Anchoring To Concrete

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INTRODUCTION

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A concrete anchor is a steel shaft either cast into concrete at placement or post-installed after the concrete has hardened.

Cast-in anchors are threaded shafts with a buried end termination of a hex head, threaded nut, or 90° (L-) or 180° (J-) hook, or headed (non-threaded) studs welded to a surface plate.

Post-installed anchors include adhesive and expansion types. Two of the expansion types are torque-controlled, where expansion is controlled by torque on the bolt, or displacement-controlled, where a plug or sleeve is impacted and the expansion is controlled by the length of travel of the plug or sleeve.

The anchors are designed to transfer the design loads from the superstructure to the foundation. In many cases, this transfer is, either from steel column base plates to the foundation, or from precast concrete members to the foundation.

An example of the connection of cast-in anchor to precast is shown in the following photograph, in the construction of a salt storage building in Western New York. The large cast-in bolts transfer the tensile force caused by the moment generated by the horizontal force of the soil against the precast walls to the foundation.
Shear is not transferred by the bolts, but by bearing between the buttress and foundation, due to the socketing of the buttress into the foundation.

The footings, anchors, buttresses, wall panels, and lintels comprising the complete foundation system for the salt storage building with arch roof under construction were designed, fabricated, and erected by the precast firm “New Eagle Silo” of Arcade, New York.
This paper describes:
● Anchor Materials
● Concrete Cracking
● General Requirements
● Bolt Bending
● Anchor Tension Reinforcement
● Anchor Shear Reinforcement
● Description of Failure Modes
● Base Plates and Anchor Bolts
● Examples
● Appendix 1 – Definitions of Terms
● Appendix 2 – Citations List
● Appendix 3 – Anc program

The basic reference is “Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary, Appendix D”, Reference 1. Citations not noted with a source refer to this specification.
ANCHOR MATERIALS

The most common steel material for cast-in anchors is ASTM F1554, Grade 36. This is generally less expensive and more readily available than other types. It is also desirable where a ductile failure rather than a concrete failure is required, and the least concrete distance restrictions are present with its low yield and ultimate strengths.

Cast-in anchors are of three (3) types, headed bolts, headed studs, and hooked bolts, installed in place prior to concrete placement. Cast-in gives greater control, but less flexibility.

Headed bolts are cylindrical threaded steel bars terminated in the concrete either by an integral head or nut, either of which may include a washer or plate. Care should be taken in the selection of a washer or plate, if used, because the stresses may exceed those allowable on conventional washers.

Headed studs are cylindrical steel bars (normally unthreaded) with an embedded head and welded to a steel plate at the surface. They are usually used to transfer shear loads between steel and concrete, typically in composite beams.

Hooked bolts refer to cylindrical steel bars with threaded connections at the ends, and possibly throughout. They are defined by the embedded end, either “L” (90°) or “J” (180°). Allowable bend diameters are not specified by ACI 318-11, only bent rebars. Appendix D does specify distance from the inner surface to the end of the hook. In projects requiring ductility, i.e., the lowest failure load is tension in the steel anchor, the concrete pullout strength must be greater than or equal to the tensile capacity of the steel anchor. This is, in general, not possible with hooked bolts as shown in the discussion of pullout strength in the pullout capacity discussion below.
Post-installed anchors material and design properties are obtained from ICC-ES Evaluation Reports such as Reference 2 for expansion anchors and Reference 3 for adhesive anchors.

Cast-in headed anchors refer to headed steel bars welded to a base plate. They are usually used to transfer shear loads between steel and concrete, typically in composite beams. See also the discussion on pullout strength for further description.

Post-installed gives greater flexibility, but less control.

CONCRETE CRACKING
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Anchor design for the concrete breakout, pullout, bond strength, and pryout failure modes depend upon judgment of cracked versus uncracked concrete in computations. Courses and control of cracking are discussed in Ref. 4 as follows:

- **Plastic Shrinkage Cracking**
  This is due to the evaporation of water near the surface, shrinking the surface layer but restrained by inner concrete, developing tensile stresses in the war surface layer. This results in a differential volume change. To slow down the evaporation, fog nozzles, plastic sheeting, windbreaks, and sun shades may be used.

- **Plastic Shrinkage Settlement Cracking**
  During the consolidation phase, the plastic concrete may be restrained by rebars, (cracking increases with rebar size), slump (increasing slump equals increasing cracking), and cover (increases with decreasing cover).

- **Hardened Concrete Drying Shrinkage**
  This is caused by volume change as the concrete shrinks, but is restrained. This may be reduced by contraction joints, proper detailing (especially no re-entrant corners), or shrinkage-compensating concrete. See Reference 5 for further details.
● Thermal stresses
Concrete has a temperature coefficient of expansion of approximately $5.5 \times 10^{-6}$. Consider two surfaces in contact with a temperature differential of 25°F. Consider one surface completely restrained, $f_{c'} = 4000$ psi

$$\text{pressure} = \text{strain} \times E = 137.5 \times 10^{-6} \times 57000 \times f_{c'}^{(1/2)}$$

pressure = 496 psi

Now $f_t = \text{modulus of rupture} = 7.5 \times f_{c'}^{(1/2)}$

$f_t = 474$ psi < 496 psi, n.g.

● $w = \text{maximum crack width} = 0.10 \times f_s (d_c A)^{(1/3)} \times 10^{-3}$

where $w$ in inches, $f_s$ in steel stress (ksi), $d_c$ = cover in inches and $A$ = area of concrete symmetric with rebars divided by the number of rebars.

● Post-installed anchors are required to perform well by tests with a crack width of 0.012 inch.

GENERAL REQUIREMENTS
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CONCRETE SPLITTING

The rules in Section D.8 refer to spacing, edge distances, and $hef$ (effective embedment depth). They apply to all failure modes and should be addressed before starting the design.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Untorqued Cast-in Anchor</th>
<th>Torqued Cast-in Anchor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum center - center spacing</td>
<td>4*da</td>
<td>6*da</td>
</tr>
<tr>
<td>Minimum edge distance</td>
<td>cover</td>
<td>6*da</td>
</tr>
<tr>
<td>Quantity</td>
<td>Adhesive Anchor</td>
<td>Expansion Anchor Torque Controlled</td>
</tr>
<tr>
<td>--------------</td>
<td>-----------------</td>
<td>------------------------------------</td>
</tr>
<tr>
<td>Min. ctr-ctr spacing</td>
<td>6*da</td>
<td>6*da</td>
</tr>
<tr>
<td>Min. edge (1) distance hef max. (2)</td>
<td>6*da</td>
<td>8*da</td>
</tr>
<tr>
<td>cac,min</td>
<td>2*hef</td>
<td>4*hef</td>
</tr>
</tbody>
</table>

cac = critical edge distance controlled by concrete breakout or bond. Unless determined by test to ACI 355.2 (mechanical anchors) or ACI 355.4 (post-installed adhesive anchors), use the following values:

- adhesive anchors → 2.0*hef
- undercut anchors → 2.5*hef
- torque-controlled expansion anchors → 4.0*hef
- displacement-controlled expansion anchors → 4.0*hef

_da_ = anchor diameter, in.

_\text{ha}_ = member depth, in.

(1) If edge distance less than that shown, substitute da’ for da that meets the requirements of minimum center-center spacing and edge distance. Forces are limited to an anchor with a diameter of da’.

(2) Values here may be reduced if tests according to the definition of cac are performed.

GROUP EFFECTS (D.3.1.1)

Group effects must be considered if anchor spacing is less than any of the following values:

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete breakout in tension</td>
<td>3*hef</td>
</tr>
<tr>
<td>Bond strength in tension</td>
<td>2*cna</td>
</tr>
<tr>
<td>Concrete breakout in shear</td>
<td>3*cal</td>
</tr>
</tbody>
</table>

cna = projects distance from the center of an anchor shaft on one side of the anchor to develop the full bond strength of an adhesive anchor.
ca1 = distance from the anchor center to the concrete edge in one direction, in. If shear is applied to the anchor, ca1 is taken in the direction of applied shear. If tension is applied to the anchor, ca1 is the minimum edge distance.

OTHER

- Loads with high fatigue or impact not covered (D.2.4) By Appendix D.
- Anchors and anchor groups can be designed by (D.3.1) elastic analysis. Plastic analysis may be used if nominal strength is controlled by ductile steel.
- Appendix D does not apply to the design of anchors in plastic hinge zones of concrete structures under earthquake loads. These zones are defined as extended from twice the member depth from any column or beam face. These zones also include any other section where yielding of reinforcement is likely to occur due to lateral displacements. (D.3.3.2)
  If anchors must be located in these plastic hinge zones, they should be designed so that the anchor forces are directly transferred to anchor reinforcement that carries these anchor forces into the member beyond the anchor region. (RD.3.3.2)
- Post-installed anchors must meet ACI 355.2 or AQCI 355-4 (D3.3.3)
- Anchors in Seismic Design Category C, D, E, and F structures must satisfy all the non-seismic requirements of Appendix D, as well as additional requirements:
  - Tensile Loading (D.3.3.4)
  - Shear Loading (D.3.3.5)
- Modification factor λa for lightweight concrete:
  - Cast-in concrete failure \( \lambda_a = 1.0 \)
  - Expansion + adhesive anchor concrete failure \( \lambda_a = 0.8 \)
  - Adhesive anchor bond failure \( \lambda_a = 0.6 \)
- \( f_{c'} \leq 10000 \) psi for cast-in anchors (D.3.7)
- \( f_{c'} \leq 8000 \) psi for post-installed anchors
- For steel and pullout failure loads, the (RD.4.11)
highly stressed anchor should be checked. For concrete breakout, the anchors should be checked as a group.

- Maximum anchor diameter = 4 inches. (D.4.2.2)
- Adhesive anchor embedment depths must be limited to $4\times da \leq hef \leq 20\times hef$ (D.4.2.3)
- Strength Reduction $\phi$ factors (D.4.3)
- Anchors governed by strength of ductile steel element - tension = 0.75 shear = 0.65
- Anchors governed by concrete breakout, side-face blowout, pullout or pryout strengths:

<table>
<thead>
<tr>
<th>Load</th>
<th>Element</th>
<th>Condition</th>
<th>Category</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear</td>
<td>---</td>
<td>A</td>
<td>---</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>---</td>
<td>B</td>
<td>---</td>
<td>0.70</td>
</tr>
<tr>
<td>Tension</td>
<td>cast-in headed</td>
<td>A</td>
<td>---</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>studs + bolts,</td>
<td>B</td>
<td>---</td>
<td>0.70</td>
</tr>
<tr>
<td>post-installed</td>
<td>A</td>
<td>1</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>anchors</td>
<td></td>
<td>A</td>
<td>2</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A</td>
<td>3</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>1</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>2</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>3</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Condition A - supplementary reinforcement is present except for pullout and pryout
Condition B - no supplementary reinforcement
And for pullout and pryout

Category - applies to post-installed anchors

<table>
<thead>
<tr>
<th>Category</th>
<th>Sensitivity</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>low</td>
<td>high</td>
</tr>
<tr>
<td>2</td>
<td>medium</td>
<td>medium</td>
</tr>
<tr>
<td>3</td>
<td>high</td>
<td>low</td>
</tr>
</tbody>
</table>

where sensitivity refers to sensitivity to installation.
BOLT BENDING

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ACI 318-11 defines stretch length as the length of anchor extending beyond the concrete, subject to tensile load. Code Section D.3.3.4.3 gives four (4) options for anchors and their attachments to structures in Seismic Design Categories C,D,E and F. Option (a), part 3, says anchors shall transmit tension loads by a ductile steel element with a stretch length of at least eight (8) bar diameters.

