



**PDHonline Course S293 (6 PDH)**

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# **Anchoring To Concrete**

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## INTRODUCTION

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A concrete anchor is a steel shaft either cast into concrete at placement or post-installed after the concrete has hardened.

Cast-in anchors are threaded shafts with a buried end termination of a hex head, threaded nut, or 90° (L-) or 180° (J-) hook, or headed (non-threaded) studs welded to a surface plate.

Post-installed anchors include adhesive and expansion types. Two of the expansion types are torque-controlled, where expansion is controlled by torque on the bolt, or displacement-controlled, where a plug or sleeve is impacted and the expansion is controlled by the length of travel of the plug or sleeve.

The anchors are designed to transfer the design loads from the superstructure to the foundation. In many cases, this transfer is, either from steel column base plates to the foundation, or from precast concrete members to the foundation.

An example of the connection of cast-in anchor to precast is shown in the following photograph, in the construction of a salt storage building in Western New York. The large cast-in bolts transfer the tensile force caused by the moment generated by the horizontal force of the soil against the precast walls to the foundation.

Shear is not transferred by the bolts, but by bearing between the buttress and foundation, due to the socketing of the buttress into the foundation.

The footings, anchors, buttresses, wall panels, and lintels comprising the complete foundation system for the salt storage building with arch roof under construction were designed, fabricated, and erected by the precast firm "New Eagle Silo" of Arcade, New York.



**This paper describes:**

- **Anchor Materials**
- **Concrete Cracking**
- **General Requirements**
- **Bolt Bending**
- **Anchor Tension Reinforcement**
- **Anchor Shear Reinforcement**
- **Description of Failure Modes**
- **Base Plates and Anchor Bolts**
- **Examples**
- **Appendix 1 - Definitions of Terms**
- **Appendix 2 - Citations List**
- **Appendix 3 - Anc program**

**The basic reference is "Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary, Appendix D", Reference 1. Citations not noted with a source refer to this specification.**

## ANCHOR MATERIALS

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The most common steel material for cast-in anchors is ASTM F1554, Grade 36. This is generally less expensive and more readily available than other types. It is also desirable where a ductile failure rather than a concrete failure is required, and the least concrete distance restrictions are present with its low yield and ultimate strengths.

Cast-in anchors are of three (3) types, headed bolts, headed studs, and hooked bolts, installed in place prior to concrete placement. Cast-in gives greater control, but less flexibility

Headed bolts are cylindrical threaded steel bars terminated in the concrete either by an integral head or nut, either of which may include a washer or plate. Care should be taken in the selection of a washer or plate, if used, because the stresses may exceed those allowable on conventional washers.

Headed studs are cylindrical steel bars (normally unthreaded) with an embedded head and welded to a steel plate at the surface. They are usually used to transfer shear loads between steel and concrete, typically in composite beams.

Hooked bolts refer to cylindrical steel bars with threaded connections at the ends, and possibly throughout. They are defined by the embedded end, either "L" (90°) or "J" (180°). Allowable bend diameters are not specified by ACI 318-11, only bent rebars. Appendix D does specify distance from the inner surface to the end of the hook. In projects requiring ductility, i.e., the lowest failure load is tension in the steel anchor, the concrete pullout strength must be greater than or equal to the tensile capacity of the steel anchor. This is, in general, not possible with hooked bolts as shown in the discussion of pullout strength in the pullout capacity discussion below.

Post-installed anchors material and design properties are obtained from ICC-ES Evaluation Reports such as Reference 2 for expansion anchors and Reference 3 for adhesive anchors.

Cast-in headed anchors refer to headed steel bars welded to a base plate. They are usually used to transfer shear loads between steel and concrete, typically in composite beams. See also the discussion on pullout strength for further description.

Post-installed gives greater flexibility, but less control.

## CONCRETE CRACKING

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Anchor design for the concrete breakout, pullout, bond strength, and pryout failure modes depend upon judgment of cracked versus uncracked concrete in computations.

Courses and control of cracking are discussed in Ref. 4 as follows :

- **Plastic Shrinkage Cracking**  
This is due to the evaporation of water near the surface, shrinking the surface layer but restrained by inner concrete, developing tensile stresses in the war surface layer. This results in a differential volume change. To slow down the evaporation, fog nozzles, plastic sheeting, windbreaks, and sun shades may be used.
- **Plastic Shrinkage Settlement Cracking**  
During the consolidation phase, the plastic concrete may be restrained by rebars, (cracking increases with rebar size), slump (increasing slump equals increasing cracking), and cover (increases with decreasing cover).
- **Hardened Concrete Drying Shrinkage**  
This is caused by volume change as the concrete shrinks, but is restrained. This may be reduced by contraction joints, proper detailing (especially no re-entrant corners), or shrinkage-compensating concrete. See Reference 5 for further details.



Quantity	Adhesive Anchor	Expansion Anchor Torque Controlled	Expansion Anchor Displacement Controlled
-----	-----	-----	-----
Min. ctr-ctr spacing	6*da	6*da	6*da
Min. edge distance	6*da	8*da	10*da
hef max. (2)		$\leq (2/3) * ha$ $\leq ha - 4$	$\leq (2/3) * ha$ $\leq ha - 4$
cac,min	2*hef	4*hef	4*hef

cac = critical edge distance controlled by concrete breakout or bond. Unless determined by test to ACI 355.2 (mechanical anchors) or ACI 355.4 (post-installed adhesive anchors), use the following values:

- adhesive anchors -> 2.0\*hef
  - undercut anchors -> 2.5\*hef
  - torque-controlled expansion anchors -> 4.0\*hef
  - displacement-controlled expansion anchors -> 4.0\*hef
- da = anchor diameter, in.  
 ha = member depth, in.

- (1) If edge distance less than that shown, substitute da' for da that meets the requirements of minimum center-center spacing and edge distance. Forces are limited to an anchor with a diameter of da'.
- (2) Values here may be reduced if tests according to the definition of cac are performed.

**GROUP EFFECTS (D.3.1.1)**



Group effects must be considered if anchor spacing is less than any of the following values:

Failure Mode	Spacing
-----	-----
Concrete breakout in tension	3*hef
Bond strength in tension	2*cna
Concrete breakout in shear	3*ca1

cna = projects distance from the center of an anchor shaft on one side of the anchor to develop the full bond strength of an adhesive anchor

$c_{al}$  = distance from the anchor center to the concrete edge in one direction, in. If shear is applied to the anchor,  $c_{al}$  is taken in the direction of applied shear. If tension is applied to the anchor,  $c_{al}$  is the minimum edge distance.

**OTHER****(D.2 - D.4)**

●●●●●

- Loads with high fatigue or impact not covered (D.2.4) By Appendix D.
- Anchors and anchor groups can be designed by (D.3.1) elastic analysis. Plastic analysis may be used if nominal strength is controlled by ductile steel.
- Appendix D does not apply to the design of anchors in plastic hinge zones of concrete structures under earthquake loads. These zones are defined as extended from twice the member depth from any column or beam face. These zones also include any other section where yielding of reinforcement is likely to occur due to lateral displacements. (D.3.3.2)  
If anchors must be located in these plastic hinge zones, they should be designed so that the anchor forces are directly transferred to anchor reinforcement that carries these anchor forces into the member beyond the anchor region. (RD.3.3.2)
- Post-installed anchors must meet ACI 355.2 or AQCI 355-4 (D3.3.3)
- Anchors in Seismic Design Category C, D, E, and F structures must satisfy all the non-seismic requirements of Appendix D, as well as additional requirements:
  - Tensile Loading (D.3.3.4)
  - Shear Loading (D.3.3.5)
- Modification factor  $\lambda_a$  for lightweight concrete : (D.3.6)
  - Cast-in concrete failure  $\lambda_a = 1.0$
  - Expansion + adhesive anchor concrete failure  $\lambda_a = 0.8$
  - Adhesive anchor bond failure  $\lambda_a = 0.6$
- $f_c' \leq 10000$  psi for cast-in anchors (D.3.7)  
 $f_c' \leq 8000$  psi for post-installed anchors
- For steel and pullout failure loads, the (RD.4.11)

highly stressed anchor should be checked.  
 For concrete breakout, the anchors should be checked as a group.

- Maximum anchor diameter = 4 inches. (D.4.2.2)
- Adhesive anchor embedment depths must be limited to  $4 \cdot d_a \leq h_{ef} \leq 20 \cdot h_{ef}$  (D.4.2.3)
- Strength Reduction  $\phi$  factors (D.4.3)
- Anchors governed by strength of ductile steel element - tension = 0.75 shear = 0.65
- Anchors governed by concrete breakout, side-face blowout, pullout or pryout strengths:

Load	Element	Condition	Category	$\phi$
Shear	---	A	---	0.75
	---	B	---	0.70
Tension	cast-in headed studs + bolts + hooked bolts,	A	---	0.75
		B	---	0.70
	post-installed anchors	A	1	0.75
		A	2	0.65
		A	3	0.55
		B	1	0.65
		B	2	0.55
		B	3	0.45

Condition A - supplementary reinforcement is present except for pullout and pryout

Condition B - no supplementary reinforcement  
 And for pullout and pryout

Category - applies to post-installed anchors

Category	Sensitivity	Reliability
1	low	high
2	medium	medium
3	high	low

where sensitivity refers to sensitivity to installation.

## BOLT BENDING

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ACI 318-11 defines stretch length as the length of anchor extending beyond the concrete, subject to tensile load. Code Section D.3.3.4.3 gives four (4) options for anchors and their attachments to structures in Seismic Design Categories C,D,E and F. Option (a), part 3, says anchors shall transmit tension loads by a ductile steel element with a stretch length of at least eight (8) bar diameters.

The following analysis is that given in Reference 10 with the exception of Z, the plastic modulus.

