^2\)
  \( f_y \) = shear reinforcement yield stress, lbf/in.\(^2\)
  \( d \) = distance from extreme compressive fiber to tensile steel, in.
  \( s \) = stirrup spacing, in.
Example 10.  
-----------
Consider a bollard, constrained at the base, with 6000 lbf applied twenty-seven (27) inches above grade (IBC 2009, Section 1607.7.3).
CFT height = 4’-0” above grade.
Pier depth = 5’-0”, with fourteen (14) inch width.
Soil type is firm sand (n = 0.48225 lbf/in.^4)

Find : (1) Deflection at pipe top on impact, moments and shears
(2) Pipe type and size
(3) Shear transfer from pipe to pier
(4) Pier design

Solution
--------

(1) \[ \tan \theta = \frac{4F0H}{(WnL^4)} = 0.0068400 \]
deflection at top of pipe = 48*\( \tan \theta \) = \( \Delta \)
\( \Delta = 0.32832 \) in. \( \cong \) 5/16 in.
nominal pipe moment = 6000*27*1.6/0.9 = \( Mn \)
\( Mn = 288000 \) lbf-in.
CFT shear above grade = \( F0 = 6000 \) lbf
\( F1 = F0(1+4H/(3L)) = 9600 \) lbf
Pier shear = \( Vn = (9600-6000)*1.6/0.65 \)
\( Vn = 8862 \) lbf

(2) Pipe type and size
Use ASTM A106, Schedule 40, Type S, 6 in. dia., Seamless carbon steel pipe.
It is industry practice with this type of pipe to hold the outside diameter, but with a +- 12.5% tolerance on thickness. So, conservatively, t=0.875*0.28=0.245 in., with outside d=6.625 in.
Consider first the shear stress, using ASD as:
\[ v=(4/3)*6000/Area=1629 \] psi<.4*fy=14000 psi,
for the pipe alone, o.k.
Use the program in Appendix 1 to find \( Mn \) for CFT,
\( Mn(\text{capacity})=398752 \) lbf-in.,> 288000 lbf-in., o.k.

(3) Use three (3) 3/8” dia. 2” long welded shear studs, at 120°.
Force each stud = \( Fu*Asa = 6737 \) lbf. Spacing at 120° gives force as (1+2*0.5)*6737 = 13474 lbf > 8862 lbf, o.k.
(4) Let Mn for pier equal or exceed Mn for CFT, say 400000 lbf-in.
Using the programs in Appendix 2, with gamma = .75 tensile strain = .01512 > .004, o.k.
Asteel = 1.0139 in.^2, use six (6) #4 bars = 1.2 in.^2
Agross = \( \pi \times d^2/4 = 201.062 \) in.^2
Asteel min. = 3*\( \sqrt{fc' \times Agross/fy} = 0.63584 \) in.^2
Asteel min. = 200*Agross/fy = 0.67021 in.^2
Vc = 2*\( \sqrt{fc' \times Agross} = 25432 \) lbf
0.5*\( \Phi \times Vc = 9537 \) lbf
Vu = 1.6*3600 = 5760 lbf, < 0.5*\( \Phi \times Vc \), no stirrups required.
To tie bars together, use geometry for tied columns, ACI 318, Section 7.10.5 with spacing less than or equal to 16 longitudinal bar diameters or 48 tie bar diameters.

In summary,
\( fc' = 4000 \) psi, CFT and pier

post = ASTM A106, Type S, Schedule 40, 6” dia. seamless carbon steel pipe, concrete filled

pier = 16 in. dia., six (6) #4 longitudinal bars at 60° spacing with #3 circular ties at eight (8) inch vertical spacing, with three (3) 3/8”, 2” long, ASTM A29 welded studs, 3 Inches below bottom surface of confinement, spaced 120° apart.
4. HIGH IMPACT BARRIERS [9 10 11 12]

High impact barriers are primarily used to prevent entry of an unauthorized vehicle into a protected area. They are characterized by very high impact kinetic energies and, hence, very high forces. This section is divided into the following parts:

4.1 Anti-Ram Barrier Standard
4.2 Types of Barriers
4.3 Estimation of Forces
4.4 Wall Design

4.1 Anti-Ram Barrier Standard [9]

The standard used by the United States Departments of State and Defense is ASTM F2656-07 “Standard Test Method for Vehicle Crash Testing of Perimeter Barriers”.