The following analysis is that given in Reference 10 with the exception of Z, the plastic modulus.

\[ z = \text{portion of moment arm above concrete, in.} \]
\[ n = 0 \text{ if clamped at concrete surface by nut and washer (required for mechanical anchors)} \]
\[ = 0.5 \text{ if not clamped at concrete surface} \]
\[ d_0 = \text{bolt diameter, in.} \]
\[ L = \text{stretch length} = z + n \times d_0, \text{ in.} \]
\[ Z = D^{3/12} \text{ in.}^3 \]
\[ M_{so} = \text{bending moment to cause rupture} = 1.2 \times f_{uta} \times Z \]
\[ N_{sa} = \text{nominal tensile strength of anchor} \]
\[ N_{ua} = \text{factored load tension} \]
\[ M_s = \text{resultant flexural resistance of anchor} \]
\[ M_s = M_{so} \times (1 - N_{ua} / \phi N_{sa}) \]
\[ \alpha = \text{adjustment factor} 1 \leq \alpha \leq 2 \]
\[ M_v = \text{factored bending moment due to factored shear} \]
\[ M_v = V_{ua} \times L, \leq M_s \Rightarrow \text{if not true, redesign.} \]
\[ V_{add} = \text{term added to factored load shear} (V_{ua}) \]
\[ = \alpha \times M_s / L \]

Check the interaction of all the governing failure loads with the addition of Vadd

The following page shows examples of stretch lengths and stretch connection.
Fig. RD.1.3—Illustrations of stretch length (see D.3.3.4.3(a)).
TENSION REINFORCEMENT
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As stated in Section D.5.2.9 anchor reinforcement strength may be used instead of concrete breakout strength (ϕNcbg) if the following conditions are met:

- Anchor reinforcement must be developed on both sides of the breakout surface.
- ϕ = 0.75
- Reinforcement should be placed as close to the surface as possible.
- Reinforcement consists of stirrups, ties, or hairpins.
- Reinforcement must be less than 0.5*hef from the anchor centerline.
- Research only done with #5 bars and smaller.
- It is good for the anchor reinforcement to enclose the surface reinforcement.
- It is generally limited to cast-in anchors.

There are three types of reinforcement given by the Code, namely hooked end, headed ends, and straight bars. Only the third is discussed here.

Conservatively, for normalweight concrete, no coating, and #6 bar or smaller,

\[
ld = \frac{fy*\psi_t*\psi_e*db}{25*\lambda*fc_1^{(1/2)}}
\]

where

- \(ld\) = development length (in.)
- \(\psi_t = 1.3\) for ≥ 12 in. Cast below bars
- 1.0 elsewhere
- \(\psi_e = 1.5\) for epoxy-coated bars with cover less than 3*db and/or clear spacing < 6*db
- 1.2 for other epoxy-coated bars
- 1.0 for no epoxy coating or galvanized
- \(\lambda\) = less than or equal to 0.75 for lightweight concrete
- \(\lambda\) = 1.0 for normalweight concrete

Two perpendicular sections are shown on the following page.
Fig. RD.5.2.9—Anchor reinforcement for tension.
SHEAR REINFORCEMENT

Section D.6.2.9 states the reinforcement should be developed on each side of the breakout surface or enclose the anchor and is developed beyond the breakout surface. If either one of these is true, the strength of the reinforcement may be used instead of $\phi V_n$, the reduced concrete shear strength. The commentary to Section D.6.2.9 gives the following details to be followed:

- Reinforcement should be properly anchored by hairpins (first page following), hooked bars (second page following), or by stirrups or ties.
- The hairpins should be in contact with the anchor, and as close to the surface as possible.
- Research on hairpins was performed on #5 or smaller bars, larger bars with increased bend radii have decreased effectiveness.
- Reinforcement can also consist of stirrups and ties enclosing the edge reinforcement, and must be placed as close to the anchors as possible. This reinforcement must be spaced less than both $0.5*ca_1$ and $0.3*ca_2$ from the anchor centerline. It must be developed on both sides of the breakout surface.
- Since the anchor reinforcement is below the source of the shear, the force in the anchor will be larger than the shear force. This may be seen by taking the sum of the moments of the shear and anchor forces about a point inward of the anchor force. Because the moment arm is shorter for the anchor force, it will be greater for balance of moments. A third force, in the same direction as the applied shear, must also be present for balance of forces.
- $\phi = 0.75$ for shear models
Fig. RD.6.2.9(b)—Edge reinforcement and anchor reinforcement for shear.
ACI FAILURE MODES
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This section provides a description of each of the eight (8) failure modes set forth in Appendix D, namely four (4) tensile, one (1) bonding, and three (3) shear. The required definitions and citations are shown in Appendices 1 and 2.

<table>
<thead>
<tr>
<th>LOADING</th>
<th>LABEL</th>
<th>ACI 318-11 SECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td>(i) Steel Failure</td>
<td>D.5.1</td>
</tr>
<tr>
<td></td>
<td>(ii) Pullout</td>
<td>D.5.3</td>
</tr>
<tr>
<td></td>
<td>(iii) Concrete Breakout</td>
<td>D.5.2</td>
</tr>
<tr>
<td></td>
<td>(iv) Concrete Splitting</td>
<td>D.5.8</td>
</tr>
<tr>
<td></td>
<td>(v) Side-Face Blowout</td>
<td>D.5.4</td>
</tr>
<tr>
<td></td>
<td>(vi) Bond Failure</td>
<td>D.5.5</td>
</tr>
<tr>
<td>Shear</td>
<td>(i) Steel Failure</td>
<td>D.6.1</td>
</tr>
<tr>
<td></td>
<td>(ii) Concrete Pryout</td>
<td>D.6.3</td>
</tr>
<tr>
<td></td>
<td>(iii) Concrete Breakout</td>
<td>D.6.2</td>
</tr>
</tbody>
</table>
1. **STEEL STRENGTH OF ANCHOR IN TENSION**

The strength of the anchor itself in tension is a function of net anchor diameter, ultimate strength, and capacity reduction factor ($\phi$). The loads, in turn, are increased by a load factor, depending on the most restrictive load combination specified by the governing code. This is the strength design method.

If the ductile failure is requested (material has minimum 14% increase in length and minimum 30% reduction in area at tensile failure), all the other tensile failure modes must have higher allowable strengths so that steel tensile failure governs.

A material commonly used for anchor bolts is ASTM F1554, Grade 36. This specification covers hooked, headed, threaded and nutted rods.

**Appendix D requires**

- $N_{sa} = A_{se,n} \cdot f_{uta}$ where
- $N_{sa} =$ nominal strength of single anchor, lbf
- $A_{se,n} =$ bolt dia., $0.7854 \cdot (D - 0.9743/n)^2$
- $D =$ nominal diameter, inches
- $n =$ number of thread turns per inch
- $f_{uta} =$ specified tensile stress, psi
- $\phi =$ 0.75

Yield stress is 36 ksi and ultimate stress varies from 58 to 80 ksi. As noted in Reference 6, two types of rods are used, threads formed by rolling or cutting. Both have the same roots, so that the root area used by each in the AISC method (Reference 7) is not changed.

For thread forming by rolling, the rod initial diameter is, for a nominal 1” diameter bolt, is 0.9067”, while that of the rod for thread cutting is 0.9755”. This leads to the following comparison:
Rolled Thread | Quantity | Cut Thread
58 ksi Ultimate | | 80 ksi Ultimate
----------------- | -------- | ---------------------
Π*(.9067)^2*futa/4 | Π*d^2*futa/4 | Π*(.9755)^2*futa/4
37.499 kip | Nsa | 59.791 kip
28.124 kip | fNSa | 44.843 kip

Appendix D requires:

Nsa = .7854*(D−.9743/nt)^2*futa

Nsa = nominal tensile capacity based on steel alone
D = nominal anchor diameter = 1 inch
nt = number of threads per inch, 8 for 1 in. dia.

Nsa = 35.133 kip

ϕNsa = 0.75*35.133
= 26.350 kip

To ensure ductility for threads cut, rather than rolled, for the highest ultimate strength, ϕN for the other capacities must exceed 44.843 kip, not 26.350 kip, as it would be if the spread in rod sizes and ultimate strengths is neglected. This, however, is not required by Appendix D.

A second specification is for headed studs, i.e., threadless headed rods fillet welded to a steel plate, is the AWS D1.1 Section 7 (Ref. 8). futa = 65000 psi

<table>
<thead>
<tr>
<th>Shank dia.(in.)</th>
<th>Head dia.(in.)</th>
<th>Head thk.(in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>1</td>
<td>9/32</td>
</tr>
<tr>
<td>5/8</td>
<td>1-1/4</td>
<td>9/32</td>
</tr>
<tr>
<td>3/4</td>
<td>1-1/4</td>
<td>3/8</td>
</tr>
<tr>
<td>7/8</td>
<td>1-3/8</td>
<td>3/8</td>
</tr>
<tr>
<td>10</td>
<td>1-5/8</td>
<td>1/2</td>
</tr>
</tbody>
</table>
2. CONCRETE BREAKOUT STRENGTH OF ANCHOR IN TENSION

The sketch above shows the basic model for concrete breakout in tension, a brittle failure which occurs before yielding of the anchor steel, if so designed. The basis of the design procedure is the Concrete Capacity Design (CCD) method, introduced in the Code Background Paper shown as Reference 9. This method assumes the shape of the fractured area is an inverted pyramid, as shown above, together with a plan view.

The model used for basic breakout strength is one where \( c_1 = c_2 = c_3 = c_4 = 1.5 \times \text{hef} \), hef equaling embedment distance, and \( s_1 = s_2 = 0 \). This means the failure planes are oriented at \( \arctan(1.0 \times \text{hef}/1.5 \times \text{hef}) = 33.7^\circ \).

For example, assume \( \text{hef} = 12 \) inches, then:

- Area each triangular side = 389.4 in.\(^2\)
- Surface area (i.e., plan view area) = 1296.0 in.\(^2\)
- Ratio of total side area to plan area = 1.20185

The plan areas are determined by \( 1/2 \) the absolute value of the cross-product of two of the side vectors. For example, to determine the area of the plane formed by nodes 0, 1, and 2, where \( \mathbf{01} \) = the vector from 0 to 2 and \( \mathbf{02} \) = the vector from 0 to 2,

\[
\text{Area} = \frac{1}{2} |\mathbf{01} \times \mathbf{02}|.
\]

See Appendix 3 for a program, Anc.c, for calculating the areas of these planes, for the interested reader.

ACI 318-11 uses the plan view in determining capacity, not the sum of the areas of the four inclined planes.

The design sequence to determine the concrete breakout in tension capacity follows as thirteen (13) steps.

1. Specify anchor type, concrete weight, \( c_1 \text{-} c_4 \), \( s_1 \text{-} s_2 \), hef, \( f_c' \), da (anchor diameter), and whether or not the concrete is cracked.

2. Check "Concrete Splitting", "Group Effects", and
"General Requirements ", for conformance. Find cac, cna and Φ.

(3) \( kc = 24 \) for cast-in, 17 for post-installed
\( \lambda = .75, .85, 1.0 \) (all-lightweight, sand-lightweight and normal weight concrete)
\( \lambda a = \lambda \), cast-in, .8\( \lambda \) for adhesive anchor

(4) If : three(3) or more edge distances (ca1-ca4)
are less than 1.5*hef, the value of hef used in
Steps 5-7 and 10-12 is the larger of ca,max/1.5
and s,max/3.
Else: continue.

(5) Calculate \( Nb = \) basic breakout strength of single anchor
\[ Nb = kc*\lambda a*(fc')^{(1/2)}*hef^{(1.5)} \]
If : type = cast-in headed studs or bolts
and 11 in. <= hef <= 25 in.
\( Nb \) may also be taken as
\[ 16*\lambda a*(fc')^{(1/2)}*hef^{(5/3)} \]
Else : continue.

(6) \( \psi_{ec,n} = \) modifier for eccentricity of loads
\[ \psi_{ec,n} = 1/(1*2*en'/(3*hef)) <= 1.0 \]
where
\( en' = \) distance from tension centroid of a group
of anchors to the resultant tension load

(7) \( \psi_{ed,n} = \) modifier based on edge conditions
If : ca,max >= 1.5*hef, then \( \psi_{ed,n} = 1.0 \)
Else : \( \psi_{ed,n} = 0.7 + 0.3*ca,min/(1.5*hef) \)

(8) \( \psi_{c,n} = \) modifier based on cracked state
\( \psi_{c,n} = 1.0, \) cracking at service loads
\( \psi_{c,n} = 1.25, \) cast-in, no cracking
\( \psi_{c,n} = 1.4, \) post-installed, no cracking

(9) \( \psi_{cp,n} = \) modifier for post-installed anchors
designed for uncracked concrete without supplementary reinforcement.
If: ca,min >= cac, then \( \psi_{cp,n} = 1.0 \)
Else : \( \psi_{cp,n} = \frac{ca_{min}}{cac} \) and \( \geq 1.5 \)hef/cac

(10) \( Anco = \) projected concrete failure of single anchor, not limited by edge distance or spacing
\( Anco = (1.5 \times hef + 1.5 \times hef) \times (1.5 \times hef + 1.5 \times hef) \)
\( Anco = 9 \times hef^2 \)

(11) \( Anc = \) projected failure area, \( \leq 1 \) anchor
\( Anc = (ca_1 + s_1 + ca_3) \times (ca_2 + s_2 + ca_4) \)

(12) \( Ncb = \) nominal concrete breakout strength, one anchor
\( Ncb = \frac{(Anc/Anco) \times \psi_{ed,n} \times \psi_{c,n} \times \psi_{cp,n} \times Nb}{\psi_{ed,n} \times \psi_{c,n} \times \psi_{cp,n} \times Nb} \)

(13) \( Ncbg = \) nominal concrete breakout strength, > 1 anchor
\( Ncbg = \frac{(Anc/Anco) \times \psi_{ec,n} \times \psi_{ed,n} \times \psi_{c,n} \times \psi_{cp,n} \times Nb}{\psi_{ec,n} \times \psi_{ed,n} \times \psi_{c,n} \times \psi_{cp,n} \times Nb} \)

3. PULLOUT STRENGTH OF CAST-IN, AND POST-INSTALLED EXPANSION ANCHORS IN TENSION

In this category, the 33.7° breakout cone does not develop, and bond is lost between the anchor shaft and concrete. Adhesive anchors are not covered in this section, and group effects are not considered. For a single headed bolt, pullout strengths are directly proportional to the head area and concrete strength.