$z$  = portion of moment arm above concrete , in.  
 $n$  = 0 if clamped at concrete surface by nut and washer (required for mechanical anchors)  
       = 0.5 if not clamped at concrete surface  
 $d_0$  = bolt diameter, in.  
 $L$  = stretch length =  $z+n*d_0$ , in.  
 $Z$  =  $D^3/12 \text{ in.}^3$   
 $M_{so}$  = bending moment to cause rupture =  $1.2*f_u t_a * Z$   
 $N_{sa}$  = nominal tensile strength of anchor  
 $N_{ua}$  = factored load tension  
 $M_s$  = resultant flexural resistance of anchor  
 $M_s$  =  $M_{so} * (1 - N_{ua} / \phi N_{sa})$   
 $\alpha$  = adjustment factor  $1 \leq \alpha \leq 2$   
 $M_v$  = factored bending moment due to factored shear  
 $M_v$  =  $V_{ua} * L$ ,  $< = M_s \rightarrow$  if not true, redesign.  
 $V_{add}$  = term added to factored load shear ( $V_{ua}$ )  
       =  $\alpha * M_s / L$

Check the interaction of all the governing failure loads with the addition of  $V_{add}$

The following page shows examples of stretch lengths and stretch connection.

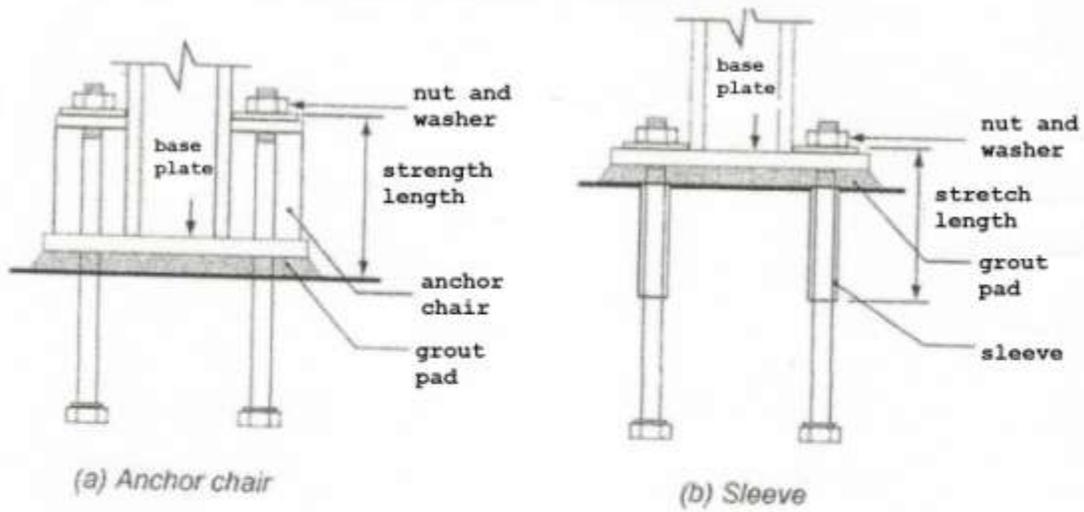
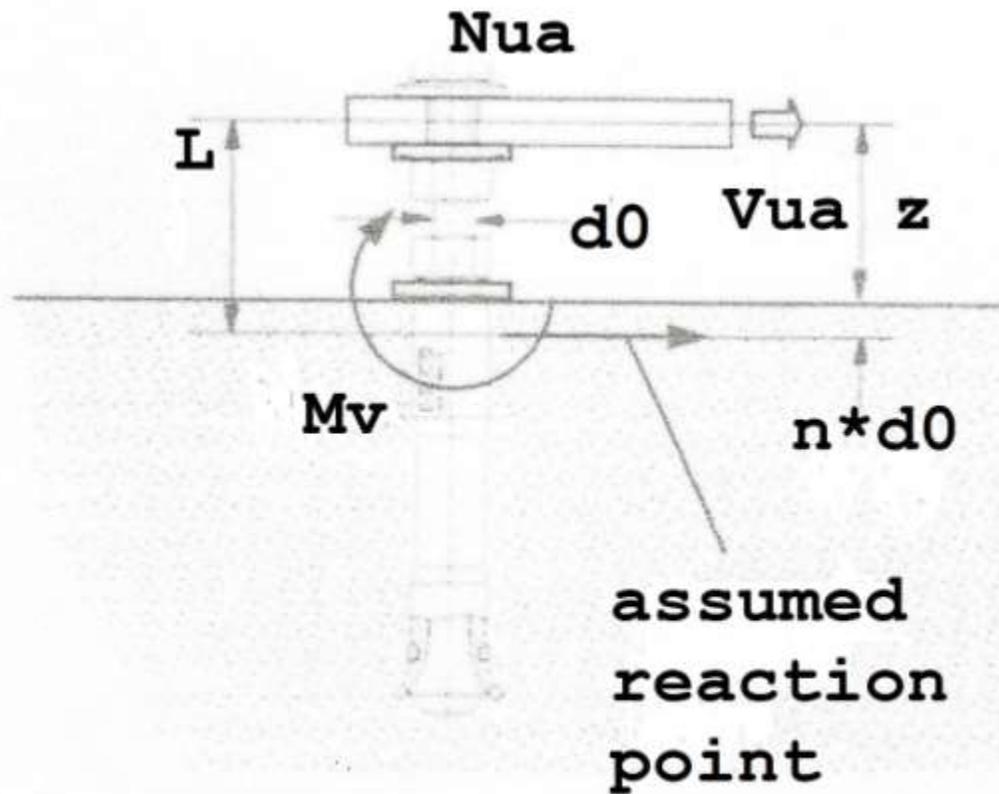


Fig. RD.1.3—Illustrations of stretch length (see D.3.3.4.3(a)).



## TENSION REINFORCEMENT

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As stated in Section D.5.2.9 anchor reinforcement strength may be used instead of concrete breakout strength ( $\phi N_{cbg}$ ) if the following conditions are met:

- Anchor reinforcement must be developed on both sides of the breakout surface.
- $\phi = 0.75$
- Reinforcement should be placed as close to the surface as possible.
- Reinforcement consists of stirrups, ties, or hairpins.
- Reinforcement must be less than  $0.5 \cdot h_{ef}$  from the anchor centerline.
- Research only done with #5 bars and smaller.
- It is good for the anchor reinforcement to enclose the surface reinforcement
- It is generally limited to cast-in anchors.

There are three types of reinforcement given by the Code, namely hooked end, headed ends, and straight bars. Only the third is discussed here. Conservatively, for normalweight concrete, no coating, and # 6 bar or smaller,

$$l_d = f_y \cdot \psi_t \cdot \psi_e \cdot d_b / 25 \cdot \lambda \cdot f_{c1}^{(1/2)} \text{ where}$$

$l_d$  = development length (in.)

$$\psi_t = 1.3 \text{ for } \geq 12 \text{ in. Cast below bars}$$

$$1.0 \text{ elsewhere}$$

$$\psi_e = 1.5 \text{ for epoxy-coated bars with cover less than } 3 \cdot d_b \text{ and/or clear spacing } < 6 \cdot d_b$$

$$1.2 \text{ for other epoxy-coated bars}$$

$$1.0 \text{ for no epoxy coating or galvanized}$$

$$\lambda = \text{less than or equal to } 0.75 \text{ for lightweight concrete}$$

$$\lambda = 1.0 \text{ for normalweight concrete}$$

Two perpendicular sections are shown on the following page.