It lists four (4) vehicle types with the following characteristics:

<table>
<thead>
<tr>
<th>Type+Weight</th>
<th>Speed</th>
<th>K.E.(ft-lbf)</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small passenger car, 2430 lbf</td>
<td>40 mph</td>
<td>131 000</td>
<td>C40</td>
</tr>
<tr>
<td></td>
<td>50 mph</td>
<td>205 000</td>
<td>C50</td>
</tr>
<tr>
<td></td>
<td>60 mph</td>
<td>295 000</td>
<td>C60</td>
</tr>
<tr>
<td>Pickup truck, 5070 lbf</td>
<td>40 mph</td>
<td>273 000</td>
<td>PU40</td>
</tr>
<tr>
<td></td>
<td>50 mph</td>
<td>426 000</td>
<td>PU50</td>
</tr>
<tr>
<td></td>
<td>60 mph</td>
<td>613 000</td>
<td>PU60</td>
</tr>
<tr>
<td>Medium duty truck, 15000 lbf</td>
<td>30 mph</td>
<td>451 000</td>
<td>M30</td>
</tr>
<tr>
<td></td>
<td>40 mph</td>
<td>802 000</td>
<td>M40</td>
</tr>
<tr>
<td></td>
<td>50 mph</td>
<td>1 250 000</td>
<td>M50</td>
</tr>
<tr>
<td>Heavy goods vehicle, 65000 lbf</td>
<td>30 mph</td>
<td>1 950 000</td>
<td>H30</td>
</tr>
<tr>
<td></td>
<td>40 mph</td>
<td>3 470 000</td>
<td>H40</td>
</tr>
<tr>
<td></td>
<td>50 mph</td>
<td>5 430 000</td>
<td>H50</td>
</tr>
</tbody>
</table>

In addition, the designation is qualified by the penetration distance, as

<table>
<thead>
<tr>
<th>Qualifier</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>less than or equal to 1 meter</td>
</tr>
<tr>
<td>P2</td>
<td>1.01 to 7 meters</td>
</tr>
<tr>
<td>P3</td>
<td>7.01 to 30 meters</td>
</tr>
<tr>
<td>P4</td>
<td>greater than 30 meters</td>
</tr>
</tbody>
</table>

And is measured from barrier inwards to reference shown:
### Vehicle Reference

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small passenger car</td>
<td>Base of &quot;A&quot; pillar</td>
</tr>
<tr>
<td>Pickup truck</td>
<td>Front leading lower edge of pickup truck bed</td>
</tr>
<tr>
<td>Medium duty truck</td>
<td>Leading lower edge of cargo bed</td>
</tr>
<tr>
<td>Heavy goods vehicle</td>
<td>Leading lower vertical edge of cargo bed</td>
</tr>
</tbody>
</table>

It should be noted that the previous DOS (Department of State) specification SD-STD-02.01 listed only three vehicle types, K4, K8, and K12, which are now M30-P1, M40-P1, and M50-P1, respectively.

Qualification of a barrier depends, in general, on both analysis and actual crash test.

#### 4.2 Types of Barriers [10]

The two general types are active (operable) and passive (fixed).

Active barriers are used only at vehicular access control points. They are mechanical devices produced by specialized manufacturers.