For a single hooked bolt, the pullout strength is directly proportional to the concrete strength, the bolt diameter, and the distance from the inner bolt surface to the outer tip of the L- or J- bolt. Expansive anchors are not calculated by formula, but must be tested to ACI 355.2. See, for example, Reference 2, for analysis and design information.

Definitions for this section:
\( Abrg = \) net bearing area, i.e., gross head or washer
plate area minus maximum shaft diameter, in.^2

\( \text{eh} \) = distance from inner surface of J- or L- bolt to outer tip, in.

\( \text{Npn} \) = nominal pullout strength in tension, 1 anchor,
\( \psi_{c,p} = 1.0 \) if cracking at service load levels
\( = 1.4 \) if no cracking at service load levels

\( \text{Npn} = 8*\psi_{c,p}*\text{Abrg}*f_{c}' \) for headed stud or headed bolt

\( \text{Npn} = 0.9*\psi_{c,p}*f_{c}'*\text{eh}*\text{da} \) for J- or L- bolt and
\( 3*\text{da} \leq \text{hef} \leq 4.5*\text{da} \) (only values tested)
See “General Requirements” for \( \phi \).

Assume that ductility is required for a J- or L-bolt. Then \( \text{Nsa} \leq \text{Npn} \). Assume cracked concrete, \( f_{c}' = 4000 \) psi, and Grade 36 steel so that \( f_{\text{uta}} = 58000 \) psi. Thus
\( (\Pi*D^2/4)*f_{\text{uta}} \leq 0.9*f_{c}'*4.5*D \)

Rearranging and solving for \( D \),
\( D \leq 16.2*f_{c}'/((\Pi*f_{\text{uta}}) = 0.356 \) in. (5/16 in.)

It is seen neither J- or L- bolts are suitable where ductility is needed.

**Headed Studs** – Defined in AWS D1.1, Chapter 7

<table>
<thead>
<tr>
<th>Shank Dia. (in.)</th>
<th>Head Dia. (in.)</th>
<th>Thk (in.)</th>
<th>Abrng (in.^2)</th>
<th>8*f_{c}'*Abrng (lbf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.500</td>
<td>1.000</td>
<td>.28125</td>
<td>.58905</td>
<td>18850</td>
</tr>
<tr>
<td>0.625</td>
<td>1.250</td>
<td>.28125</td>
<td>.15625</td>
<td>29452</td>
</tr>
<tr>
<td>0.750</td>
<td>1.250</td>
<td>.37500</td>
<td>.12500</td>
<td>25133</td>
</tr>
<tr>
<td>0.875</td>
<td>1.375</td>
<td>.37500</td>
<td>.12500</td>
<td>28274</td>
</tr>
<tr>
<td>1.000</td>
<td>1.625</td>
<td>.50000</td>
<td>.15625</td>
<td>41235</td>
</tr>
</tbody>
</table>

**Heavy Hex Bolts and Nuts**

The the plan dimensions of heavy hex bolts and nuts are equal, so they obtain the same net bearing area, \( \text{Abrg} \). The nuts, however, are much thicker than the bolt heads. Another difference is the bolts are
usually Grade 36 steel with an ultimate stress of 58 ksi, while the nuts steel has a proof load (no distortions after removal of force) of 100 ksi, as per ASTM A563 (Reference 11 )

Tensile and Shear Strengths

Reference 12, “Unified Inch Screw Threads”, is the American standard for bolt and nut thread dimensions. UNC (UN Coarse) is the series used for inside threads (nuts) and UNCR for external threads (bolts). The profile shown below defines the location of the basic dimensions.

Tensile strength is the same as that as tensile steel strength, i.e., futa*Ase,n.

The shear strengths, on the other hand, are different for the internal and external threads as,

Shear strength internal threads = .55*fult*ASn
Shear strength external threads = .55*fult*ASs

\[
\begin{align*}
ASn &= 3.1416*n*LE*d1,\text{min}*(1/2n+.57735*(d1,\text{min}-D_2,\text{max})) \\
ASs &= 3.1416*n*LE*D1,\text{max}*(1/2n+.57735*(d2,\text{min}-D1,\text{max})
\end{align*}
\]

where, in inches,

d1,\text{min} = minimum major diameter of external threads
d2,\text{min} = minimum pitch diameter of external threads
D1,\text{max} = maximum minor diameter of internal threads
D2,\text{max} = maximum pitch diameter of internal threads
LE = length of engagement
n = number of threads/inch

Two (2) examples are chosen, nom. 1/2” and 2” dia.

<table>
<thead>
<tr>
<th>Dia.(in.)</th>
<th>Tensile Bolt Threads</th>
<th>Nut Threads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(kip)</td>
<td>Shear (kip)</td>
</tr>
<tr>
<td>1/2</td>
<td>8.233</td>
<td>13.240</td>
</tr>
<tr>
<td>2</td>
<td>144.897</td>
<td>220.550</td>
</tr>
</tbody>
</table>

Washer Design

A design may require a larger Abrg than that available from the hex nut or hex head. In this case, a washer is tack welded to the embedded nut or head. Let d0 = shank diameter
d1 = equivalent diameter of nut or head

d2 = washer o.d.

A = plan area of heavy hex nut or head (hexagon)

thk = thickness of washer (to be designed)

(1) Find required Abrg \(\phi\ast8\ast\text{Abrg}\ast f_{c'} = f_{\text{acity}}\)

(2) Find plan area of hexagon

\[ A = 1.5\ast F^2\ast \tan(30^\circ) \text{ where} \]

\( F = \text{flat-opposite flat distance, in.} \)

\[ \text{Abrg, nut or head} = A - (\Pi/4)\ast d0^2 \]

If : Abrg, nut or head \(\geq\) Abrg, required, exit.

Else : Continue.

(3) Find outside diameter of washer to provide sufficient Abrg.

\[ (\Pi/4)\ast d2^2 = (\Pi/4)\ast d0^2 + \text{Abrg, required} \]

Solve for \(d2\).

(4) Find equivalent diameter of nut or head

Solve \(A = (\Pi/4)\ast d1^2\) for \(d1\)

(5) Find \(N_{s\alpha} = \text{capacity of single anchor} = f_{\text{uta}}\ast A_{s\alpha}, n_e\)

(6) Find load on washer \((d2^2-d1^2)/(d2^2-d0^2)\ast N_{s\alpha}\)

(7) The cantilever load in (6) is spread over a distance of \(\Pi\ast(d2+d1)/2\) with a moment arm of \((d2-d1)/4\).

Moment = force in (6)\ast moment arm

(8) \(Z = \text{plastic section modulus} = \text{length of strip beyond } d1\ast \text{thk}^2/4\)

(9) Moment = stress\ast Z, where stress = f_y

Solve (9) for \(\text{thk}\).

4. CONCRETE SIDE-FACE BLOWOUT STRENGTH OF HEADED ANCHORS IN TENSION

The single anchor and group anchor formulas in this category cover the situation where embedment length is much greater than the nearest edge distance, with a ratio of 2-1/2 times, i.e., \(\text{hef} \geq 2.5\ast \text{ca1}\). These requirements are applicable to headed anchors, which are usually cast-in.

\(N_{sb} = \text{nominal side-face blowout strength, 1 anchor}\)

\(N_{sb\alpha} = \text{nominal side-face blowout strength, > 1 anchor}\)

\(s = \text{distance between outer anchors along the edge}\)
Three (3) cases exist for single anchors:

1. \( ca2 \geq 3\times ca1 \)
   \[ Nsb = 160\times ca1\times (Abrg)^{(1/2)}\times \lambda a\times (fc')^{(1/2)} \]

2. \( 1 \leq ca2/ca1 < 3.0 \)
   Multiply \( Nsb \) above by \( (1+ca2/ca1)/4 \)

3. \( ca2 < ca1 \rightarrow \) interchange roles of \( ca2 \) and \( ca1 \)

For multiple headed anchors with \( hef \geq 2.5\times ca1 \) and \( s \) less than \( 6\times ca1 \),
\[ Nsbg = (1+s/6\times ca1)\times Nsb \]

5. **BOND STRENGTH OF ADHESIVE ANCHORS IN TENSION**

This category includes only adhesive anchors and are analyzed both as single anchors and groups of anchors. Appendix D characterizes the minimum bond stress as:

<table>
<thead>
<tr>
<th>Installation Environment</th>
<th>Moisture at Install.</th>
<th>Peak Service Temp.</th>
<th>( \tau_{cr} ) psi</th>
<th>( \tau_{uncr} ) psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outdoor</td>
<td>Dry to 175°</td>
<td>200</td>
<td>650</td>
<td></td>
</tr>
<tr>
<td>Indoor</td>
<td>Dry 110°</td>
<td>300</td>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

These values may only be used if:

(a) Tested to ACI 355.4
(b) Holes drilled only with rotary impact drills or rock drills
(c) At installation concrete strength \( \geq 2500 \) psi
(d) Concrete age at installation \( \geq 24 \) days
(e) Temperature at installation \( \geq 50°F \)

Reference 13 gives four major categories of factors influencing bond strength:

(a) In-Service
   - Possibility of creep at high temperatures
● In-service moisture – can degrade adhesion by moisture penetration into adhesive to soften it, between adhesive and substrate destroying bond, and penetrating into porous substrates causing swelling and detrimental movement

● Freeze-thaw

(b) Adhesive
● Curing time when first loaded – 24 hour/7 day loading = 81% of bond strength
● Bond line thickness – the smaller this dimension, the lesser potential for creep

(c) Installation
● Hole orientation – vertical and upwardly inclined holes are difficult to fill with adhesive
● Hole drilling – diamond-core drills not recommended as they produce a very smooth-sided hole, as increased surface roughness increases bond strength
● Hole cleaning – non-metallic brushes should be used as metallic brushes tend to polish the side of the hole

(d) Concrete
● Harder coarse aggregates produce higher bond strengths
● Cracked concrete – significantly reduces bond strength

Analysis of the bond strength uses the following terms:
Ana = projected influence area of one or more anchors, for calculation of bond strength in tension, in.^2
Anao = projected influence area of a single adhesive anchor, for calculation of bond strength in tension, not limited by edge distance or spacing, in.^2
cac = critical edge distance controlled by concrete breakout or bond, uncracked concrete, no supplementary reinforcement
ca, min = minimum distance from anchor bolt center to concrete edge, in.

cna = projected distance from center of an anchor shaft on one side of the anchor required to develop the full bond strength of an adhesive anchor, in.

Na = nominal bond strength in tension of a single anchor, lbf

Nag = nominal bond strength in tension of a group of adhesive anchors, lbf

Nba = basic bond strength of a single adhesive anchor in cracked concrete, lbf

ψcp,na = modifier for uncracked concrete, no supplementary reinforcement
if : ca, min >= cac, ψcp,na = 1
else : ψcp,na = ca, min/cac but not less than cna/cac

ψec,na = modifier for eccentricity
= 1/(1+2*e’n/3*hef)

ψed,na = modifier for edge distance
if : ca, min >= cna, ψed,na = 1
else : ψed,na = 0.7+0.3*(ca, min/can)

τcr = characteristic bond stress of adhesive anchor in cracked concrete, psi
= 200 psi, outdoor, 175 °F max.

τuncr = characteristic bond stress of adhesive anchor in uncracked concrete
= 650 psi, outdoor, 175°F max.