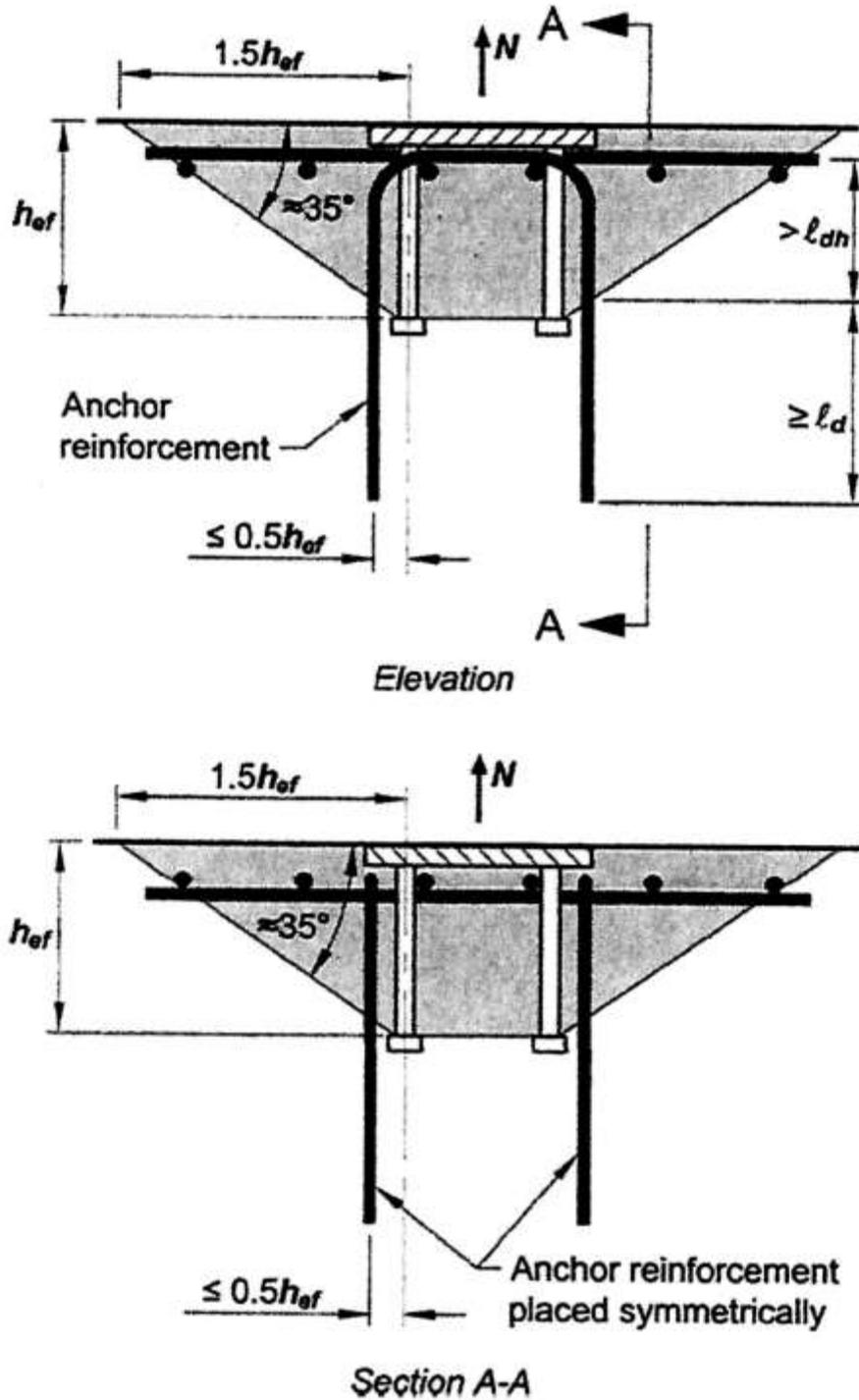


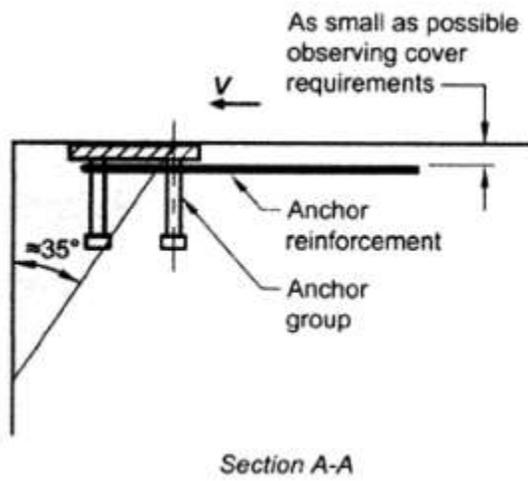
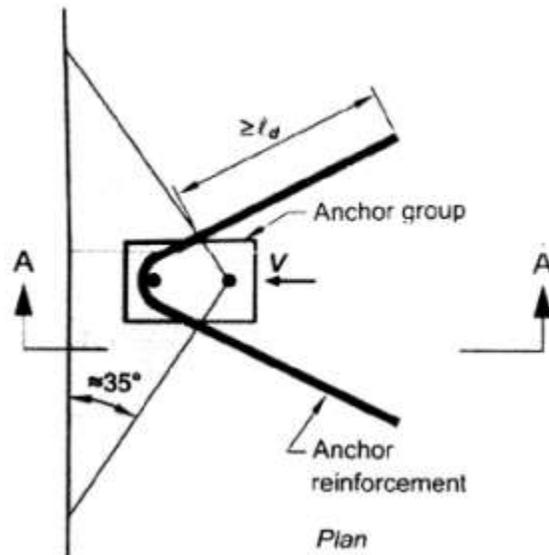
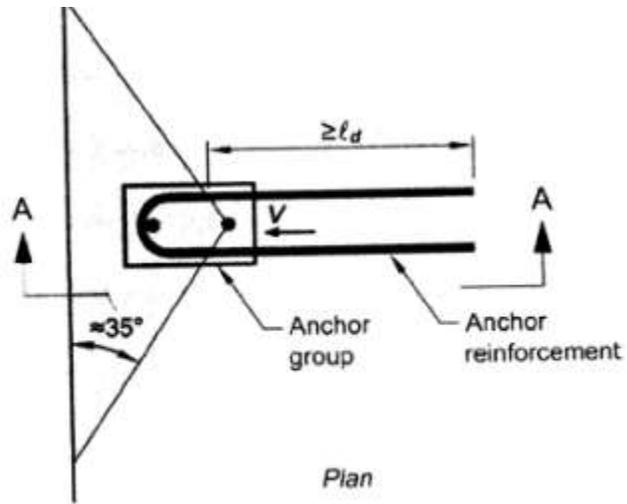
Fig. RD.5.2.9—Anchor reinforcement for tension.

## SHEAR REINFORCEMENT

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Section D.6.2.9 states the reinforcement should be developed on each side of the breakout surface or enclose the anchor and is developed beyond the breakout surface. If either one of these is true, the strength of the reinforcement may be used instead of  $\phi V_n$ , the reduced concrete shear strength. The commentary to Section D.6.2.9 gives the following details to be followed:

- Reinforcement should be properly anchored by hairpins (first page following), hooked bars (second page following), or by stirrups or ties.
- The hairpins should be in contact with the anchor, and as close to the surface as possible.
- Research on hairpins was performed on #5 or smaller bars, larger bars with increased bend radii have decreased effectiveness.
- Reinforcement can also consist of stirrups and ties enclosing the edge reinforcement, and must be placed as close to the anchors as possible. This reinforcement must be spaced less than both  $0.5 \cdot c_{a1}$  and  $0.3 \cdot c_{a2}$  from the anchor centerline. It must be developed on both sides of the breakout surface.
- Since the anchor reinforcement is below the source of the shear, the force in the anchor will be larger than the shear force. This may be seen by taking the sum of the moments of the shear and anchor forces about a point inward of the anchor force. Because the moment arm is shorter for the anchor force, it will be greater for balance of moments. A third force, in the same direction as the applied shear, must also be present for balance of forces.
- $\phi = 0.75$  for shear models



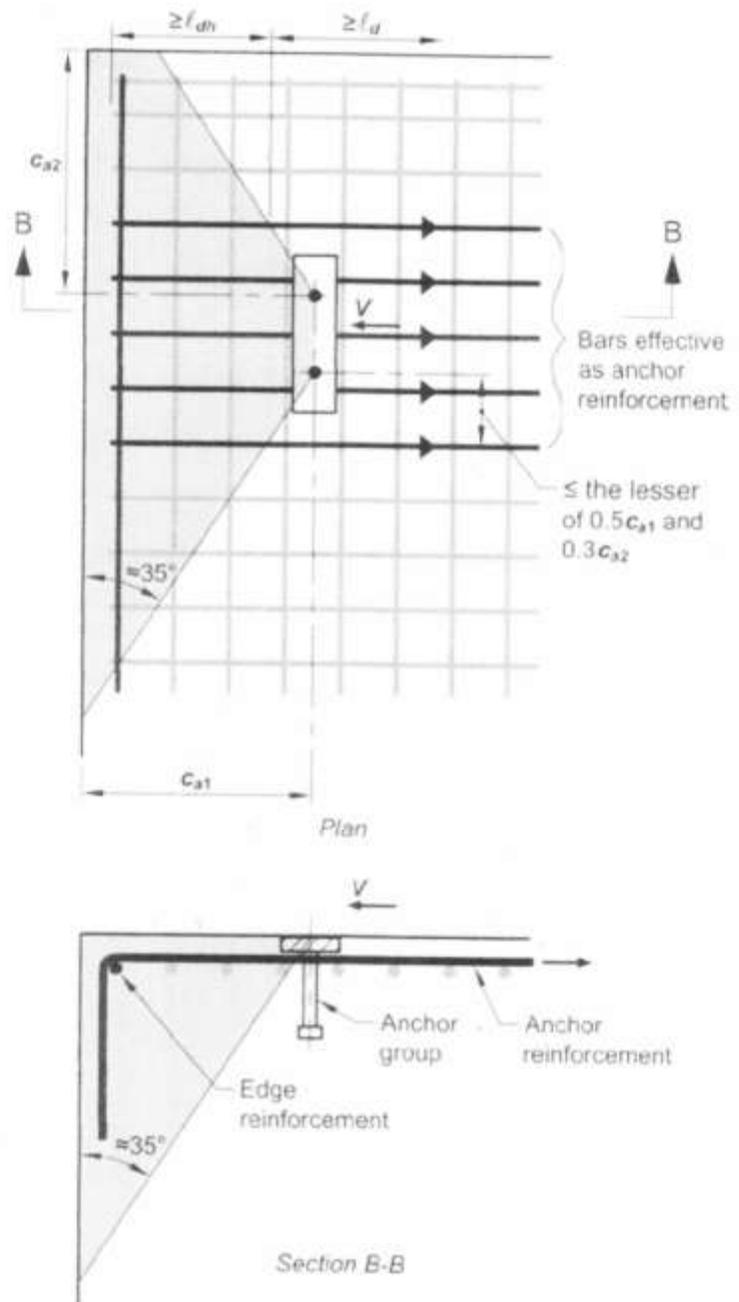
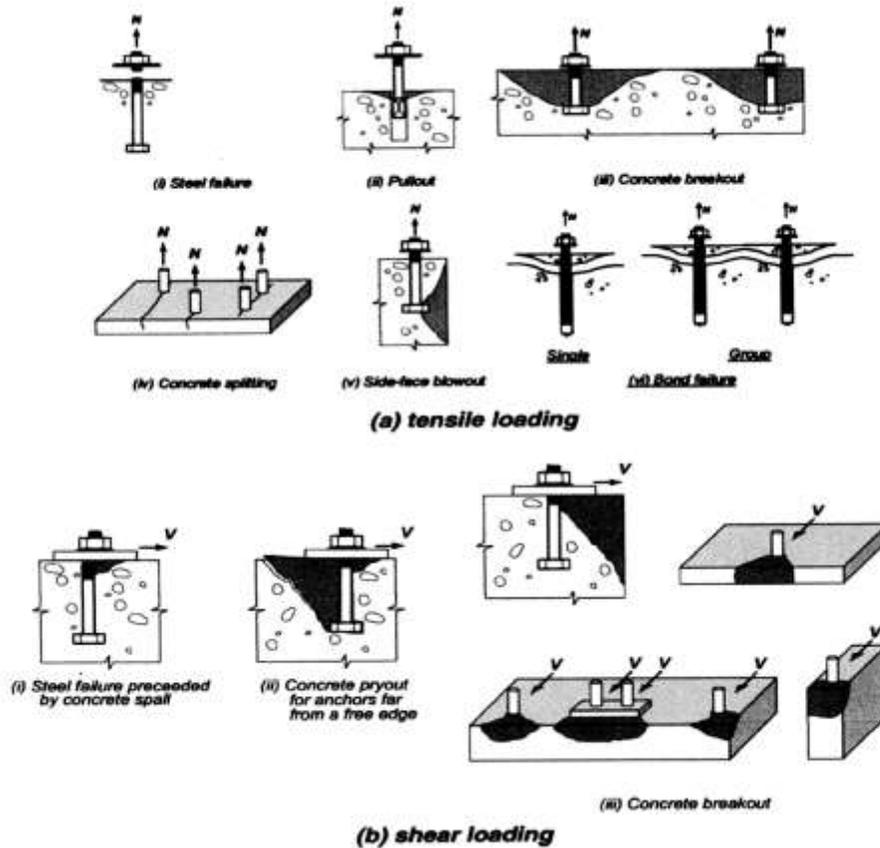


Fig. RD.6.2.9(b)—Edge reinforcement and anchor reinforcement for shear.