Passive barriers include:

1. Walls – see Section 4.4
2. Engineered planters – these are relatively massive planters which depend upon the friction between the planter base and grade, or near grade, to dissipate kinetic energy.
   - Some guidelines are:
     - Rectangular planters $\leq 2$ foot wide
     - Circular planters $\leq 3$ foot diameter
     - Four (4) maximum clear distance between planters
     - Landscaping $\leq 2'-6"$ in height
     - Vegetation should not hide a 6 inch thick package, nor a briefcase, nor a knapsack
     - Clean regularly

3. Bollards – See Section 3.3 above for design. For this use, they generally have continuous strip footings, rather than individual pier footings. If underground utilities make the installation of conventional footings impractical, one solution is to use a shallow system of beams or deep slabs underneath the pavement to prevent overturning.

4. Excavations, berms, and ditches – the following diagram is taken from Fig.4-11 in reference [10]:

---

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4.3 Estimation of Forces

The following section only approximates the forces for rigid and flexible barriers, and should not be used for final design.

4.3.1 Rigid Barrier [11]

Here the vehicle is represented by a linear single degree-of-freedom linear system, composed of a linear spring and a mass with an initial velocity. Terms are defined as:

- **B** = stiffness coefficient that represents the ratio of the force per unit width of contact area per unit depth of residual crush, psi
- **f** = force on rigid barrier
- **g** = gravitational acceleration, 386.1 in./sec^2
- **K** = spring constant, taken as = B*wd lbf./in.
- **m** = vehicle mass, lbf-sec^2/in.
- **t** = elapsed time, seconds
- **tf** = total time of crushing, seconds
- **v0** = vehicle impact velocity, 17.6 in./sec = 1 mile/hour
- **wgt** = vehicle weight, lbf
- **wd** = vehicle width, in.
- **wn** = natural frequency, radians/sec = sqrt(K/m)
- **x** = vehicle crush depth, in.
- **x’** = vehicle velocity, as a function of time, in./sec
- **x’’** = vehicle acceleration, as a function of time, in./sec^2

For a single degree-of-freedom linear system, with an initial velocity v0,

- **x** = \( \frac{v0}{wn} \cdot \sin(wn \cdot t) \)
- **x’** = \( v0 \cdot \cos(wn \cdot t) \)
- **x’’** = \( -v0 \cdot wn \cdot \sin(wn \cdot t) \)

From the expression for \( x’ \), we can solve for the time of end of crush, namely \( x’ = 0 \). Thus \( wn \cdot tf = \Pi/2 \) and \( tf = \Pi/(2*wn) \).
Example 11
----------
A rigid barrier is impacted by a pickup truck with the followings parameters:
B = 110 psi
v₀ = 50 mph = 880 in./sec
w₀ = 80 in.
wgt = 5070 lbf
Find average force on deceleration, and peak force.

Solution
--------
K = B*w = 8800 lbf/in.
m = wgt/g = 13.131 lbf-sec^2/in.
wn = sqrt(K/m) = 25.887 radians/second
tf = Π/(2*wn) = 0.06078 seconds
average deceleration = v₀/tf
= 14503 in./sec^2
= 14503/386.1 = 37.562g
average force = wgt*deceleration in g’s
= 190 440 lbf
maximum force = wn*m*v₀ = 299 140 lbf

4.3.2 Flexible Barrier

With a flexible barrier, we assume the vehicle kinetic energy is equal to an average force times the penetration distance,
(1/2)*m*v₀^2 = average force times distance

Example 12
----------
A flexible barrier is used for the vehicle in Example 11, with a distance of 10 feet.

Solution
--------
K.E. = (1/2)*(13.131)*(880^2)
= 5 084 400 in. lbf
Average force = K.E./120 = 42 370 lbf
4.4 Wall Design

This design procedure is based on the derivation in reference [12], diagram and formulas reproduced below. It is based on a yield line analysis of a concrete slab, not conventional reinforced concrete design. In this case, no top rail is used, so $M_b = 0$.

$M_c$ is the nominal strength of the vertical reinforcement, and $M_w$ the nominal strength of the horizontal reinforcement. Both are ultimate moments. We solve here for $\Phi(wl)$, the load causing wall failure, multiplied by the capacity reduction factor.