λ = .75, .85, 1 for all-lightweight concrete, sand-lightweight concrete, and normalweight concrete, respectively

λa = 1.0λ for cast-in, 0.8λ for concrete failure, adhesive anchor, 0.6λ for concrete bond failure, adhesive anchor

Now using the definitions above and illustrations
From tension concrete breakout, the capacity equations may now be solved:
 cac = 2*hef or tests to ACI 355.4.
cna = 10*da*(τuncrack/1100)^(1/2)
Nba = λa*τcr*Π*da*hef
if anchor designed to resist sustained loads:
0.55*ϕ*Nba > = Nua,a
else : continue

Na = (Ana/Anao)*ψed,na*ψcp,na*Nba
Nag = (Ana/Anao)*ψec,na,ψed,na*ψcp,na*Nba
See “General Requirements” for ϕ.

6. STEEL STRENGTH OF ANCHOR IN SHEAR

In this category headed studs are welded to a base plate, developing a higher steel strength in shear than headed bolts, hooked bolts, or post-installed anchors by themselves, due to the fixity given by the welds between stud and base plate.

\[ A_{se,v} = 0.7854 \times (D-0.9743/nt)^2 \]
\[ \phi = 0.65 \]

<table>
<thead>
<tr>
<th>Type of Anchor</th>
<th>Vsa</th>
</tr>
</thead>
<tbody>
<tr>
<td>cast-in headed stud anchor</td>
<td>1.0<em>Ase,v</em>fult</td>
</tr>
<tr>
<td>cast-in headed bolt and hooked anchors</td>
<td>0.6<em>Ase,v</em>fult</td>
</tr>
<tr>
<td>and for post-installed anchors where sleeves do not extend through the shear plane</td>
<td></td>
</tr>
<tr>
<td>post-installed anchors where sleeves extend through the shear plane</td>
<td>ACI 355.2 tests</td>
</tr>
</tbody>
</table>

where anchors are used with built-up grout pads, multiply values above by 0.80

7. CONCRETE BREAKOUT STRENGTH IN SHEAR

The formulas in this section are based on a 33.7° breakout angle, and use fracture mechanics theory. Breakout in shear depends on
(1) number of anchors
(2) spacings
(3) edge distances
(4) thickness of concrete

The following terms used in capacity calculations are:
Avc = projected failure area for shear, in.^2
Avco = projected failure area for shear, 1 anchor, not limited by edge or concrete depth
cal = distance from anchor at surface perpendicular to edge (vector $05 \perp 12$ on the following page), in.
ha = concrete thickness, in.
le = load bearing length for anchor in shear, in.
le $\leq 8*da$
Vb = basic concrete breakout strength in shear, one anchor, cracked concrete, lbf
Vcb = nominal concrete breakout strength in shear, one anchor, lbf
Vcbg = nominal concrete breakout strength in shear, greater than one anchor, lbf

A design sequence follows:

(1) Using the diagrams on parts 2. and this part find cal, ca2, ca4, ha, s1, and s2.
   Note that:
   In tension cal = min. edge distance, and in shear cal = distance to edge in direction of shear.
   Height of vertical block = ha if ha < 1.5*cal
   Else height of vertical block = 1.5*cal
   Width of vertical block left distance the lesser of 1.5*cal and ca2 and right distance the lesser of ca4 and 1.5*cal.
   If s $\geq$ cal, evaluate cases 1 and 2 on second diagram in this section.
   Else evaluate case 3.

(2) $\psi_{ec,v} =$ modification factor for eccentricity, $1/(1+2*ev'/3*cal)$

(3) $\psi_{ed,v} =$ modification factor for edge distance,
   if : ca2 and ca4 $\geq$ 1.5*cal, $\psi_{ed,v} = 1$
   else : $\psi_{ed,v} = 0.7+0.3*(lower of ca2,ca4/1.5*cal$

(4) $\psi_{c,v} =$ modification factor for cracks =
   $= 1.4$ no cracking
1.2 cracking and #4 bar between anchor And edge
1.0 cracking and bar smaller than #4 bar

$$\psi_{h,v} = \text{modification factor for concrete thickness}$$

$$= (1.5*ca1/ha)^{(1/2)} \text{ where } ha \leq 1.5*ca1$$

and not less than 1

$$Avco = 4.5*ca1^2$$

See the diagram this section for Calculation of Avc.

Avc shall not exceed number of anchors*Avco

For a single anchor:

\[ \begin{array}{cccc}
ca2 & ca4 & ha & Avc \\
\geq 1.5ca1 & \geq 1.5ca1 & \geq 1.5ca1 & 1.5*ca1*(3*ca1) \\
\geq 1.5ca1 & \geq 1.5ca1 & < 1.5ca1 & ha*(3*ca1) \\
\geq 1.5ca1 & < 1.5ca1 & \geq 1.5ca1 & 1.5*ca1*(1.5*ca1+ca4) \\
\geq 1.5ca1 & < 1.5ca1 & < 1.5ca1 & ha*(1.5*ca1+ca4) \\
< 1.5ca1 & \geq 1.5ca1 & \geq 1.5ca1 & 1.5*ca1*(1.5*ca1+ca2) \\
< 1.5ca1 & \geq 1.5ca1 & < 1.5ca1 & ha*(1.5*ca1+ca2) \\
< 1.5ca1 & < 1.5ca1 & \geq 1.5ca1 & 1.5*ca1*(ca2+ca4) \\
< 1.5ca1 & < 1.5ca1 & < 1.5ca1 & ha*(ca2+ca4) \\
\end{array} \]

\[ V_b = \text{smaller of:} \]

\[ 7*(le/do)^{(0.2)}*do^{(1/2)}*(fc')^{(1/2)}*ca1^{(1.5)} \]

and

\[ 9*\lambda_a*(fc')^{(1/2)}*ca1^{(1.5)} \]

\[ V_{cb} = (Avc/Avco)*\psi_{ed,v}*(\psi_{c,v})*V_{b} \]

\[ V_{cbg} = (Avc/Avco)*\psi_{ec,v}*(\psi_{ed,v})*V_{b} \]

See GENERAL REQUIREMENTS for $\phi$. 
Case 1: One assumption of the distribution of forces indicates that half of the shear force would be critical on the front anchor and the projected area. For the calculation of concrete breakout, c_{st,1} is taken as c_{st,1}.

\[ A_{vc} = 2(1.5c_{st})h_a \]
\[ c_{st} < 1.5c_{st} \]

\[ A_{vc} = 2(1.5c_{st,1})h_a \]
\[ h_a < 1.5c_{st} \]

Case 2: Another assumption of the distribution of forces indicates that the total shear force would be critical on the rear anchor and its projected area. Only this assumption needs to be considered when anchors are welded to a common plate independent of s. For the calculation of concrete breakout, c_{st} is taken as c_{st,2}.

Note: For s ≥ c_{st,1}, both Case 1 and Case 2 should be evaluated to determine which controls for design except as noted for anchors welded to a common plate.

Case 3: Where s < c_{st,1}, apply the entire shear load V to the front anchor. This case does not apply for anchors welded to a common plate. For the calculation of concrete breakout, c_{st} is taken as c_{st,1}.

\[ A_{vc} = 2(1.5c_{st,1})h_a \]
\[ h_a < 1.5c_{st} \]
8. CONCRETE PRYOUT STRENGTH OF ANCHOR IN SHEAR

This may govern if the anchor is short, and is reflected in the design equations by halving the capacity if hef, the effective embedment depth, is less than 2-1/2 inches.

\[ K_{cp} = \text{coefficient for pryout strength} \]
\[ = 1.0 \text{ for } hef < 2.5 \text{ inches and} \]
\[ = 2.0 \text{ for } hef \geq 2.5 \text{ inch} \]

\[ V_{cp} = \text{nominal pryout strength, 1 anchor, lbf} \]
\[ V_{cpg} = \text{nominal pryout strength, > 1 anchor, lbf} \]

\[ V_{cp} = k_{cp}N_{cp} \text{ where:} \]
\[ = \text{use } N_{cb} \text{ (part 2) for cast-in, expansion, or undercut anchors and the lesser of } N_{cb} \text{ (part 2) and } N_{a} \text{ (part 5) for adhesive anchors} \]

\[ V_{cpg} = k_{cp}N_{cpg} \text{ where:} \]
\[ = \text{use } N_{cbg} \text{ (part 2) for cast-in, expansion, or undercut anchors and the lesser of } N_{cbg} \text{ (part 2) and } N_{ag} \text{ (part 5) for adhesive anchors} \]

See GENERAL REQUIREMENTS for \( \phi \).

9. INTERACTION OF TENSILE AND SHEAR FORCES

An interaction formula for the ratio of factored load to nominal strength times the appropriate capacity reduction factor is given for both tension and shear ratios greater than 0.2. It is called the trilinear interaction approach, although any other formula verified by test data may be used.

Determine load factors from applicable Code.

\[ N_{ua} = \text{factor*service load} \]
\[ V_{ua} = \text{factor*service noad} \]

Using the lowest values of \( \phi N_{n} \) and \( \phi W_{n} \) for all combinations of parts 1 through 8,
From the Appendix D body,
if  \( \frac{V_{ua}}{\phi V_n} \leq 0.2 \), use \( \phi N_n \geq N_{ua} \)
and if
if  \( \frac{N_{ua}}{\phi N_n} \leq 0.2 \), use \( \phi V_n \geq V_{ua} \)
else
\( \frac{N_{ua}}{\phi N_n} + \frac{V_{ua}}{\phi V_n} \leq 1.2 \)

From the Appendix D commentary,
(\( \frac{N_{ua}}{\phi N_n} \))^{5/3} + (\( \frac{V_{ua}}{\phi V_n} \))^{5/3} \leq 1.0

**BASE PLATES AND ANCHOR BOLTS**

Many applications of anchor bolts involve the connection of a steel base plate to a concrete foundation. Here we discuss three considerations necessary for design, namely anchor rod force and plate thickness, shear transfer, and practical considerations.
Reference 14, the AISC Design Guide for Base Plate and Anchor Rod Design is the basis for the development here.

**ANCHOR ROD AND BASE PLATE THICKNESS**

The following definitions aid in the calculation of anchor bolt force and plate thickness are used in conjunction with the diagram on the following page. This analysis uses the LRFD method.

- \( A_1 \) = area of base plate
- \( A_2 \) = area of concrete foundation surface
- \( B \) = plate dimension perpendicular to the plane of bending
- \( d \) = depth of column
- \( e \) = eccentricity of load
- \( e_{crit} \) = maximum eccentricity for no tension
- \( f \) = distance from the tension anchor bolt to the center of the base plate
- \( f_y \) = yield stress of steel
- \( f_{c'} \) = specified concrete compressive strength
- \( f_{pmax} \) = maximum bearing pressure on concrete
- \( M_r \) = factored moment in column
- \( m \) = distance from column flange outer surface to
near base plate outer edge

$N = \text{plate dimension parallel to plane of moment}$

$Pr = \text{factored vertical force in column}$

$q_{\text{max}} = \text{maximum bearing force per unit length in the N direction. This is the basic assumption of this method, i.e., that the compressive force is constant throughout the contact area}$

$t_{f} = \text{flange thickness of column}$
Column moment base using stool

Base plate with large moment
reqd = required base plate thickness with two values, one at the compressive side and one at the tension side

Tu = ultimate tension load on member

x = used in calculation of base plate yield on tension end

Y = length of compression pressure parallel to plane of moment

Now this procedure may be used:

1. Establish A1, A2, B, d, f, fc', Fy, N, service loads, tf
2. Pr = 1.2*DL force + 1.6*LL force
   Mr = 1.2*Dl moment + 1.6*LL moment
3. e = Mr/Pr
4. fbmax = \( \phi \cdot 0.8f \cdot fc' \cdot (A2/A1)^{1/2} \) A2 \( \leq 2 \cdot A1 \)
   \( \phi = 0.65 \)
5. Find qmax = fbmax*B
6. Is \((f+N/2)^2 > = 2 \cdot Pr \cdot (f+e)/qmax \) ?
   If so, continue
   If not, design with parameters used is not possible, base plate must probably be larger
7. Find \( ecrit \) N/2 \( - Pr/(2 \cdot qmax) \)
8. If e > ecrit, need tension anchor
   If not, go to step (15)
9. Solve for Y
   Y = \((f+N/2) \pm \sqrt{(f+N/2)^2 - 2 \cdot Pr \cdot (f+e)/qmax})^{1/2}\)
10. Tu = qmax*Y - Pr
11. m = \((N-.95*d)/2 \)
12. If Y > = m :
    reqd for compression end = 1.5\( m \cdot (fmax/Fy)^{1/2} \)
    If Y < m :
    reqd for compression end = 2.11\( (fmax \cdot (m-Y/2)/Fy)^{1/2} \)
13. x = f - d/2 + tf/2
14. reqd for tension end = 2.11\( (Tu*x/(B*Fy))^{1/2} \)
15. End of process – proceed to design anchor
16. Tension anchor not required
SHEAR TRANSFER

There are four methods of shear transfer from a column to the foundation. They are friction between the base plate and concrete bearing of the column and base plate and/or shear lug, shear by the anchor bolt strength without hairpin reinforcement, and shear through the anchor bolts to hairpin reinforcement. The latter two are shown above.