## ACI FAILURE MODES

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 This section provides a description of each of the eight (8) failure modes set forth in Appendix D, namely four (4) tensile, one (1) bonding, and three (3) shear. The required definitions and citations are shown in Appendices 1 and 2.



LOADING	LABEL	ACI 318-11 SECTION
Tension	(i) Steel Failure	D.5.1
	(ii) Pullout	D.5.3
	(iii) Concrete Breakout	D.5.2
	(iv) Concrete Splitting	D.8
	(v) Side-Face Blowout	D.5.4
	(vi) Bond Failure	D.5.5
Shear	(i) Steel Failure	D.6.1
	(ii) Concrete Pryout	D.6.3
	(iii) Concrete Breakout	D.6.2

## 1. STEEL STRENGTH OF ANCHOR IN TENSION

The strength of the anchor itself in tension is a function of net anchor diameter, ultimate strength, and capacity reduction factor ( $\phi$ ). The loads, in turn, are increased by a load factor, depending on the most restrictive load combination specified by the governing code. This is the strength design method.

If the ductile failure is requested (material has minimum 14% increase in length and minimum 30% reduction in area at tensile failure), all the other tensile failure modes must have higher allowable strengths so that steel tensile failure governs.

A material commonly used for anchor bolts is ASTM F1554, Grade 36. This specification covers hooked, headed, threaded and nutted rods.

Appendix D requires

$$N_{sa} = A_{se,n} \cdot f_{uta} \text{ where}$$

$$N_{sa} = \text{nominal strength of single anchor, lbf}$$

$$A_{se,n} = \text{bolt dia.}, 0.7854 \cdot (D - 0.9743/n)^2$$

D is nominal diameter, inches  
n is number of thread turns per inch

$$f_{uta} = \text{specified tensile stress, psi}$$

$$\phi = 0.75$$

Yield stress is 36 ksi and ultimate stress varies from 58 to 80 ksi. As noted in Reference 6, two types of rods are used, threads formed by rolling or cutting. Both have the same roots, so that the root area used by each in the AISC method (Reference 7) is not changed.

For thread forming by rolling, the rod initial diameter is, for a nominal 1" diameter bolt, is 0.9067", while that of the rod for thread cutting is 0.9755". This leads to the following comparison:

Rolled Thread	Quantity	Cut Thread
58 ksi Ultimate		80 ksi Ultimate
-----	-----	-----
$\pi \cdot (.9067)^2 \cdot f_{uta} / 4$	$\pi \cdot d^2 \cdot f_{uta} / 4$	$\pi \cdot (.9755)^2 \cdot f_{uta} / 4$
37.499 kip	N <sub>sa</sub>	59.791 kip
28.124 kip	$\phi N_{sa}$	44.843 kip

Appendix D requires:

$N_{sa} = .7854 \cdot (D - .9743/nt)^2 \cdot f_{uta}$  where

N<sub>sa</sub> = nominal tensile capacity based on steel alone

D = nominal anchor diameter = 1 inch

nt = number of threads per inch, 8 for 1 in. dia.

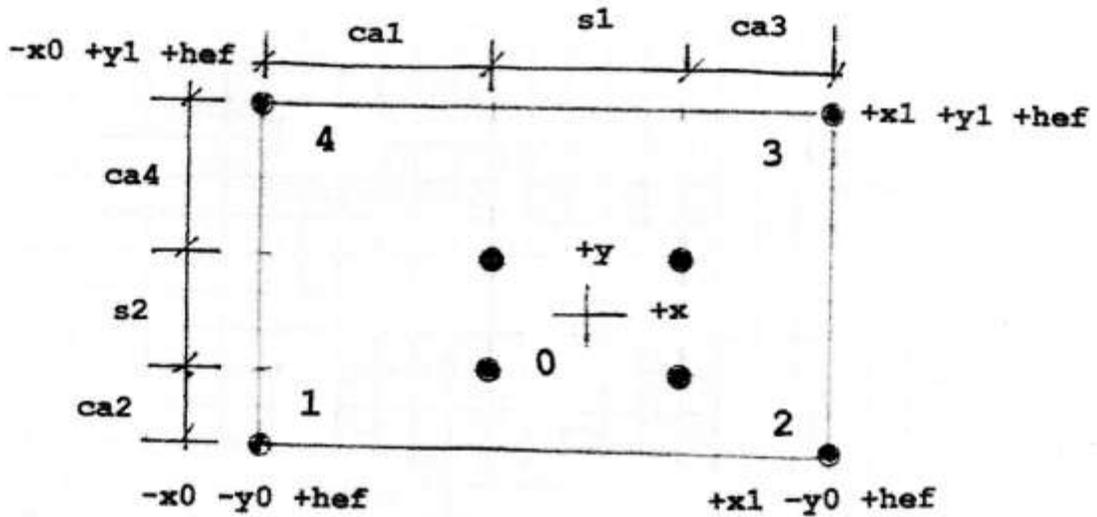
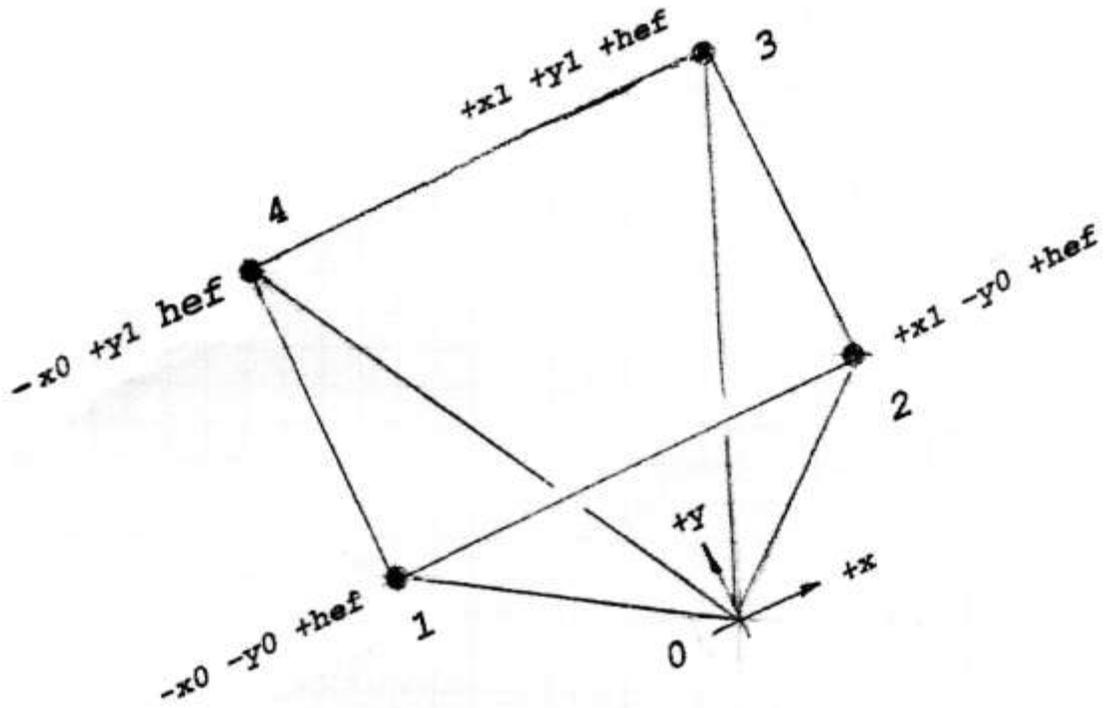
$N_{sa} = 35.133 \text{ kip}$

$\phi N_{sa} = 0.75 \cdot 35.133$   
 $= 26.350 \text{ kip}$

To ensure ductility for threads cut, rather than rolled, for the highest ultimate strength,  $\phi N$  for the other capacities must exceed 44.843 kip, not 26.350 kip, as it would be if the spread in rod sizes and ultimate strengths is neglected. This, however, is not required by Appendix D.

A second specification is for headed studs, i.e., threadless headed rods fillet welded to a steel plate, is the AWS D1.1 Section 7 (Ref. 8).  $f_{uta} = 65000 \text{ psi}$

Shank dia. (in.)	Head dia. (in.)	Head thk. (in.)
-----	-----	-----
1/2	1	9/32
5/8	1-1/4	9/32
3/4	1-1/4	3/8
7/8	1-3/8	3/8
10	1-5/8	1/2



## 2. CONCRETE BREAKOUT STRENGTH OF ANCHOR IN TENSION

The sketch above shows the basic model for concrete breakout in tension, a brittle failure which occurs before yielding of the anchor steel, if so designed. The basis of the design procedure is the Concrete Capacity Design (CCD) method, introduced in the Code Background Paper shown as Reference 9. This method assumes the shape of the fractured area is an inverted pyramid, as shown above, together with a plan view.

The model used for basic breakout strength is one where  $ca_1=ca_2=ca_3=ca_4=1.5*hef$ ,  $hef$  equaling embedment distance, and  $s_1=s_2=0$ . This means the failure planes are oriented at  $\arctan(1.0*hef/1.5*hef) = 33.7^\circ$ .