This analysis can be used as a starting point for a more detailed finite element approach, specific to the vehicle(s) being stopped, the site layout, and soil conditions. The total structure (wall, foundation, and soil system) must resist overturning and excessive sliding.
(wL) = \frac{8M_b}{L - \frac{L}{2}} + \frac{8M_WH}{L - \frac{L}{2}} + \frac{M_cL^2}{H(L - \frac{L}{2})}

L = \frac{L}{2} + \sqrt{\left(\frac{L}{2}\right)^2 + \frac{8H(M_b + M_WH)}{M_c}}

H = \text{height of wall, ft}
L = \text{critical length of wall failure, ft}
(wL) = \text{total ultimate load capacity of walls, kips}
M_b = \text{ultimate moment capacity of beam at top of wall, kip-ft}
M_W = \text{ultimate moment capacity of wall per ft of wall height, kip-ft/ft}
M_c = \text{ultimate moment capacity of wall cantilever up from bridge deck per ft of length of wall, kip-ft/ft}
L = \text{length of distributed impact load, ft}
Example 13.
-------------
Find \( \Phi(wl) \) for the wall shown below, with the following parameters:

- \( fc' = 4 \) ksi
- \( fy = 60 \) kksi
- \( Es = 29000 \) psi
- \( wl = 300 \) kip
- \( \Phi = 0.9 \)
Solution
--------
The following equations are used to solve for moment capacity:

\[ M_n = f_y A_s (d - d') \]

\[ a = f_y A_s \]

\[ a = f_y A_s \]

\[ \varepsilon_c = 0.003 \]

\[ \varepsilon_c = 0.003 \]

\[ a = f_y A_s \]

\[ a = f_y A_s \]

\[ \varepsilon_c = 0.004 \]

\[ \varepsilon_c = 0.004 \]

\[ a = f_y A_s \]

\[ a = f_y A_s \]

\[ c = 0.85 f_c' b \]

\[ c = 0.85 f_c' b \]

\[ F \text{ may be } T \text{ or } C \]

\[ F = 0.85 f_c' (f_y x) b \]

\[ F = 0.85 f_c' (f_y x) b \]

\[ F = 0.85 f_c' (f_y x) b \]

\[ F = 0.85 f_c' (f_y x) b \]

\[ 0.85 f_c' b x^2 + (\varepsilon_c E_s - f_y) A_s x - \varepsilon_c d' E_s A_s = 0 \]

\[ 0.85 f_c' b x^2 + (\varepsilon_c E_s - f_y) A_s x - \varepsilon_c d' E_s A_s = 0 \]

\[ M_n = T (d - x) + |F(x - d')| + C x (1 - f_y x) \]

\[ M_n = T (d - x) + |F(x - d')| + C x (1 - f_y x) \]
Consider first tension reinforcement only.

Vertical steel
\[ d = 13.125'' \]
\[ Mc = 54.902 \text{ kip-ft/ft} \]

Horizontal steel
\[ d = 12.375'' \]
\[ Mw = 51.603 \text{ kip-ft/ft} \]

From the wall equations,
\[ (wl) = 329.80 \text{ kip} \]
\[ \Phi(wl) = 294.82 \text{ kip, marginal} \]

Now consider equal reinforcement both faces

Vertical steel
\[ x = 1.72060'' \]
\[ Mc = 55.187 \text{ kip-ft/ft} \]

Horizontal steel
\[ x = 2.08807'' \]
\[ Mw = 53.388 \text{ kip-ft/ft} \]

From the wall equations,
\[ wl = 335.89 \text{ kip} \]
\[ \Phi(wl) = 302.30 \text{ kip} > 300 \text{ kip, o.k.} \]
\[ L = 12.173 \text{ ft} \]

Shear strength = concrete + shear friction
\[ Vc = 2\sqrt{fc'}b*d + 0.6*fy*As \]
\[ Vc = 242.51 + 1267.2 = 1509.71 \]
\[ \Phi Vc = 0.75*Vc = 1132.28 \text{ kip} > 300 \text{ kip, o.k.} \]
APPENDIX 1