FRICITION

This method depends upon the magnitude of the vertical load above the base plate. This load may just be the dead load. Here we have

\[ \phi V = \mu P \leq 0.2\phi f_{c'} Ap \]

where

- \( V \) = factored shear load = 1.6*service load
- \( \phi \) = capacity reduction factor = 0.65 in bearing
- \( \mu \) = coefficient of friction, .55 for steel on grout, 0.7 for steel on concrete
- \( Ap \) = base plate area
- \( P \) = minimum vertical load

SHEAR LUG

This method is shown by the top diagram on the page following. Also shown on this sheet are elevations of possible hairpin placements.

The following definitions are needed in the calculations.

- \( Ab \) = area of lug contacting concrete in compression
- \( Av \) = projected area in vertical plane used to calculate concrete tensile strength to resist concrete shear failure
- \( F_{exx} \) = specified tensile strength of weld filler material
- \( G \) = thickness of grout layer between the bottom of the base plate and the top surface of concrete foundation
Shear Lug Detail

Column Embedment Detail

Transfer of base shears through bearing

Typical detail using hairpin bars

Alternate hairpin detail
\[ H = \text{total lug height} \]
\[ H - G = \text{effective lug height} \]
\[ L = \text{length of lug, placed in the middle of the pier, perpendicular to the direction of shear} \]
\[ Mu = \text{ultimate moment load at base of lug} \]
\[ L = \text{length of lug} \]
\[ t = \text{lug thickness} \]
\[ t_{pl} = \text{base plate thickness, take equal to } t \]
\[ V_u = \text{factored design shear} \]
\[ W_{pier} = \text{lug width (depth)} \]
\[ Z = \text{plastic modulus of lug, weak direction} \]
\[ \Phi = \text{capacity reduction factor, depends on process} \]

The criteria to be evaluated here are the compressive strength of the concrete in front of the lug, the shear strength evaluated on the projection of a plane at 45° from the bearing area of the lug to the face of the pier (not including the bearing area of the lug) and the strength of the weld from the lug to the base plate.

1. Collect information on baseplate plan dimensions, \( f_{c'} \), \( F_{exx} \), \( G \), service shear load, steel grade
   Assume \( L, G, H, \) and \( t \), to be verified.

2. Find \( V_u = 1.6 \times \text{service shear load} \)

3. Check that the effective lug compressive area, \( A_b = L \times (H - G) \), is sufficient
   Required \( A_b \geq V_u / (\Phi \times 0.85 \times f_{c'}) \), \( \Phi = 0.75 \)

4. The available shear area is approximated as a rectangle with width equal to \( L \) times \( L/2 - \text{the effective area of the lug} \)
   \( A_v = L \times L/2 - L \times (H - G) \)

5. Take the tensile allowable tensile stress as \( 4 \times \Phi \times (f_{c'})^{(1/2)} \). Check that this stress \( A_v > V_u \), \( \Phi = 0.75 \)

6. Calculate moment at connection of lug to base plate
   \[ Mu = V_u \times (G + (H - G)/2) = V_u \times (H + G)/2 \]

7. Find required plastic modulus and lug
thickness
\[ Z = \frac{Mu}{(\phi*fy)}, \quad \phi = 0.90 \]

Check lug thickness
Solve \( Z = I*t^2/4 \) for \( t \)

Find size of welds (one on each side) attaching lug to base plate. There are two shear \( \perp \) loads on the welds, namely \( V_u/2 \) on each side and the moment-caused shear from \( M_u \) above, i.e., \( M_u = Mu/t \), conservatively. Call these weld 1 (both sides) and weld 2 (each side), respectively.

Total shear each side= (weld 1^2 + weld 2^2)^(1/2)

Strength of weld in shear =
\[ (L-2*throat)*throat*\phi*0.60*F_{exx}, \quad \phi = .75 \]
Solve for throat
For a symmetrical fillet weld (height=base) size= \( 2^{(1/2)}*throat \)
Solve for weld size and round up to nearest sixteenth.
Do not use weld-all-around symbol, stopping one weld size each end of lug.

PRACTICAL CONSIDERATIONS
-----------------------------------------

The figure shown below, from Reference 15, “Practical Design and Detailing of Steel Column Base Plates”, provides a very good checklist for column base plates.
FIGURE 1 - SUGGESTED BASE PLATE DETAILS

1. Use square plate and hole pattern dimensions where possible to avoid problems associated with mis-placed anchor bolts, rotated anchor bolt patterns or plates that are accidentally rotated 90 degrees during fabrication.

2. Try to reduce numerous base plate variations by sizing typical plate based on the largest column in a size group (e.g. W10's, W12's or W14's). Reducing the number of variations will reduce the chance for error during erection and fabrication, and allow for simpler verification in the field. Provide maximum edge distance to bolt to allow base plate slotting if bolts are mislocated.

3. When additional bolts are required, add additional holes to make double symmetric bolt patterns. This is useful even if not all holes and bolts are needed. Four bolts is the suggested minimum for any base plate.

4. Anchor bolts should be at least 1" diameter. This is beneficial for erection safety and the anchor bolts are harder to accidentally bend in the field. Specify A307 or A36 material when possible. Both are easier to obtain and weldable.

5. Oversize holes in base plates should be used wherever possible to accommodate anchor bolt placement tolerances.

6. Plate washers with field welds should be used in conjunction with oversize holes to resist nut pull-through and to transfer shear from the base plate to the anchor bolts. Special attention should be directed toward weld access. Plate washer should have hole 1/16" larger than bolt diameter. Welds may not be needed if the column is for "gravity only" and there are no shear forces at the base of the column.

7. Leveling nuts are recommended in lieu of leveling plates or shims for ease of construction, safety and efficiency.

8. The thickness of grout specified should accommodate the leveling nuts and be in proportion to the dimensions of the base plate (for example do not specify 3 inches of grout under a W6 column).

9. Specify an additional bolt extension above the top of the base plate to accommodate bolts that are set too low. Also specify extra threaded length to accommodate bolts set too high.

10. Specify fillet welds whenever possible. Partial penetration welds and complete penetration welds should only be specified when required.

11. Avoid specifying all-around welds. There should be no weld at the ends of the flanges and in the fillet (k region) of the column.

12. If a grout hole is needed, specify the same diameter as the anchor bolt holes to reduce the number of drill bit sizes required during fabrication.
EXAMPLE 1 – SHEAR LUG

See text section “SHEAR TRANSFER, BEARING”

(1) \( f_c' = 4000 \text{ psi} \)
Feex = 70 ksi (E70XX filler weld metal)
\( G = 1-1/2 \text{ inch grout depth} \)
F1554 Grade 36 steel
Pier = 24 in. x 24 in.
Service shear load = 23 kip
Try \( L = 9'' \), \( H = 4'' \), \( t = \text{tp1} = 2'' \)

(2) \( V_u = 1.6*23 = 36.8 \text{ kip} \)

(3) \( A_b = L*(H-G) = 9*(4-1.5) = 22.5 \text{ in.}^2 \)
Compressive strength = \( .85*.65*4.000 = 49.725 \text{ k, o.k.} \)

(4) \( A_v = 24*24/2 - 22.5 = 265.5 \text{ in.}^2 \)

(5) Resisting shear stress \( = 4*.75*4000^{(1/2)}/1000 \)
\( = 0.1897 \text{ ksi} \)
Resisting shear strength = 265.5*.1897
\( = 50.375 \text{ k, o.k.} \)

(6) Moment arm = \( H/2+G/2 = 2.75'' \)
Mu = 36.8*2.75

(7) \( Z = 101.2/(.9*36) = 3.12345 \text{ in.}^2 \)

(8) \( t \) solves to 1.178”, o.k.

(9) Weld 1 shear = 36.8/2 = 18.4 kip (each side)
Weld 2 shear = 101.2/2 = 50.6 kip (+-)
Total weld load, each side = \((18.4^2+50.6^2)^{(1/2)}\)
\( = 53.774 \text{ kip} \)
\((L-2*\text{throat})*\text{throat}*.75*6*70/2^{(1/2)} = 53.774 \)
This quadratic is solved for throat = .1984
Size of symmetric fillet weld = throat*2^{(1/2)}
\( = .2806'' \)
Say 5/16” symmetric fillet weld, 8-3/8” long, each side.
Note: \( t \) and \text{tp1} may be reduced, but weld size increases
EXAMPLE

Single stud, combined tension and shear

Design an embedment using a stud welded to an embedded plate.

Given:
- Edge
  - \( c_1 = 12 \text{ in.} \)
  - \( c_2 = 20 \text{ in.} \)
  - \( h = 18 \text{ in.} \)

- Concrete
  - \( f'_{c} = 4000 \text{ psi} \)

- Stud material (A108)
  - \( f_y = 51 \text{ ksi} \)
  - \( f_{ult} = 65 \text{ ksi} \)

- Plate
  - \( 3 \times 3 \times 3/8 \text{ in. thick} \)
  - \( F_y = 36 \text{ ksi} \)

- Loads
  - \( N_u = 8 \text{ kips} \)
  - \( V_u = 6 \text{ kips} \)

Where \( N_u \) and \( V_u \) are the applied factored external loads

Assumptions:
- Concrete is cracked
- \( 
\) factors are based on Condition B in D.4.5 of the code
  (no supplementary reinforcement)

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Try 3/4" dia., hef = 8", fc' = 4000psi
Normal weight concrete
See text “ACI FAILURE MODES”

1. Steel Strength
\[ A_{se,n} = 0.7854 \times (0.75 - 0.9743/10)^2 \]
\[ = 0.33460 \text{ in.}^2 \]
\[ N_{sa} = 0.33460 \times 65000 \]
\[ = 21740 \text{ lbf} \]
\[ \phi = 0.75 \]
\[ \phi N_{sa} = 16305 \text{ lbf} \]

2. Concrete Breakout in Tension
(1) headed anchor welded to an embedded plate
see AWS D1.1, Chapter 7, Stud Welding
head diameter = 1.25 in.
concrete is cracked at service loading
(2) \( \phi = 0.7 \), Condition B
(3) \( k_c = 24 \)
\( \lambda_a = 1.0 \) (applies to lightweight concrete
(4) not applicable
(5) \[ N_b = 24 \times 1 \times 4000^{(1/2)} \times 8^{(1.5)} = 34346 \text{ lbf} \]
or
\[ N_b = 16 \times 1 \times 4000^{(1/2)} \times 8^{(5/3)} = 32382 \text{ lbf} \]
(6) \( \psi_{ec,n} \) doesn’t apply
(7) \( \psi_{ed,n} = 1 \)
(8) \( \psi_{c,b} = 1 \)
(9) not applicable
(10) \[ Anco = 9 \times 8^2 = 576 \text{ in.}^2 \]
(11) \[ Anc = (3 \times 8) \times (3 \times 8) = 576 \text{ in.}^2 \]
(12) \[ N_{cb} = (1/1) \times 1 \times 34346 = 34346 \text{ lbf} \]
(13) not applicable
\[ \phi N_{cb} = 24842 \text{ lbf} \]

3. Pullout Strength
\[ \psi_{cb} = 1 \]
\[ A_{brg} = (\Pi/4) \times (1.25^2 - 0.75^2) = 0.78540 \text{ in.}^2 \]
\[ N_{pn} = 8 \times 1 \times 0.78540 \times 4000 = 25133 \text{ lbf} \]
\[ \phi N_{pn} = 0.7 \times N_{pn} = 17593 \text{ lbf} \]

4. Side-Face Blowout - not applicable
5. Bond Strength – not applicable

6. Steel Strength in Shear
   \[ A_{se,v} = 0.33460 \text{ in.}^2 \]
   \[ V_{sa} = 1.0 \times 0.33460 \times 65000 = 21749 \text{ lbf} \]
   \[ \phi V_{sa} = 0.65 \times 21749 = 14137 \text{ lbf} \]