For example, assume  $hef = 12$  inches, then :

Area each triangular side = 389.4 in.<sup>2</sup>

Surface area (i.e., plan view area) = 1296.0 in.<sup>2</sup>

Ratio of total side area to plan area = 1.20185

The plan areas are determined by 1/2 the absolute value of the cross-product of two of the side vectors. For example, to determine the area of the plane formed by nodes 0,1, and 2, where **01** = the vector from 0 to 1 and **02** = the vector from 0 to 2,

Area =  $1/2|\mathbf{01} \times \mathbf{02}|$ . See Appendix 3 for a program, Anc.c, for calculating the areas of these planes, for the interested reader.

ACI 318-11 uses the plan view in determining capacity, not the sum of the areas of the four inclined planes.

The design sequence to determine the concrete breakout in tension capacity follows as thirteen(13) steps.

- (1) Specify anchor type, concrete weight,  $ca_1-ca_4$ ,  $s_1-s_2$ ,  $hef$ ,  $fc'$ ,  $da$  (anchor diameter), and whether or not the concrete is cracked.
- (2) Check "Concrete Splitting", "Group Effects", and

"General Requirements ", for conformance. Find  $c_{ac}$ ,  $c_{na}$  and  $\phi$ .

- (3)  $k_c = 24$  for cast-in, 17 for post-installed  
 $\lambda = .75, .85, 1.0$  (all-lightweight, sand-lightweight and normal weight concrete)  
 $\lambda_a = \lambda$ , cast-in,  $.8\lambda$  for adhesive anchor
- (4) If : three(3) or more edge distances ( $c_{a1}$ - $c_{a4}$ ) are less than  $1.5 \cdot h_{ef}$ , the value of  $h_{ef}$  used in Steps 5-7 and 10-12 is the larger of  $c_{a,max}/1.5$  and  $s_{,max}/3$ .  
 Else: continue.
- (5) Calculate  $N_b$  = basic breakout strength of single anchor  

$$N_b = k_c \cdot \lambda_a \cdot (f_c')^{1/2} \cdot h_{ef}^{1.5}$$
 If : type = cast-in headed studs or bolts and  $11 \text{ in.} \leq h_{ef} \leq 25 \text{ in.}$   
 $N_b$  may also be taken as  

$$16 \cdot \lambda_a \cdot (f_c')^{1/2} \cdot h_{ef}^{5/3}$$
 Else : continue.
- (6)  $\psi_{ec,n}$  = modifier for eccentricity of loads  
 $\Psi_{ec,n} = 1 / (1 + 2 \cdot e_n' / (3 \cdot h_{ef})) \leq 1.0$  where  
 $e_n'$  = distance from tension centroid of a group of anchors to the resultant tension load
- (7)  $\psi_{ed,n}$  = modifier based on edge conditions  
 If :  $c_{a,max} \geq 1.5 \cdot h_{ef}$ , then  $\psi_{ed,n} = 1.0$   
 Else :  $\psi_{ed,n} = 0.7 + 0.3 \cdot c_{a,min} / (1.5 \cdot h_{ef})$
- (8)  $\psi_{c,n}$  = modifier based on cracked state  
 $\Psi_{c,n} = 1.0$ , cracking at service loads  
 $\Psi_{c,n} = 1.25$ , cast-in, no cracking  
 $\Psi_{c,n} = 1.4$ , post-installed, no cracking
- (9)  $\psi_{cp,n}$  = modifier for post-installed anchors designed for uncracked concrete without supplementary reinforcement.  
 If:  $c_{a,min} \geq c_{ac}$ , tehen  $\psi_{cp,n} = 1.0$

Else :  $\psi_{cp,n} = c_{a,min}/c_{ac}$  and  $\geq 1.5 \cdot h_{ef}/c_{ac}$

(10)  $A_{nc}$  = projected concrete failure of single anchor , not limited by edge distance or spacing

$$A_{nc} = (1.5 \cdot h_{ef} + 1.5 \cdot h_{ef}) \cdot (1.5 \cdot h_{ef} + 1.5 \cdot h_{ef})$$

$$A_{nc} = 9 \cdot h_{ef}^2$$

(11)  $A_{nc}$  = projected failure area,  $\leq 1$  anchor

$$A_{nc} = (c_{a1} + s_1 + c_{a3}) \cdot (c_{a2} + s_2 + c_{a4})$$

(12)  $N_{cb}$  = nominal concrete breakout strength, one anchor

$$N_{cb} = (A_{nc}/A_{nco}) \cdot \psi_{ed,n} \cdot \psi_{c,n} \cdot \psi_{cp,n} \cdot N_b$$

(13)  $N_{cbg}$  = nominal concrete breakout strength,  $> 1$  anchor

$$N_{cbg} = (A_{nc}/A_{nco}) \cdot \psi_{ec,n} \cdot \psi_{ed,n} \cdot \psi_{c,n} \cdot \psi_{cp,n} \cdot N_b$$

### 3. PULLOUT STRENGTH OF CAST-IN, AND POST-INSTALLED EXPANSION ANCHORS IN TENSION

In this category, the  $33.7^\circ$  breakout cone does not develop, and bond is lost between the anchor shaft and concrete. Adhesive anchors are not covered in this section, and group effects are not considered. For a single headed bolt, pullout strengths are directly proportional to the head area and concrete strength.

For a single hooked bolt, the pullout strength is directly proportional to the concrete strength, the bolt diameter, and the distance from the inner bolt surface to the outer tip of the L- or J- bolt. Expansive anchors are not calculated by formula, but must be tested to ACI 355.2. See, for example, Reference 2, for analysis and design information.

Definitions for this section:

$A_{brg}$  = net bearing area, i.e., gross head or washer





- d1 = equivalent diameter of nut or head  
d2 = washer o.d.  
A = plan area of heavy hex nut or head (hexagon)  
thk = thickness of washer (to be designed)
- (1) Find required  $A_{brg} \rightarrow \phi^8 * A_{brg} * f_c' = \text{capacity}$
  - (2) Find plan area of hexagon  
 $A = 1.5 * F^2 * \tan(30^\circ)$  where  
F = flat-opposite flat distance, in.  
 $A_{brg, \text{nut or head}} = A - (\pi/4) * d_0^2$   
If :  $A_{brg, \text{nut or head}} \geq A_{brg, \text{required}}$ , exit.  
Else : Continue.
  - (3) Find outside diameter of washer to provide sufficient  $A_{brg}$ .  
 $(\pi/4) * d_2^2 = (\pi/4) * d_0^2 + A_{brg, \text{required}}$   
Solve for d2.
  - (4) Find equivalent diameter of nut or head  
Solve  $A = (\pi/4) * d_1^2$  for d1
  - (5) Find  $N_{sa} = \text{capacity of single anchor} = f_{uta} * A_{s, ne}$
  - (6) Find load on washer  $(d_2^2 - d_1^2) / (d_2^2 - d_0^2) * N_{sa}$
  - (7) The cantilever load in (6) is spread over a distance of  $\pi * (d_2 + d_1) / 2$  with a moment arm of  $(d_2 - d_1) / 4$ .  
Moment = force in (6) \* moment arm
  - (8)  $Z = \text{plastic section modulus} = \text{length of strip beyond } d_1 * thk^2 / 4$
  - (9) Moment = stress \* Z, where stress =  $f_y$   
Solve (9) for thk.

#### 4. CONCRETE SIDE-FACE BLOWOUT STRENGTH OF HEADED ANCHORS IN TENSION

The single anchor and group anchor formulas in this category cover the situation where embedment length is much greater than the nearest edge distance, with a ratio of 2-1/2 times, i.e.,  $h_{ef} \geq 2.5 * c_{a1}$ . These requirements are applicable to headed anchors, which are usually cast-in.

$N_{sb}$  = nominal side-face blowout strength, 1 anchor  
 $N_{sbg}$  = nominal side-face blowout strength, > 1 anchor  
s = distance between outer anchors along the edge

Three (3) cases exist for single anchors:

- (1)  $ca2 \geq 3*ca1$   
 $Nsb = 160*ca1*(A_{brg})^{(1/2)}*\lambda*(f'c')^{(1/2)}$
- (2)  $1 \leq ca2/ca1 < 3.0$   
 Multiply Nsb above by  $(1+ca2/ca1)/4$
- (3)  $ca2 < ca1 \rightarrow$  interchange roles of ca2 and ca1

For multiple headed anchors with  $hef \geq 2.5*ca1$  and  $s$  less than  $6*ca1$ ,

$$Nsb_g = (1+s/6*ca1)*Nsb$$

5. BOND STRENGTH OF ADHESIVE ANCHORS IN TENSION

This category includes only adhesive anchors and are analyzed both as single anchors and groups of anchors. Appendix D characterizes the minimum bond stress as:

Installation Environment	Moisture at Install.	Peak Service Temp.	$\tau_{cr}$ psi	$\tau_{uncr}$ psi
Outdoor	Dry to Fully Saturated	175°	200	650
Indoor	Dry	110°	300	1000

These values may only be used if:

- (a) Tested to ACI 355.4
- (b) Holes drilled only with rotary impact drills or rock drills
- (c) At installation concrete strength  $\geq 2500$  psi
- (d) Concrete age at installation  $\geq 24$  days
- (e) Temperature at installation  $\geq 50^\circ F$

Reference 13 gives four major categories of factors influencing bond strength :

- (a) In-Service
  - Possibility of creep at high temperatures

- In-service moisture - can degrade adhesion by moisture penetration into adhesive to soften it, between adhesive and substrate destroying bond, and penetrating into porous substrates causing swelling and detrimental movement
  - Freeze-thaw
- (b) Adhesive
- Curing time when first loaded - 24 hour/7 day loading = 81% of bond strength
  - Bond line thickness - the smaller this dimension, the lesser potential for creep
- (c) Installation
- Hole orientation - vertical and upwardly inclined holes are difficult to fill with adhesive
  - Hole drilling - diamond-core drills not recommended as they produce a very smooth-sided hole, as increased surface roughness increases bond strength
  - Hole cleaning - non-metallic brushes should be used as metallic brushes tend to polish the side of the hole
- (d) Concrete
- Harder coarse aggregates produce higher bond strengths
  - Cracked concrete - significantly reduces bond strength

Analysis of the bond strength uses the following terms:

Ana = projected influence area of one or more anchors, for calculation of bond strength in tension, in.<sup>2</sup>

Anao = projected influence area of a single adhesive anchor, for calculation of bond strength in tension, not limited by edge distance or spacing, in.<sup>2</sup>

cac = critical edge distance controlled by concrete breakout or bond, uncracked concrete, no supplementary reinforcement

- $c_{a,min}$  = minimum distance from anchor bolt center to concrete edge, in.
- $c_{na}$  = projected distance from center of an anchor shaft on one side of the anchor required to develop the full bond strength of an adhesive anchor, in.
- $N_a$  = nominal bond strength in tension of a single anchor, lbf
- $N_{ag}$  = nominal bond strength in tension of a group of adhesive anchors, lbf
- $N_{ba}$  = basic bond strength of a single adhesive anchor in cracked concrete, lbf
- $\psi_{cp,na}$  = modifier for uncracked concrete, no supplementary reinforcement  
 if :  $c_{a,min} \geq c_{ac}$ ,  $\psi_{cp,na} = 1$   
 else :  $\psi_{cp,na} = c_{a,min}/c_{ac}$  but not less than  $c_{na}/c_{ac}$
- $\psi_{ec,na}$  = modifier for eccentricity  
 $= 1/(1+2*e'n/3*hef)$
- $\psi_{ed,na}$  = modifier for edge distance  
 if :  $c_{a,min} \geq c_{na}$ ,  $\psi_{ed,na} = 1$   
 else :  $\psi_{ed,na} = 0.7+0.3*(c_{a,min}/c_{na})$
- $\tau_{cr}$  = characteristic bond stress of adhesive anchor in cracked concrete, psi  
 $= 200$  psi, outdoor, 175 °F max.
- $\tau_{uncr}$  = characteristic bond stress of adhesive anchor in uncracked concrete  
 $= 650$  psi, outdoor, 175°F max.
- $\lambda$  = .75, .85, 1 for all-lightweight concrete, sand-lightweight concrete, and normalweight concrete, respectively
- $\lambda_a$  =  $1.0\lambda$  for cast-in,  $0.8\lambda$  for concrete failure, adhesive anchor,  $0.6\lambda$  for concrete bond failure, adhesive anchor

Now using the definitions above and illustrations  
 From tension concrete breakout, the capacity equations  
 may now be solved :

$$c_{ac} = 2*hef \text{ or tests to ACI 355.4.}$$

$$c_{na} = 10*d_a*(\tau_{uncr}/1100)^{(1/2)}$$

$$N_{ba} = \lambda_a*\tau_{cr}*\Pi*d_a*hef$$

if anchor designed to resist sustained loads:  
 $0.55*\phi*Nba \geq Nua,a$   
 else : continue

$Na = (Ana/Anao)*\psi_{ed,na}*\psi_{cp,na}*Nba$

$Nag = (Ana/Anao)*\psi_{ec,na}*\psi_{ed,na}*\psi_{cp,na}*Nba$

See "General Requirements" for  $\phi$ .

## 6. STEEL STRENGTH OF ANCHOR IN SHEAR

In this category headed studs are welded to a base plate, developing a higher steel strength in shear than headed bolts, hooked bolts, or post-installed anchors by themselves, due to the fixity given by the welds between stud and base plate.

$A_{se,v} = 0.7854*(D-.9743/nt)^2$

$\phi = 0.65$

Type of Anchor	Vsa
-----	-----
cast-in headed stud anchor	$1.0*A_{se,v}*f_{ult}$
cast-in headed bolt and hooked anchors and for post-installed anchors where sleeves do not extend through the shear plane	$0.6*A_{se,v}*f_{ult}$
post-installed anchors where sleeves extend through the shear plane	ACI 355.2 tests

where anchors are used with built-up grout pads, multiply values above by 0.80

## 7. CONCRETE BREAKOUT STRENGTH IN SHEAR

The formulas in this section are based on a 33.7° breakout angle, and use fracture mechanics theory. Breakout in shear depends on

- (1) number of anchors
- (2) spacings
- (3) edge distances

## (4) thickness of concrete

The following terms used in capacity calculations are:

- $A_{vc}$  = projected failure area for shear, in.<sup>2</sup>  
 $A_{vco}$  = projected failure area for shear, 1 anchor, not limited by edge or concrete depth  
 $ca_1$  = distance from anchor at surface perpendicular to edge (vector **05**  $\perp$  **12** on the following page), in.  
 $ha$  = concrete thickness, in.  
 $le$  = load bearing length for anchor in shear, in.  
 $le < = 8 \cdot da$   
 $V_b$  = basic concrete breakout strength in shear, one anchor, cracked concrete, lbf  
 $V_{cb}$  = nominal concrete breakout strength in shear, one anchor, lbf  
 $V_{cbg}$  = nominal concrete breakout strength in shear. greater than one anchor, lbf

A design sequence follows:

- (1) Using the diagrams on parts 2. and this part find  $ca_1$ ,  $ca_2$ ,  $ca_4$ ,  $ha$ ,  $s_1$ , and  $s_2$ .

Note that:

In tension  $ca_1$  = min. edge distance, and in shear  $ca_1$  = distance to edge in direction of shear.

Height of vertical block =  $ha$  if  $ha < 1.5 \cdot ca_1$

Else height of vertical block =  $1.5 \cdot ca_1$

Width of vertical block left distance the lesser of  $1.5 \cdot ca_1$  and  $ca_2$  and right distance the lesser of  $ca_4$  and  $1.5 \cdot ca_1$ .

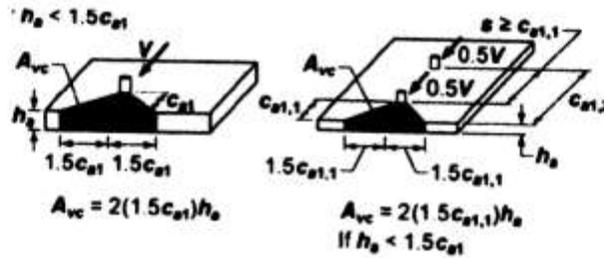
If  $s \geq ca_1$ , evaluate cases 1 and 2 on second diagram in this section.

Else evaluate case 3.

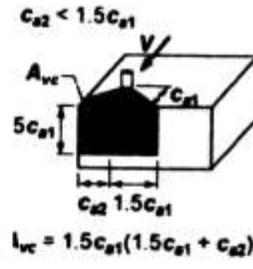
- (2)  $\psi_{ec,v}$  = modification factor for eccentricity,  
 $1 / (1 + 2 \cdot e_v' / 3 \cdot ca_1)$
- (3)  $\psi_{ed,v}$  = modification factor for edge distance,  
 if :  $ca_2$  and  $ca_4 \geq 1.5 \cdot ca_1$ ,  $\psi_{ed,v} = 1$   
 else :  $\psi_{ed,v} = 0.7 + 0.3 \cdot (\text{lower of } ca_2, ca_4 / 1.5 \cdot ca_1)$
- (4)  $\psi_c$  = modification factor for cracks =  
 = 1.4 no cracking

- = 1.2 cracking and #4 bar between anchor  
And edge
- = 1.0 cracking and bar smaller than #4 bar
- (5)  $\psi_{h,v}$  = modification factor for concrete  
thickness  
=  $(1.5 \cdot ca_1 / ha)^{(1/2)}$  where  $ha \leq 1.5 \cdot ca_1$   
and not less than 1
- (6)  $Avco = 4.5 \cdot ca_1^2$
- (7) See the the diagram this section for  
Calculation of  $Avc$ .  
 $Avc$  shall not exceed number of anchors  $\cdot Avco$
- (8) For a single anchor:
- | $ca_2$         | $ca_4$         | $ha$           | $Avc$  |
|----------------|----------------|----------------|--|
| $\geq 1.5ca_1$ | $\geq 1.5ca_1$ | $\geq 1.5ca_1$ | $1.5 \cdot ca_1 \cdot (3 \cdot ca_1)$          |
| $\geq 1.5ca_1$ | $\geq 1.5ca_1$ | $< 1.5ca_1$    | $ha \cdot (3 \cdot ca_1)$                      |
| $\geq 1.5ca_1$ | $< 1.5ca_1$    | $\geq 1.5ca_1$ | $1.5 \cdot ca_1 \cdot (1.5 \cdot ca_1 + ca_4)$ |
| $\geq 1.5ca_1$ | $< 1.5ca_1$    | $< 1.5ca_1$    | $ha \cdot (1.5 \cdot ca_1 + ca_4)$             |
| $< 1.5ca_1$    | $\geq 1.5ca_1$ | $\geq 1.5ca_1$ | $1.5 \cdot ca_1 \cdot (1.5 \cdot ca_1 + ca_2)$ |
| $< 1.5ca_1$    | $\geq 1.5ca_1$ | $< 1.5ca_1$    | $ha \cdot (1.5 \cdot ca_1 + ca_2)$             |
| $< 1.5 ca_1$   | $< 1.5ca_1$    | $\geq 1.5ca_1$ | $1.5 \cdot ca_1 \cdot (ca_2 + ca_4)$           |
| $< 1.5 ca_1$   | $< 1.5ca_1$    | $< 1.5ca_1$    | $ha \cdot (ca_2 + ca_4)$                       |
- (9)  $V_b$  = smaller of:  
 $7 \cdot (l_e / d_o)^{(0.2)} \cdot d_o^{(1/2)} \cdot (f_c')^{(1/2)} \cdot ca_1^{(1.5)}$   
and  
 $9 \cdot \lambda_a \cdot (f_c')^{(1/2)} \cdot ca_1^{(1.5)}$
- (10)  $V_{cb} = (Avc / Avco) \cdot \psi_{e,d,v} \cdot \psi_{c,v} \cdot \psi_{h,v} \cdot V_b$
- (11)  $V_{cbg} = (Avc / Avco) \cdot \psi_{e,c,v} \cdot \psi_{e,d,v} \cdot \psi_{c,v} \cdot \psi_{h,v} \cdot V_b$

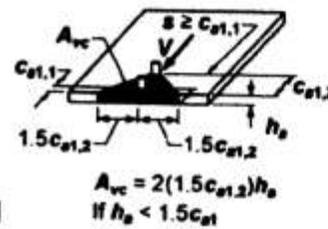
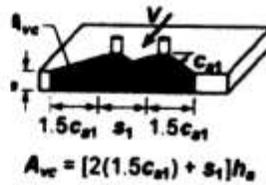
See GENERAL REQUIREMENTS for  $\phi$ .