/************************************************************************* 
* cft.c : see p.I-6,7 of "design Examples V14.0", AISC, 2011 : 
* 
* d = outside diameter, in. 
* h = inside diameter, in. 
* t = nominal wall thickness x 0.93, in. 
* fc' = specified concrete strength, psi 
* fy = specified pipe yield, psi 
* Kc,Ks = derived parameters to find é, lbf 
* é = central angle, center to neutral axis, radians 
* Zcb,Zsb = derived parameters to find Mb, in.^3 
* Mb = plastic moment, no axial load 
* 
* 05-13-12 : ml 
*************************************************************************/

#include<math.h>
#include<stdio.h>
#include<stdlib.h>

int main(void)
{
    double d,fcp,fy,h,t;
    double Kc,Ks,theta,Zcb,Zsb,Mb;
    double arg1,arg2,arg3,zin;
    double pi;
    FILE *inn;
    FILE *out;
    inn = fopen("cft.in","r");
    out = fopen("cft.out","w+");
    fscanf(inn,"%lf %lf %lf %lf %lf", &d,&fcp,&fy,&h,&t);
    fclose(inn);
    t = 0.875*t;
    Kc = fcp*h*h;
    Ks = fy*(d-t)*t/2.0;
    arg1 = (0.026*Kc-2.0*Ks)/(0.0848*Kc);
    arg2 = (0.026*Kc+2.0*Ks)*(0.026*Kc+2.0*Ks)+0.857*Kc*Ks;
    arg3 = sqrt(arg2)/(0.0848*Kc);
    theta = arg1+arg3;
    if((sin(pi-theta)/2.0)>0.5)
    {
        printf("(sin(pi-theta)/2.0)>0.5\n");
        exit(0);
    }
    else
    {
    
    }
    zin = sin(theta/2.0);
    Zcb = h*h*h*zin*zin*zin/6.0;
    Zsb = (d*d*d-h*h*h)*zin/6.0;
    Mb = fy*Zsb+0.475*fcp*Zcb;
    fprintf(out,"input data\n");
    fprintf(out,"------------------------------\n");
    fprintf(out,"d = ");fprintf(out,"%14.6e",d);fprintf(out," in.\n");
    fprintf(out,"fcp = ");fprintf(out,"%14.6e",fcp);fprintf(out," psi\n");
    fprintf(out,"fy = ");fprintf(out,"%14.6e",fy);fprintf(out," psi\n");
    fprintf(out,"h = ");fprintf(out,"%14.6e",h);fprintf(out," in.\n");
    fprintf(out,"t = ");fprintf(out,"%14.6e",t);fprintf(out," in.\n");
    fprintf(out,"output data\n");
    fprintf(out,"------------------------------\n");
    fprintf(out,"Kc = ");fprintf(out,"%14.6e",Kc);fprintf(out," lbf\n");
    fprintf(out,"Ks = ");fprintf(out,"%14.6e",Ks);fprintf(out," lbf\n");
    fprintf(out,"theta = ");fprintf(out,"%14.6e",theta);fprintf(out," rad\n");
    fprintf(out,"Zcb = ");fprintf(out,"%14.6e",Zcb);fprintf(out," in.^3\n");
    fprintf(out,"Zsb = ");fprintf(out,"%14.6e",Zsb);fprintf(out," in.^3\n");
    fprintf(out,"Mb = ");fprintf(out,"%14.6e",Mb);fprintf(out," lbf-in.\n");
    fclose(out);
    return 0;
}
APPENDIX 2

\[
d = h \left( 1 - \frac{1 - \gamma}{2} \right)
\]