7. Concrete Breakout in Shear
   \[ x_1, x_2, h_a \geq 1.5 \text{cal} \]
   thus \[ A_{vc} = A_{vco} = 1.5 \times 8 \times (3 \times 8) = 288 \text{ in.}^2 \]
   \[ l_e \leq 8 \times d_a = 6 \text{ in.} \]
   \[ \psi_{ed,v} = \psi_{c,v} = \psi_{h,v} = 1 \]
   \[ V_b = \text{smaller of} \]
   \[ 7 \times (6/0.75)^{0.2} \times 0.75^{1/2} \times 12^{1.5} = 24157 \text{ lbf} \]
   \[ \text{and} \]
   \[ *1 \times 4000^{91/2} \times 12^{1.5} = 23662 \text{ lbf} \]
   \[ V_{cb} = 1 \times V_b = 23662 \text{ lbf} \]
   \[ \phi = 0.7 \]
   \[ \phi V_{cb} = 16563 \text{ lbf} \]

8. Concrete Pryout in Shear
   \[ k_{cp} = 2, h_{ef} > 2.5 \text{ in.} \]
   \[ V_{cp} = 2 \times 34346 = 68692 \text{ lbf, from step 2} \]

9. Summary
   \[ \begin{array}{ccc}
   \text{Step} & \phi N_n (\text{lbf}) & \phi V_n (\text{lbf}) \\
   \hline
   1 & 16305 & - \\
   2 & 24842 & - \\
   3 & 17593 & - \\
   6 & - & 14137 \\
   7 & - & 16563 \\
   8 & - & 48024 \\
   \end{array} \]
   \[ N_{ua} / \phi N_n + V_{ua} / \phi V_n = 8000 / 16305 + 6000 / 14137 \]
   \[ = 0.491 + 0.424 = 0.915 < 1.2, \text{o.k.} \]
EXAMPLE 3 – TWO ADHESIVE ANCHORS

Given: Service load = 10000 lbf shear (not sustained)
Anchor steel = Grade 36
f_{c'} = 4000 psi
Installed with hammer drill
Dry concrete
Max. short term temperature = 130°F
Max. sustained temperature = 110°F
Concrete cracked under service load
Installation Condition B, Category 2
ca1 = 12 in.
ca2 = ca4 = 15 in.
ca3 = 84 in., use 1.5*hef
s1 = 0, s2 = 18 in.
h = 48 in.

Try: 1 in. dia., hef = 12 in.

5. BOND STRENGTH OF ADHESIVE ANCHORS IN TENSION

Λ_a = 0.6
Anco = (2*ca1)^2 = 576 in.^2
Anc = (ca1+1.5*hef)*(ca2+s2+ca4) = 1440 in.^2
   = greater than 2*Anco, use 2*576 = 1152 in.^2

From ESR-2322 (reference __) for the above conditions:
τ_{uncr} = 1365 psi
τ_{cr} = 600 psi
ϕ = .55
 cac = hef*(τ_{uncr}/1160)^0.4*(3.1-.7*h/hef)
      use 2.5*hef < h, for h
 cac = 12*(1365/1160)^0.4*(3.1-.7*2.5)
cac = 17.2896 in.
cna = 10*1*(165/1100)^0.5
   = 11.1396 in.
ψcp,na = 12/17.2896 but not less than
   18/17.2896, use ψcp,na = 1
Ψec,na = 1
Ψed,na = 1
\[ N_{ba} = 0.6 \times 600 \times \Pi \times 1 \times 12 = 13572 \text{ lbf} \]
\[ N_a = 2 \times 1 \times 13572 = 27144 \text{ lbf} \]
\[ N_{ag} = N_a = 27144 \text{ lbf} \]
\[ \phi_{Nag} = 14929 \text{ lbf} \text{ (need this number for step 8)} \]

2. **CONCRETE BREAKOUT STRENGTH IN TENSION**

This failure mode included because it is used in Step 8, **CONCRETE PRYOUT IN SHEAR**

1. See given statement above
2. From above, cac = 17.2896 in.
3. \( K_c = 17, \lambda_a = 0.8 \)
4. Not applicable
5. \[ N_b = 17 \times 0.8 \times 4000^{(1/2)} \times 12^{(1.5)} = 35755 \text{ lbf} \]
6. \( \psi_{ec,n} = 1 \)
7. \[ \psi_{ed,n} = 0.7 + 0.3 \times (12/18) \]
   \[ \psi_{ed,n} = 0.9 \]
8. \( \psi_{c,n} = 1 \)
9. \( \psi_{cp,n} \) not less than \( 18/17.2896 \), use \( \psi_{cp,n} = 1 \)
10. \( A_{nco} = 9 \times 12^2 = 1296 \text{ in.}^2 \)
11. \( A_{nc} = (12+0+18) \times (15+18+15) = 1440 \text{ in.}^2 \)
12. \[ N_{cb} = (1440/1296) \times 0.9 \times 35755 = 35795 \text{ lbf} \]
    \[ N_{cbg} = N_{ch} \]
13. \[ \phi_{Ncbg} = 0.55 \times 35795 = 19687 \text{ lbf} \]

6. **STEEL STRENGTH OF ANCHOR IN SHEAR**

\[ A_{se,v} = 0.7854 \times (1-0.9743/8)^2 = 0.6057 \text{ in.}^2 \]
\[ V_{sa} = 2 \times (0.6057) \times 58000 = 70261 \text{ lbf} \]
\[ \phi_{Vsa} = 0.65 \times V_{sa} = 45670 \text{ lbf} \]

7. **CONCRETE BREAKOUT IN SHEAR**

\[ A_{vco} = 4.5 \times (ca1)^2 = 4.5 \times 12^2 = 648 \text{ in.}^2 \]
\[ A_{vc} = 1.5 \times ca1 \times (ca2_s2+ca4) = 864 \text{ in.}^2 \]
\[ l_e = 8 \times 1 = 8 \text{ in.} \]
\[ \psi_{ec,v} = 1 \]
\[ \psi_{ed,v} = 0.7 + 0.3 \times (15/18) = 0.95 \]
\[ \psi_{c,v} = 1 \]
\[ (1.5 \times ca1/ha)^{(1/2)} = (18,48)^{(1/2)} = 0.6124 \]
But not less than 1, use 1
\[ V_b \text{ is the smaller of:} \]
\[7 \times 8^{(0.2)} \times 1^{(1/2)} \times 4000^{(1/2)} \times 12^{(1.5)} = 26335 \text{ lbf}\]
\[9 \times 0.8 \times 4000^{(1/2)} \times 12^{(1.5)} = 18929 \text{ lbf}\]
\[V_{cb} = V_{cbg} = (864/648) \times 0.95 \times 18929 = 23977 \text{ lbf}\]
\[\phi V_{cb} = \phi V_{cbg} = 16783 \text{ lbf}\]

8. **CONCRETE PRYOUT IN SHEAR**

\[K_{cp} = 2.0 \text{ since } hef > 2.5 \text{ in.}\]
\[V_{cp} = K_{cp} \times N_{cp} \text{ where}\]
\[N_{cp} = \text{ lesser of } N_{cb} (\text{step 2, } 19687 \text{ lbf}) \text{ and } N_{a} (\text{step 5, } 14929 \text{ lbf}).\]
\[V_{cp} = 2 \times 14929 \text{ lbf}\]
\[V_{cp} = 29858 \text{ lbf}\]
\[\phi V_{cp} = 0.7 \times 29858 = 20901 \text{ lbf}\]

9. **SUMMARY**

<table>
<thead>
<tr>
<th>Step</th>
<th>(\phi V(\text{lbf}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>45670</td>
</tr>
<tr>
<td>7</td>
<td>16763</td>
</tr>
<tr>
<td>8</td>
<td>20901</td>
</tr>
</tbody>
</table>

\[Vu/\phi V = 1.6 \times 10000/16763 = 0.953 < 1, \text{ o.k.}\]
EXAMPLE 4 – TWO BOLTS IN TENSION

GIVEN:  The following drawing from the classic text “Design of Welded Structures”, by Omer Blodgett (Ref. 17).
Base plate = PL 3-1/4”x19”x2’-5” (scaled)
Foundation = 8’-2” x 7’-2” x 60”
Base plate embedded in foundation
fc’ = 4000 psi
Concrete cracked under service loads
Service load = 84600 tension on 2 bolts
cal = ca2 = ca3 = ca4 = 39”
s1 = 0  s2 = 8”
Anchors cast-in with heavy hex nut
Anchor steel = Grade 36 (futa = 58000 lbf)
Installation Condition B

FIND:  A method for calculating anchor(s) diameter and hef given a tension load and ductility requirement. In this method, tensile anchor capacity is given by $\phi A_{se,n} \cdot \text{no. bolts} \cdot \text{futa}$. This is set equal to the factored tensile load to find anchor diameter. The tensile load $N_u$ is load factor*service load = $\phi \cdot \text{no.bolts} \cdot A_{se,n} \cdot f_y$, for ductility.

METHOD:  Failures 4 and 8 do not govern.
Failure 5 is not applicable.
Abrg in Failure 3 can be adjusted by an oversized washer so it does not govern.
Assume $(\text{Anc}/\text{Anco}) \cdot \psi \cdot \psi \cdot \psi \cdot \psi = 1$
$\lambda a = 1$
$le/da = 8$
$1.6 \times 84600 = 0.72 \times A_{se,n} \cdot 58000$
This solves to $A_{se,n} = 1.667 \text{ in.}^2$
Use $da = 1.75 \text{ in.}^2$
Anchor steel capacity governs in tension
USE OF WING PLATES

When large wing plates are used to increase the leverage of an anchor bolt, the detail should always be checked for weakness in bearing against the side of the column flange.

Problem

![Diagram of wing plate detail](image)

FIGURE 29

Figure 29 illustrates a wing-plate type of column base detail that is not limited with respect to size of bolts or strength of column flange. A similar detail, with bolts as large as 4\(\frac{3}{4}\)" diameter, has been used on a large terminal project.

The detail shown is good for four 2\(\frac{3}{4}\)"-dia. anchor bolts. Two of these bolts have a gross area of 6.048 in.\(^2\) and are good for 84,800 lbs tension at a stress of 14,000 psi.
Tension to cause yielding = 1.90*36000
= 68400 lbf

Nu = 1.6*68400 = 109440 lbf

Now the anchor steel tensile capacity must govern in tension so the hef value used must give a higher capacity than the anchor steel, i.e., Nsa.

Nsa = no. bolts*tensile stress area of
1-3/4” bolt*futa
Nsa = 2*1.90*58000 = 228400 lbf
ϕNsa = .75*Nsa = 165300 lbf

Note that because of Condition B, all capacity reduction factors, except for anchor steel in tension (0.75) and in shear (0.65), in both tension and shear are 0.70.

Two values for hef are given by
ϕ*24*1*(fc’)^{(1/2)}*hef^1.5 and
ϕ*16*1*(fc’)^{(1/2)}*hef^{(5/3)}, ϕ = 0.70

Setting these equal to ϕNsa obtains:
hef = 28.926” and 26.352”, respectively.
Use hef = 26”

There are three (3) possibilities for capabilities in shear,, one corresponding to steel strength and two possibilities for concrete breakout. Pryout in shear does not govern.

ϕVsa = Steel strength = .65*.6*Nsa
ϕVsa = 85956 lbf

The breakout strength is the lesser of
7*(1e/do)^.2*d0^{(1/2)}*(fc’)^{(1/2)}*ca1^{(1.5)}
and 9*1*(fc’)^{(1/2)}*ca1^{(1.5)}
The former obtains 200165 lbf and the latter 138634 lbf.
Thus steel shear strength governs, i.e., is
The lowest shear capacity of the shear failure loads.

The available shear fraction, 1.2-\(\frac{\nu}{\phi Nsa}\) is 1.2-..66207 = .53793
Then max. \(Vu = .53793*85956 = 45163\) lbf.

This does not, however, include the additional shear due to bolt bending, or speak to what part of the shear is resisted by the bolts and that resisted by other constructions as reinforcement, bearing of edge of base plate, shear lugs, or others.

Additional shear may also be induced by displacement of the top of the stretch bolts by flexure of the column, rotation of the base plate, or construction error.

Consider a cantilever with a point load at the end.
\[\Delta = \frac{P*L^3}{3*E*I}\]
This may be solved for \(P\) giving
\[P = 50608\] lbf per inch of displacement of tip

Now the base plate scaled as
PL 3-1/4"x19"x2'-5" so that the bolt length is 9.25 in.