**Case 1:** One assumption of the distribution of forces indicates that half of the shear force would be critical on the front anchor and the projected area. For the calculation of concrete breakout,  $c_{d1}$  is taken as  $c_{d1,1}$ .

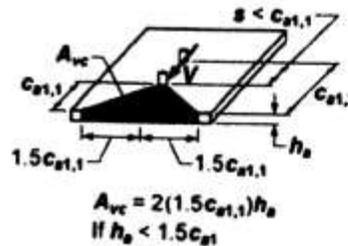


$h_a < 1.5c_{d1}$  and  $s_1 < 3c_{d1}$



**Case 2:** Another assumption of the distribution of forces indicates that the total shear force would be critical on the rear anchor and its projected area. Only this assumption needs to be considered when anchors are welded to a common plate independent of  $s$ . For the calculation of concrete breakout,  $c_{d1}$  is taken as  $c_{d1,2}$ .

**Note:** For  $s \geq c_{d1,1}$ , both Case 1 and Case 2 should be evaluated to determine which controls for design except as noted for anchors welded to a common plate.



**Case 3:** Where  $s < c_{d1,1}$ , apply the entire shear load  $V$  to the front anchor. This case does not apply for anchors welded to a common plate. For the calculation of concrete breakout,  $c_{d1}$  is taken as  $c_{d1,1}$ .

## 8. CONCRETE PRYOUT STRENGTH OF ANCHOR IN SHEAR

This may govern if the anchor is short, and is reflected in the design equations by halving the capacity if  $h_{ef}$ , the effective embedment depth, is less than 2-1/2 inches.

$K_{cp}$  = coefficient for pryout strength  
= 1.0 for  $h_{ef} < 2.5$  inches and  
= 2.0 for  $h_{ef} \geq 2.5$  inch

$V_{cp}$  = nominal pryout strength, 1 anchor, lbf

$V_{cpg}$  = nominal pryout strength, > 1 anchor, lbf

$V_{cp} = k_{cp} * N_{cp}$  where:

= use  $N_{cb}$  (part 2) for cast-in, expansion, or undercut anchors and the lesser of  $N_{cb}$  (part 2) and  $N_a$  (part 5) for adhesive anchors

$V_{cpg} = k_{cp} * N_{cpg}$  where:

= use  $N_{cbg}$  (part 2) for cast-in, expansion, or undercut anchors and the lesser of  $N_{cbg}$  (part 2) and  $N_{ag}$  (part 5) for adhesive anchors

See GENERAL REQUIREMENTS for  $\phi$ .

## 9. INTERACTION OF TENSILE AND SHEAR FORCES

An interaction formula for the ratio of factored load to nominal strength times the appropriate capacity reduction factor is given for both tension and shear ratios greater than 0.2. It is called the trilinear interaction approach, although any other formula verified by test data may be used.

Determine load factors from applicable Code.

$N_{ua} = \text{factor} * \text{service load}$

$V_{ua} = \text{factor} * \text{service load}$

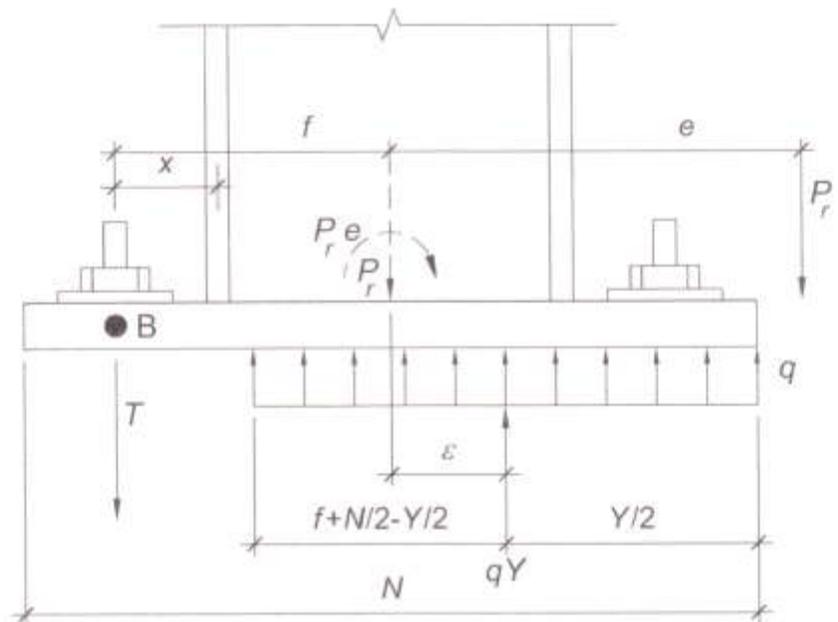
Using the lowest values of  $\phi N_n$  and  $\phi V_n$  for all combinations of parts 1 through 8,



near base plate outer edge  
N = plate dimension parallel to plane of moment  
Pr = factored vertical force in colimn  
qmax = maximum bearing force per unit length in the  
N direction. This is the basic assumption of  
this method, i.e., that the compressive  
force is constant throughout the contact  
area  
tf = flange thickness of column



*Column moment base using stool*



*Base plate with large moment*

reqd = required base plate thickness with two values, one at the compressive side and one at the tension side

Tu = ultimate tension load on member

x = used in calculation of base plate yield on tension end

Y = length of compression pressure parallel to plane of moment

Now this procedure may be used:

- (1) Establish  $A_1, A_2, B, d, f, f_c', F_y, N, \text{service loads}, t_f$
- (2)  $P_r = 1.2 \cdot DL \text{ force} + 1.6 \cdot LL \text{ force}$   
 $M_r = 1.2 \cdot D_l \text{ moment} + 1.6 \cdot LL \text{ moment}$
- (3)  $e = M_r / P_r$
- (4)  $f_{bmax} = \phi \cdot .8f \cdot f_c' \cdot (A_2 / A_1)^{(1/2)}$   $A_2 \leq 2 \cdot A_1$   
 $\phi = 0.65$
- (5) Find  $q_{max} = f_{bmax} \cdot B$
- (6) Is  $(f + N/2)^2 \geq 2 \cdot P_r \cdot (f + e) / q_{max}$  ?  
 If so, continue  
 If not, design with parameters used is not possible, base plate must probably be larger
- (7) Find  $e_{crit} = N/2 - P_r / (2 \cdot q_{max})$
- (8) If  $e > e_{crit}$ , need tension anchor  
 If not, go to step (15)
- (9) Solve for Y  

$$Y = f + N/2 \pm \left( (f + N/2)^2 - 2 \cdot P_r \cdot (f + e) / q_{max} \right)^{(1/2)}$$
- (10)  $T_u = q_{max} \cdot Y - P_r$
- (11)  $m = (N - .95 \cdot d) / 2$
- (12) If  $Y \geq m$  :  
 $reqd \text{ for compression end} = 1.5 \cdot m \cdot (f_{pmax} / F_y)^{(1/2)}$   
 If  $Y < m$  :  
 $reqd \text{ for compression end} = 2.11 \cdot (f_{pmax} \cdot (m - Y/2) / F_y)^{(1/2)}$
- (13)  $x = f - d/2 + t_f/2$
- (14)  $reqd \text{ for tension end} = 2.11 \cdot (T_u \cdot x / (B \cdot F_y))^{(1/2)}$   
 End of process - proceed to design anchor
- (15) Tension anchor not required

## SHEAR TRANSFER

●●●●●●●●●●●●●●●●

There are four methods of shear transfer from a column to the foundation. They are friction between the base plate and concrete bearing of the column and base plate and/or shear lug, shear by the anchor bolt strength without hairpin reinforcement, and shear through the anchor bolts to hairpin reinforcement. The latter two are shown above.

### FRICTION

.....

This method depends upon the magnitude of the vertical load above the base plate. This load may just be the dead load. Here we have

$$\phi V = \mu * P < = 0.2 * \phi * f_c' * A_p \text{ where}$$

V = factored shear load = 1.6 \* service load

$\phi$  = capacity reduction factor = 0.65 in bearing

$\mu$  = coefficient of friction, .55 for steel on grout,  
0.7 for steel on concrete

$A_p$  = base plate area

P = minimum vertical load

### SHEAR LUG

.....

This method is shown by the top diagram on the page following . Also shown on this sheet are elevations of possible hairpin placements.

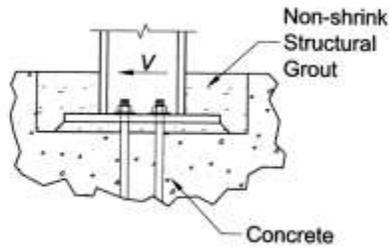
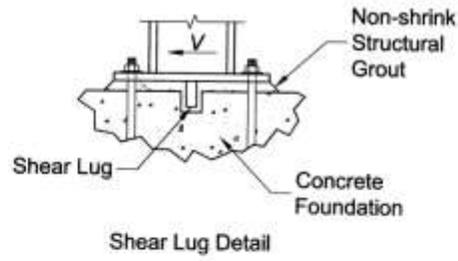
The following definitions are needed in the calculations.