\[
\text{PNA} = \text{plastic neutral axis}
\]
/***************************************************************************
* pier1.c : compile as 'bcc32 -tC pier1.c'                                 *
*                                                                          *
*       e1      =       extreme compression concrete fiber strain,         *
*                       rectangular stress block, usually .003 in./in.     *
*       Es      =       modulus of elasticity of reinforcing steel, psi    *
*       fcp     =       28 day strength of concrete, psi                   *
*       fy      =       yield strength of reinforcement, psi               *
*       gamma   =       dia. circular reinforcement/dia. pier              *
*       h       =       dia. pier, in.                                     *
*       ftarget =       input Mn, lbf-in.                                  *
*       x_1     =       first estimate of extreme tensile strain           *
*       x_0     =       second   "     "    "        "      "              *
*       f_1     =       Mn based on x_1, lbf-in.                           *
*       f_0     =       Mn based on x_0, lbf-in.                           *
*       n       =       (x_0-x_1)/(f_0-f_1)                                *
*       xnew    =       n*(ftarget-f_0)+x_0                                *
*       Mn,out  =       last calculated Mn, lbf-in.                        *
*       As      =       circular reinforcement total area to balance       *
*                       forces, in.^2                                      *
*                                                                          *
*       This program checked against ACI Sp-17, Chapter 3, -09-07,         *
*       pages 70 (for gamma = 0.6, for rho from 0.1 to 0.3) and 73         *
*       (for gamma = 0.9, for rho = 01. to 0.8) for values at Pn =0        *
*       horizontal axis.                                                   *
*       Error defined as (Aprogram-Agraph)/Agraph                          *
*       Average error = +0.133%                                            *
*       Least conseravtive error = -1.721%                                 *
*       Most conservative error  = +2.571%                                 *
*                                                                          *
*                                               05-13-12 : ml              *
***************************************************************************/

#include<math.h>
#include<stdio.h>
#include<stdlib.h>

int main(void)
{
    int num = 1;
    double e1,Es,fcp,fy,gamma,h;
    double x_1,x_0,xnew,f_1,f_0,ftarget,wrkf;
    double *gMn,*gAs;
    void getMnAs(double,double,double,double,double,double,double,
                 double*,double*);
    gMn = calloc(1,sizeof(double));
    gAs = calloc(1,sizeof(double));
    *(gMn+0) = 0.0;
    *(gAs+0) = 0.0;
    printf("input e1 Es fcp fy gamma h\n");
    scanf("%lf %lf %lf %lf %lf %lf",&e1,&Es,&fcp,&fy,&gamma,&h);
    printf("input x_1 x_0 ftarget\n");
    scanf("%lf %lf %lf",&x_1,&x_0,&ftarget);
    do
    {
        getMnAs(e1,x_1,Es,fcp,fy,gamma,h,gAs,gMn);
        f_1 = *(gMn+0);
        getMnAs(e1,x_0,Es,fcp,fy,gamma,h,gAs,gMn);
        f_0 = *(gMn+0);
        n = (x_0-x_1)/(f_0-f_1);
        xnew = +n*(ftarget-f_0)+x_0;
        getMnAs(e1,xnew,Es,fcp,fy,gamma,h,gAs,gMn);
        fnew = *(gMn+0);
        printf("xnew = ");printf("%20.9f\n",xnew);
        printf("fnew = ");printf("%20.9f\n",fnew);
        printf("   hit 1 to continue\n");
        scanf("%d",&num);
        if(num!=1)}
wrkf = *(gAs+0);
printf("\n\n");
printf("As = ");
printf("%14.6f",wrkf);
printf(" in.^2\n\n");
exit(0);
} 
else 
{
    ;
}
x_1 = x_0;
f_1 = f_0;
x_0 = xnew;
f_0 = fnew;
}
while(num==1);
return 0;
}