Using the bolt bending calculations in the Text,
\[L = 9.25 + .5*1.75 = 10.125\] in.
\[Z = 1.75^3/6 = .89323\] in.^3
\[Mao = 1.2*58000*.89323 = 62169\] lbf-in.
\[Ms = 62169*(1-.709360) = 18869\] lbf-in.
\[\alpha = 2\]
\[Vadd = 2*18869/10.125 = 1864\] lbf
\[Mv = Vu*L, where Vu here is the force caused by displacement.\]
The critical point is where $M_v = M_s$, or

\[ \Delta * 50608 * 10.125 = 18869, \] which solves for

\[ \Delta = 0.037 \text{ in.}, \] which must be addressed in design.

\[ \phi_{Nsa} = 165300 \text{ lbf} \]

Concrete breakout strength is $\phi_{Ncbg}$

\[ \phi_{Ncbg} = 178275 \text{ lbf} > \phi_{Nsa}, \text{ o.k.} \]

Abrg 1-3/4” heavy hex nut is 4.144 in.$^2$

\[ \phi_{Npn} = \text{ pullout strength} = 92836 \text{ lbf/anchor} \]

\[ \phi_{Npng} = 183672 \text{ lbf}, > \phi_{Nsa}, \text{ o.k.} \]

Thus steel strength governs in tension, and

The connection is ductile, Q.E.D.
REFERENCES

1. “Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary”, American Concrete Institute, www.concrete.org


8. “Structural Welding Code – Steel” AWS D1.1 www.google.com


APPENDIX 1

ACI 318-11, Section 2.1, Code Notation
Units of length in inches, areas in square inches, forces
in pounds force (lbf), pressures in pounds force/square
inch (psi) unless noted otherwise.

Abrg = net bearing area of headed stud, headed anchor bolt,
or headed deformed bar
Ana = projected influence area of ≥ 1 adhesive anchors,
For calculation of bond strength in tension
Anao = projected influence area of a single adhesive anchor
for calculation of bond strength in tension, not
limited by edge distance or spacing
Anc = projected concrete failure area for tension,
> = 1 anchor
Anco = projected failure area, 1 bolt, not limited by edge
distance or spacing for calculation of strength in
tension
Ase,n = effective cross-sectional area of anchor bolt in
tension
Ase,v = effective cross-sectional area of anchor bolt in
shear
Avc = projected concrete failure area for shear.
Avco = projected concrete failure area for shear, single
anchor bolt, not limited by corners, spacing, or
thickness
cal = distance from the anchor center to the concrete
edge in one direction, in. If shear is applied to
anchor, cal is taken in direction of applied shear.
If tension is applied to the anchor, cal is the
minimum edge distance, in.
ca2 = distance from the anchor center to the concrete
edge perpendicular to cal, in.

cac = critical edge distance required to develop the basic
strength as controlled by concrete breakout or bond
of a post-tensioned anchor in tension in uncracked
concrete without supplementary reinforcement to
control splitting
ca,max = max. distance from a.b. center, to concrete edge
ca,min = min. distance from a.b. center to concrete edge

cna = projected distance from center of an anchor shaft

one side of the anchor required to develop the full

full bond strength of an adhesive anchor

da = outside diameter anchor or shaft diameter of headed

stud, headed bolt, or hooked bolt

da’ = value substituted for da when an oversized anchor is

used

eh = distance from inner surface of shaft of J- or L-bolt
to outer tip

en’ = distance from resultant tension load to tension

centroid of group of anchors

ev’ = distance from resultant shear load on a group of

anchors loaded in shear in the same direction, and the

centroid of the group loaded in the same direction

fc’ = specified compressive strength of concrete

futa = specified tensile strength of anchor steel

fya = specified yield strength of anchor steel

ha = thickness of member in which anchor is located,

measured parallel to the anchor axis

hef = effective embedment depth

kc = coefficient for concrete breakout strength in
tension

kcp = coefficient for pryout strength

le = load bearing length of anchor for shear

n = number of items in group

Na = nominal bond strength in tension of single adhesive

anchor

Nag = nominal bond strength in tension of a group of

adhesive anchors

Nb = basic concrete breakout strength in tension,

cracked concrete, 1 anchor

Nba = basic bond strength of a single adhesive anchor in

cracked concrete

Nbagg = nominal bond strength for a group of adhesive

Anchors in tension

Ncb = nominal concrete breakout strength in tension, 1 a.b.

Ncbbg = nominal concrete breakout strength in tension,

>1 a.b.

Nn = nominal strength in tension

Np = nominal pullout strength in tension, 1 anchor,
cracked concrete

Npn = nominal pullout strength in tension, 1 anchor

Nsa = nominal strength of > = 1 anchor, in tension, governed by steel

Nsb = side face blowout strength, 1 anchor

NsbG = side face blowout strength, > 1 anchor

Nua = factored tensile factor applied to single anchor or group

s = center-to-center spacing of anchors

Vb = basic concrete breakout strength in shear, 1 anchor, cracked concrete

Vcb = nominal concrete breakout strength in shear, 1 anchor

VcBG = nominal concrete breakout strength in shear >1 anchor

Vcp = nominal concrete pryout strength, 1 anchor

VcpG = nominal concrete pryout strength, > 1 anchor

Vn = nominal concrete shear strength

Vsa = nominal strength in shear 1 anchor governed by steel

Vua = factored shear force, > = 1 anchor

Ψc,n = modifier for tensile strength for cracked versus uncracked concrete

Ψc,v = modifier for shear strength in anchors based on presence or absence of concrete cracking and presence or absence of supplementary reinforcement

Ψcp,n = modifier for tensile strength of post-installed anchors intended for use in uncracked concrete without supplementary reinforcement

Ψcp,na = modify tensile strength of adhesive anchors, uncracked concrete, no supplementary reinforcement

Ψec,n = modifier for tensile strength of anchors based on eccentricity of loads

Ψec,na = modify tensile strength of adhesive anchors, eccentric loads

Ψec,v = modifier for shear strength of anchors based on eccentricity of loads

Ψed,n = modifier for tensile strength of anchors based on edge distances

Ψed,na = modifier for shear strength of anchors based on edge distances
ψ_{ed,na} = modify tensile strength of adhesive anchors, based on edge distances
ψ_{h,v} = modify shear strength for concrete thickness
τ_{cr} = characteristic bond stress of adhesive anchor in cracked concrete
τ_{uncr} = characteristic bond stress of adhesive anchor in uncracked concrete
λ = modification factor reflecting the reduced properties of lightweight concrete relative to normalweight concrete.
λ_{a} = modification reflecting reduced mechanical properties of lightweight concrete in certain applications
ϕ = strength reduction factor
## APPENDIX 2

### ANCHOR BOLTS DESIGN CITATION LIST

Reference: ACI 318-11, appendix D.

<table>
<thead>
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<th>Section Title</th>
<th>Step</th>
<th>Section</th>
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<tbody>
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<td>text</td>
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<tr>
<td>Steel strength in tension</td>
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<td>D.5.1</td>
</tr>
<tr>
<td>Concrete breakout strength</td>
<td>2</td>
<td>D.5.2</td>
</tr>
<tr>
<td>Pull-out strength cast-in and post-installed expansion anchor</td>
<td>3</td>
<td>D.5.3</td>
</tr>
<tr>
<td>Concrete side-face blowout strength of headed anchor</td>
<td>4</td>
<td>D.5.4</td>
</tr>
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<td>Bond strength of adhesive anchor in tension</td>
<td>5</td>
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<tr>
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<td>7</td>
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<tr>
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<td>9</td>
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</table>

![Table D.4.1.1 + above]

<table>
<thead>
<tr>
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<th>Group</th>
<th>Cast-in Effect</th>
<th>Expansion Anchor</th>
<th>Adhesive Anchor</th>
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<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
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<td></td>
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<tr>
<td>2</td>
<td>■</td>
<td></td>
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</tr>
<tr>
<td>3</td>
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<td>9</td>
<td>■</td>
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</tbody>
</table>
STEP 1: STEEL STRENGTH OF ANCHOR IN TENSION (D.5.1)

\[ A_{en} = 0.7854 \times (D-.9743/nt)^2 \]
where
\[ nt = \text{no. turns/inch} \] (AISC)
\[ \text{futa} \leq 1.9 \times \text{fya} \text{ and } \leq 125000 \text{ psi} \]
\[ N_{sa} = n \times A_{en} \times \text{futa} \] (D-2)

STEP 2: CONCRETE BREAKOUT STRENGTH OF ANCHOR IN TENSION (D.5.2)

\[ k_c = 24 \text{ for cast-in and 17 for post-installed, the post-installed may be increased above 17 by product-specific tests but in no case greater than 24} \]
\[ \lambda = 0.75, 0.85, 1.0 \text{ for all-lightweight concrete, sand-lightweight, concrete, normalweight concrete, respectively} \] (D.6.1)
\[ \lambda_a = 1.0 \lambda \text{ cast-in.} 0.8 \lambda \text{ for concrete failure, adhesive anchors,} \]
\[ 0.6 \lambda \text{ for concrete bond failure, adhesive anchors} \]
If an additional plate or washer is added at the head of the anchor the projected area may be (D.5.2.8) calculated from the perimeter of the plate or washer. The effective perimeter should not exceed the thickness of the washer or plate.
Where anchors are located less than 1.5*hef from three or more edges, the value of hef used for the calculation of Anc in accordance with D.5.2.1, as well as in Equations (D-3) through (D-10) shall be the larger of ca,max/1.5 and s/3, where s is the maximum spacing of anchors in a group (D.5.2.3)

For a single anchor in tension in cracked concrete
\[ N_b = k_c \times \lambda_a \times (f'c)^{(1/2)} \times \text{hef}^{(1.5)} \] (D-6)
Alternatively, for cast-in headed studs and headed bolts where 11 in. \( \leq \text{hef} \leq 25 \text{ in.}, \)
\[ N_b = 16 \times \lambda a \times (f'c)^{(1/2)} \times \text{hef}^{(5/3)} \] (D-7)
\[ \Psi_{ec,n} = 1/(1+2*en'/3*hef) \leq 1.0 \] (D-8)
If ca,min \( \geq 1.5 \times \text{hef} \) then \( \Psi_{ed,n} = 1.0 \) (D-9)
If ca,min <1.5*hef, then
ψ_{ed,n} = 0.7 + 0.3*ca_{min}/1.5*hef  \hspace{1cm} (D-10)
ψ_{c,n} = 1.0, cracking at service loads \hspace{1cm} (D.5.2.6)
ψ_{c,n} = 1.25, cast-in-anchors, no cracking \hspace{1cm} (D.5.2.6)
ψ_{c,n} = 1.40, post-installed anchors, no cracking \hspace{1cm} (D.5.2.6)

cac for adhesive anchors = 2*hef \hspace{1cm} (D.8.6)
cac for undercut anchors = 2.5 hef \hspace{1cm} (D.8.6)
unless determined by tests (eg., ESR-2322)

If cracking:
If ca_{min} > cac then Ψ_{cp,n} = 1.0 \hspace{1cm} (D-11)
If ca_{min} < cac and > 1.5*hef/cac then Ψ_{cp,n} = ca_{min}/cac but not less than 1.5*hef/cac for post-installed anchors
Ψ_{cp,n} for cast-in = 1.0

See Fig. RD.5.2.1 (a) for calculation of Anco and (b) for calculation of Anc. Anc < = n*Anco \hspace{1cm} (D-5)
Anco = 9*hef^2 if edge distance > = 1.5*hef
1 anchor Ncb = (Anc/Anco)*Ψ_{ed,n}*Ψ_{c,n}*Ψ_{cp,n}*N_b \hspace{1cm} (D-3)
else Ncbg = (Anc/Anco)*Ψ_{ec,n}*Ψ_{ed,n}*Ψ_{c,n}*Ψ_{cp,n}*N_b \hspace{1cm} (D-4)

STEP 3: PULLOUT STRENGTH OF CAST-IN AND EXPANSION ANCHORS \hspace{1cm} (D.5.3)

for single headed stud or headed bolt,  
N_p = 8*A_{brg}*f_{c'} \hspace{1cm} (D-14)
for single hooked bolt,  
N_p = 0.9*(f_{c'})*eh*do where 3*do<=eh<=4.5*do \hspace{1cm} (D-15)
Expansion anchors pullout strength obtained by tests  
t0 ACI 355.2 \hspace{1cm} (D.5.3.2)
Ψ_{c,p} = 1.4 no cracking, 1.0 if cracking \hspace{1cm} (D-13)
N_{pn} = Ψ_{c,p}*N_p

STEP 4: CONCRETE SIDE-FACE BLOWOUT STRENGTH OF ANCHOR IN TENSION \hspace{1cm} (D.5.4)

λ_a - see Step 2
A_{brg} - see Step3
for a single headed anchor with deep embedment close to an edge, (hef > 2.5*ca):
N_{sb} = 160*ca1*A_{brg}^{(0.5)}*λ_a*(f_{c'})^{(1/2)} \hspace{1cm} (D-16)
Nsbg = \((1+s/6*ca1)*Nsb\) \hspace{1cm} \text{(D-17)}

for multiple headed anchors with deep embedment close to an edge \((hef>2.5ca1)\) and anchor spacing less than \(6*ca1\).