void getMnAs(double e1,double e5,double Es,double fcp,double fy,double gamma,
    double h,double *As,double *Mn)
{
    double a,area,beta1,c,d;
    double etop,ey,r1,r2,theta1,theta2,theta3,theta4,y0;
    double pi = 3.1415926536;
    double arm1,arm2,arm3,arm4,arm5;
    double C1,C2,C3,d2,d4,T1,T2,t;
    if(e5<.004)
    {
        printf("e5<.004\n");
        exit(0);
    }
    else 
    {
    ;
    }
    t = 1.0;
betal = 0.85;
    if((fcp>4000)&&(fcp<=8000))
    {
        betal = 0.85-0.05*(fcp-4000.0)/1000.0;
    }
    else 
    {
    ;
    }
    if(betal<0.65)
    {
        betal = 0.65;
    }
    else 
    {
    ;
    }
    d = h*(1.0-(1.0-gamma)/2.0);
c = d*{(e1/(e1+e5))};
a = betal*c;
r1 = h/2.0;
area = pi*r1*r1/2.0-(r1-a)*sqrt(r1*r1-(r1-a)*(r1-a))-r1*r1*asin((r1-a)/r1);
arm1 = 2.0*pow(a*(2.0*r1-a),1.5)/(3.0*area)-(r1-c);
C1 = .85*fcp*area;
r2 = gamma*h/2.0;
etop = e1-(r1-r2)*(e1+e5)/(r1+r2);
    if(etop<=0.0)
    {
        printf("no compression steel\n");
        }
```c
}
else
{
  
}
  
ey  =  fy/Es;
y0  =  r1-c;
if((r1-c)>=r2)
{
  printf("r1-c >= r2\n");
  exit(0);
}
else
{
  
}
theta1  =  asin((r1-c)/r2);
y0  =  r1-c;
if(etop>ey)
{
  printf("CASE 1\n");
d2  =  ey*c/e1;
theta2  =  asin((d2+y0)/r2);
C3  =  2.0*fy*r2*t/d2;
C3 *=  r2*(cos(theta1)-cos(theta2)) - y0*(theta2-theta1);
arm3  =  r2*(theta2-theta1)/2.0;
arm3 +=  r2*(sin(2.0*theta1)-sin(2.0*theta2))/4.0;
arm3 +=  y0*(cos(theta2)-cos(theta1));
arm3 /=  cos(theta1)-cos(theta2)+y0*(theta1-theta2)/r2;
arm3 -=  y0;
C2  =  2.0*fy*r2*t*(pi/2.0-theta2);
arm2  =  r2*cos(theta2)/(pi/2.0-theta2)-y0;
}
else
{
  printf("CASE 2\n");
d2  =  theta2  =  0.0;
C2  =  arm2  =  0.0;
C3  =  2.0*etop*Es*r2*t*
  (r2*cos(theta1)-y0*(pi/2.0-theta1))/(r2-y0);
arm3  =  r2*r2*(pi/4.0-theta1/2.0+sin(2.0*theta1)/4.0);
arm3 -=  r2*y0*cos(theta1);
arm3 /=  r2*cos(theta1)-y0*(pi/2.0-theta1);
arm3 -=  y0;
}
d4  =  (r2+y0)*ey/e5-y0;
theta3  =  asin(y0/r2);
theta4  =  asin(d4/r2);
T1  =  2.0*fy*r2*t*(r2*(-cos(theta4)+cos(theta3))+y0*(theta4+theta3));
T1 /=  d4+y0;
arm4  =  r2*r2*((theta4+theta3)/2.0-(sin(2.0*theta4)+sin(2.0*theta3))/4.0);
arm4 -=  r2*y0*(cos(theta4)-cos(theta3));
arm4 /=  r2*(-cos(theta4)+cos(theta3))+y0*(theta4+theta3);
arm4 +=  y0;
T2  =  2.0*fy*r2*t*(pi/2.0-theta4);
arm5  =  r2*cos(theta4)/(pi/2.0-theta4)+y0;
t  =  C1/(T1+T2+C2-C3);
*(As+0)  =  2*pi*r2*t;
*(Mn+0)  =  C1*arm1+t*(C2*arm2+C3*arm3+T1*arm4+T2*arm5);
}
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