**STEP 5: BOND STRENGTH OF ADHESIVE ANCHOR** \hspace{1cm} \text{(D.5.5)}

\[ \Lambda a = \text{see Step 2} \]

for outdoor work, dry to fully saturated moisture content, at installation, and maximum temperature of \(175^\circ\text{F}\),

\[ \tau_{cr} = 200 \text{ psi and } \tau_{uncr} = 650 \text{ psi} \hspace{1cm} \text{(Table D.5.5.2)} \]

\[ Nba = \Lambda a \times \tau_{cr} \times \Pi \times da \times hef \hspace{1cm} \text{(D-22)} \]

If adhesive anchor designed to resist sustained loads,

\[ 0.55 \times \phi \times Nba \geq Nu_{,\text{sustained}} \hspace{1cm} \text{(D-1)} \]

\[ c_{na} = 10 \times da \times (\tau_{uncr}/1100)^{(1/2)} \hspace{1cm} \text{(D-21)} \]

\[ An_{ao} = (2 \times c_{na})^2 \hspace{1cm} \text{(D-20)} \]

\[ An_{ao} \leq n \times An_{ao} , \text{ n= number of anchors in group} \]

See Fig. RD.5.5.1 for calculation of \(An_{ao}, An\)

if \(ca_{,\text{min}} \leq c_{na}\), \(An = (2 \times c_{,\text{min}})^2 \) (author's opinion)

else \(An = (2 \times c_{na})^2\)

\[ \psi_{ec,na} = 1 / (1 + 2 \times \frac{e'n}{3 \times hef}) \hspace{1cm} \text{(D-23)} \]

\[ \psi_{ed,na} = \text{if } ca_{,\text{min}} \geq c_{na} \text{ then } =1 \hspace{1cm} \text{(D-24)} \]

else \(\psi_{ed,na} = 0.7 + 0.3 \times (ca_{,\text{min}}/cna) \hspace{1cm} \text{(D-25)} \]

\[ cac = 2 \times hef, \text{or tests to ACI 355.2 or ACI 355.4} \hspace{1cm} \text{(D.8.6)} \]

\[ \psi_{cp,na} = \text{if } ca_{,\text{min}} \geq cac \text{ then } = 1 \hspace{1cm} \text{(D-26)} \]

else \(\psi_{cp,na} = ca_{,\text{min}} / cac \text{ but not less than } c_{na} / cac \hspace{1cm} \text{(D-25)} \]

\[ Na = (An / An_{bo}) \times \psi_{ed,na} \times \psi_{cp,na} \times Nba \hspace{1cm} \text{(D-18)} \]

\[ Nag = (An / An_{bo}) \times \psi_{ec,na} \times \psi_{ed,na} \times \psi_{cp,na} \times Nba \hspace{1cm} \text{(D-19)} \]

**STEP 6: STEEL STRENGTH OF ANCHOR IN SHEAR** \hspace{1cm} \text{(D.6.1)}

Where concrete breakout is a potential failure mode, the required steel shear strength shall be consistent with the assumed breakout surface \hspace{1cm} \text{(D.6.2.1)}

\[ A_{se,v} = 0.7854 \times (D - 0.9743/nt)^2 \]

<table>
<thead>
<tr>
<th>Type of Anchor</th>
<th>Vsa</th>
</tr>
</thead>
<tbody>
<tr>
<td>cast-in headed stud anchor</td>
<td>1.0*A_{se,v}*f_{uta}</td>
</tr>
</tbody>
</table>
cast-in headed bolt and hooked anchors $0.6 \cdot A_{se,v} \cdot f_{u,t,a}$
for post-installed anchors where sleeves do not extend through the shear plane (D-29)
post-installed anchors where sleeves extend through the shear plane
ACI 355.2 tests
where anchors are used with built-up grout pads, multiply values above by 0.80 (D.6.1.3)

STEP 7: CONCRETE BREAKOUT STRENGTH IN SHEAR (D.6.2)

$le = hef$ for anchors with constant stiffness over the full length of the anchor
$le \leq 8 \cdot da$

$\lambda = 0.75$, 0.85, and 1.0 for all-lightweight, sand-lightweight, and normalweight concrete, respectively (8.6.1)

$\lambda_a = 0.6 \cdot \lambda$ adhesive concrete bond failure, $0.8 \cdot \lambda$ expansion and adhesive anchor concrete failure, and $1.0 \lambda$ for cast-in and undercut anchor concrete failure.

Figure Used to find
---------------------------
RD.6.2.1 (a) and (b) $ca_1$, $A_{vc}$
RD.6.2.1(c) shear force parallel to edge
RD.6.2.1(d) shear force at corner, $ca_1, ca_2$
RD.6.2.9(a) hairpin anchor reinforcement
RD.6.2.9(b) edge and anchor reinforcement for shear

$V_b =$ is the smaller of (a) and (b):
(a) $V_b = 7 \cdot (((le/da) \cdot 0.2) \cdot \lambda a) \cdot (f_{c'}^{(1/2)}) \cdot (ca_1)^{1.5}$ (D-33)
(b) $V_b = 9 \cdot \lambda a \cdot f_{c'}^{(1/2)} \cdot (ca_1)^{1.5}$ (D-34)

$A_{vco} = 4.5 \cdot (ca_1)^2$ (D-32)

$A_{vc} \leq$ number of anchors in group $\cdot A_{vco}$

$\Psi_{ec,v} = 1 / (1 + 2 \cdot ev' / 3 \cdot ca_1) \leq 1$ (D-36)

$\Psi_{ed,v} = 1.0 \text{ if } ca_2 \geq 1.5 \cdot ca_1$ (D-37)

$\Psi_{ed,v} = 0.7 + 0.3 \cdot ca_2 / (1.5 \cdot ca_1)$ (D-38)

$\psi_{c,v} = 1.4 \text{ if no cracking }$ (D.6.2.7)

$= 1.2 \text{ cracked and #4 bar between between anchor and edge}$
= 1.0 if bar smaller than #4 or no bar between anchor and edge

ψh,v = (1.5*ca1/ha)^(1/2) where ha < 1.5*ca1    (D-39)
else ψh,v = 1.0

for shear force perpendicular to edge of a single anchor:
Vcb = (Avc/Avco)*ψed,v*ψc,v*ψh,v*ψh,v*Vb         (D-30)

And for a group of anchors
Vcbg = (Avc/Avco)*ψec,v*ψed,v*ψc,v*ψh,v*ψh,v*Vb    (D-31)
For shear force parallel to edge, use twice the values for Vcb and Vcbg above.
For corner locations, calculate Vcb above and Vcbg above for both directions, using the smaller value.

STEP 8: CONCRETE PRYOUT STRENGTH IN SHEAR    (D.6.3)

Kcp = 1.0 for hef < 2.5 inches and
     = 2.0 for hef >= 2.5 inch               (D.6.3.1)
Vcp = kcp*Ncp where:
     = use Ncb (step 2) for cast-in, expansion, or undercut anchors and the lesser of Ncb (step 2) and Na (step 5) for adhesive anchors
Vcpg = kcp*Ncpg where:
     = use Ncbg (step 2) for cast-in, expansion, or undercut anchors and the lesser of Ncbg (step 2) and Nag (step 5) for adhesive anchors

STEP 9: INTERACTION OF TENSILE AND SHEAR FORCES   (D.7)

Determine load factors from applicable Code.
Nua = factor*service load
Vua = factor*service noad
Using the lowest values of φNn and φVn for all combinations of Steps 0 through 9,
From the Code body (D.7),
if \( \frac{V_{ua}}{\phi V_n} \leq 0.2 \), use \( \phi N_n = U_{ua} \) (D.7.1)
and if
if \( \frac{N_{ua}}{\phi N_n} \leq 0.2 \), use \( \phi V_n = U_{ua} \) (D.7.2)
else
\( \frac{N_{ua}}{\phi N_n} + \frac{V_{ua}}{\phi V_n} \leq 1.2 \) (D-42)

From the commentary (RD.7)
\( (\frac{N_{ua}}{\phi N_n})^{(5/3)} + (\frac{V_{ua}}{\phi V_n})^{(5/3)} \leq 1.0 \)
/********************************************************************************

* Anc.c : 12-02-15 : ml

* ca1  s1  ca3  ca4

* -------------------------

* ca4 | | | | |
| | | | |
* -------------------------

* s2 | | | | |
| | | | |
* -------------------------

* ca2 | | | | |
| | | | |
* -------------------------

********************************************************************************/

#include<math.h>
#include<stdio.h>
#include<stdlib.h>

int main(void)
{
    int i;
    double gca1,gca2,gca3,gca4,hef,gs1,gs2; /* inputs */
    double x0,x1,y0,y1,xbar,ybar;
    double gbar[2],Area[4];
    double Areatotal,Darea,ratio; /* outputs */
    FILE *inn;
    FILE *out;
    void centroid
    double,double,double,double,double,double,double[2]);
    inn = fopen("Anc.in","r");
    out = fopen("Anc.out","w+");
    fscanf(inn,"%lf %lf %lf %lf %lf %lf %lf %lf
%lf",&gca1,&gca2,&gca3,&gca4,&hef,&gs1,&gs2);
    fclose(inn);

    gbar[0] = 0.0;
gbar[1] = 0.0;
for(i=0;i<=3;i++)
{
    Area[i] = 0.0;
}

centroid(gca1,gca2,gca3,gca4,gs1,gs2,gbar);

xbar = gbar[0];
ybar = gbar[1];
fprintf(out,"xbar      = ");
fprintf(out,"%19.6e\n",xbar);
fprintf(out,"ybar      = ");
fprintf(out,"%19.6e\n",ybar);

x0 = xbar;
x1 = gca1+gs1+gca3-xbar;
y0 = ybar;
y1 = gca2+gs2+gca4-ybar;

Area[0] = sqrt(4*y0*y0*(hef*hef+x0*x0))/2;
Area[1] = sqrt((x1+x0)*(x1+x0)*(hef*hef+y1*y1))/2;
Area[2] = sqrt((y1+y0)*(y1+y0)*(hef*hef+x1*x1))/2;
Area[3] = sqrt((x1+x0)*(x1+x0)*(hef*hef+y0*y0))/2;

Areatotal = 0.0;

for(i=0;i<=3;i++)
{
    fprintf(out,"Area");fprintf(out,"%1d",i);
    fprintf(out," = ");
    fprintf(out,"%16.6e\n",Area[i]);
    Areatotal += Area[i];
}

fprintf(out,"Areatotal = ");
fprintf(out,"%19.6e\n",Areatotal);
Darea = (gca1+gs1+gca3)*(gca2+gs2+gca4);
fprintf(out,"DArea     = ");
fprintf(out,"%19.6e\n",Darea);
ratio     = Areatotal/Darea;
fprintf(out,"ratio     = ");
fprintf(out,"%19.6e\n",ratio);
fclose(out);
return 0;
}

void centroid(double ca1,double ca2,double ca3,double ca4,double s1,double s2,double bar[2])
{
  int i,j;
  double xx[3],yy[3],A[3][3];
  double zzarea,xxbar,yybar;

  xx[0]     =  ca1/2.0;
  xx[1]     =  ca1+s1/2.0;
  xx[2]     =  ca1+s1+ca3/2.0;
  yy[0]     =  ca2/2.0;
  yy[1]     =  ca2+s2/2.0;
  yy[2]     =  ca2+s2+ca4/2.0;

  A[0][0]     =  ca2*ca1;
  A[0][1]     =  ca2*s1;
  A[0][2]     =  ca2*ca3;
  A[1][0]     =  s2*ca1;
  A[1][1]     =  s2*s1;
  A[1][2]     =  s2*ca3;
  A[2][0]     =  ca4*ca1;
  A[2][1]     =  ca4*s1;
  A[2][2]     =  ca4*ca3;

  xxbar     =  0.0;
  yybar     =  0.0;
  zzarea     =  (ca1+s1+ca3)*(ca2+s2+ca4);

  for(i=0;i<=2;i++)
  {
    for(j=0;j<=2;j++)
    {

\[
\begin{align*}
\text{xxbar} &\; \; + = \; \; \text{xx}[i] \ast \text{A}[j][i] \\
\text{yybar} &\; \; + = \; \; \text{yy}[i] \ast \text{A}[i][j] \\
\end{align*}
\]

} 

} 

bar[0] = \text{xxbar}/\text{zzarea}; 

bar[1] = \text{yybar}/\text{zzarea}; 

}