$A_b$  = area of lug contacting concrete in compression

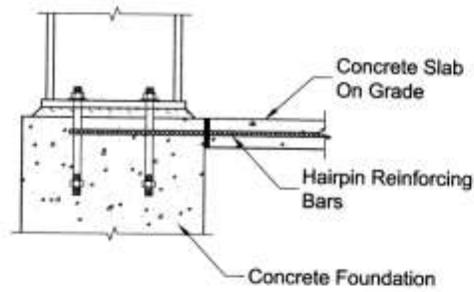
$A_v$  = projected area in vertical plane used to calculate concrete tensile strength to resist concrete shear failure

$F_{exx}$  = specified tensile strength of weld filler material

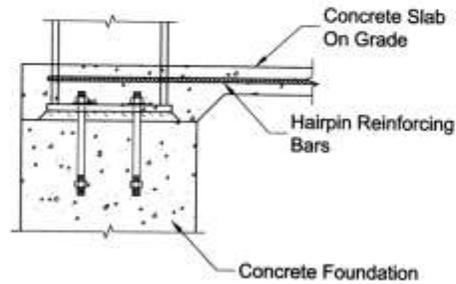
G = thickness of grout layer between the bottom of the base plate and the top surface of concrete foundation



*Transfer of base shears through bearing*



*Typical detail using hairpin bars*



*Alternate hairpin detail*

H	=	total lug height
H-G	=	effective lug height
L	=	length of lug, placed in the middle of the pier, perpendicular to the direction of shear
Mu	=	ultimate moment load at base of lug
L	=	length of lug
t	=	lug thickness
tp1	=	base plate thickness, take equal to t
Vu	=	factored design shear
Wpier	=	lug width (depth)
Z	=	plastic modulus of lug, weak direction
$\phi$	=	capacity reduction factor, depends on process

The criteria to be evaluated here are the compressive strength of the concrete in front of the lug, the shear strength evaluated on the projection of a plane at 45° from the bearing area of the lug to the face of the pier (not including the bearing area of the lug) and the strength of the weld from the lug to the base plate.

- (1) Collect information on baseplate plan dimensions,  $f_c'$ ,  $F_{exx}$ ,  $G$ , service shear load, steel grade  
Assume  $L$ ,  $G$ ,  $H$ , and  $t$ , to be verified.
- (2) Find  $V_u = 1.6 \times \text{service shear load}$
- (3) Check that the effective lug compressive area,  $A_b = L \times (H-G)$ , is sufficient  
Required  $A_b \geq V_u / (\phi \times 0.85 \times f_c')$ ,  $\phi = 0.75$
- (4) The available shear area is approximated as a rectangle with width equal to  $L$  times  $L/2 - \text{the effective area of the lug}$   
 $A_v = L \times L/2 - L \times (H-G)$
- (5) Take the tensile allowable tensile stress as  $4 \times \phi \times (f_c')^{1/2}$ . Check that this stress  
 $A_v > V_u$ ,  $\phi = 0.75$
- (6) Calculate moment at connection of lug to base plate  
 $M_u = V_u \times (G + (H-G)/2) = V_u \times (H+G)/2$
- (7) Find required plastic modulus and lug

- thickness
- $Z = \frac{M_u}{(\phi * f_y)}, \quad \phi = 0.90$
- (8) Check lug thickness
- Solve  $Z = \frac{L * t^2}{4}$  for t
- (8) Find size of welds (one on each side) attaching lug to base plate. There are two shear  $\perp$  loads on the welds, namely  $V_u/2$  on each side and the moment-caused shear from  $M_u$  above, i.e.,  $M_u = M_u/t$ , conservatively. Call these weld 1 (both sides) and weld 2 (each side), respectively.

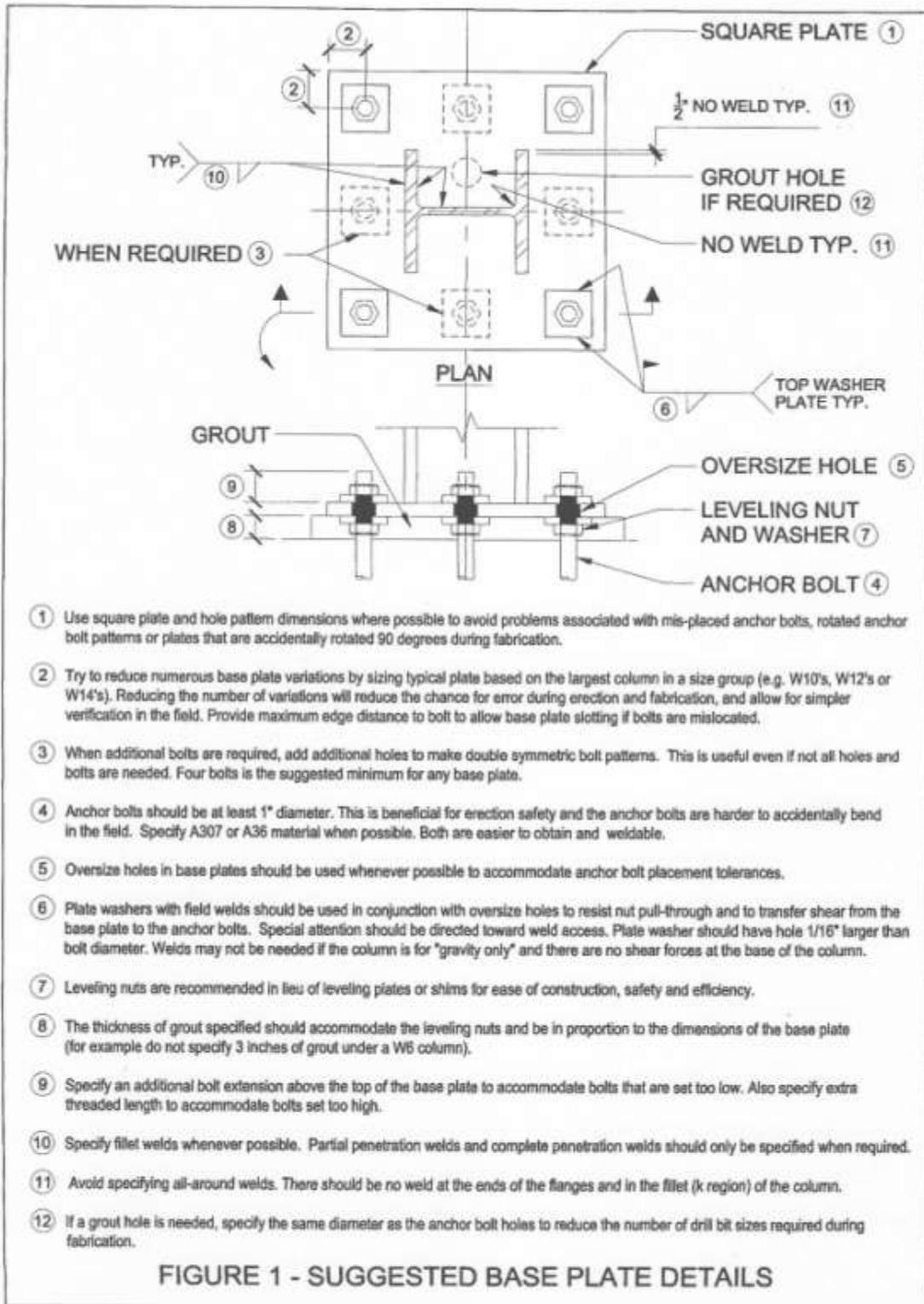
$$\text{Total shear each side} = (\text{weld 1}^2 + \text{weld 2}^2)^{(1/2)}$$

Strength of weld in shear =  
 $(L - 2 * \text{throat}) * \text{throat} * \phi * 0.60 * F_{exx}, \quad \phi = .75$   
 Solve for throat  
 For a symmetrical fillet weld (height=base)  
 $\text{size} = 2^{(1/2)} * \text{throat}$   
 Solve for weld size and round up to nearest sixteenth.  
 Do not use weld-all-around symbol, stopping one weld size each end of lug.

## PRACTICAL CONSIDERATIONS

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The figure shown below, from Reference 15, "Practical Design and Detailing of Steel Column Base Plates", provides a very good checklist for column base plates.



- ① Use square plate and hole pattern dimensions where possible to avoid problems associated with mis-placed anchor bolts, rotated anchor bolt patterns or plates that are accidentally rotated 90 degrees during fabrication.
- ② Try to reduce numerous base plate variations by sizing typical plate based on the largest column in a size group (e.g. W10's, W12's or W14's). Reducing the number of variations will reduce the chance for error during erection and fabrication, and allow for simpler verification in the field. Provide maximum edge distance to bolt to allow base plate slotting if bolts are mislocated.
- ③ When additional bolts are required, add additional holes to make double symmetric bolt patterns. This is useful even if not all holes and bolts are needed. Four bolts is the suggested minimum for any base plate.
- ④ Anchor bolts should be at least 1" diameter. This is beneficial for erection safety and the anchor bolts are harder to accidentally bend in the field. Specify A307 or A36 material when possible. Both are easier to obtain and weldable.
- ⑤ Oversize holes in base plates should be used whenever possible to accommodate anchor bolt placement tolerances.
- ⑥ Plate washers with field welds should be used in conjunction with oversize holes to resist nut pull-through and to transfer shear from the base plate to the anchor bolts. Special attention should be directed toward weld access. Plate washer should have hole 1/16" larger than bolt diameter. Welds may not be needed if the column is for "gravity only" and there are no shear forces at the base of the column.
- ⑦ Leveling nuts are recommended in lieu of leveling plates or shims for ease of construction, safety and efficiency.
- ⑧ The thickness of grout specified should accommodate the leveling nuts and be in proportion to the dimensions of the base plate (for example do not specify 3 inches of grout under a W6 column).
- ⑨ Specify an additional bolt extension above the top of the base plate to accommodate bolts that are set too low. Also specify extra threaded length to accommodate bolts set too high.
- ⑩ Specify fillet welds whenever possible. Partial penetration welds and complete penetration welds should only be specified when required.
- ⑪ Avoid specifying all-around welds. There should be no weld at the ends of the flanges and in the fillet (k region) of the column.
- ⑫ If a grout hole is needed, specify the same diameter as the anchor bolt holes to reduce the number of drill bit sizes required during fabrication.

## EXAMPLE 1 - SHEAR LUG

-----  
 See text section "SHEAR TRANSFER, BEARING"

- (1)  $f_c' = 4000$  psi  
 $F_{eex} = 70$  ksi (E70XX filler weld metal)  
 $G = 1\text{-}1/2$  inch grout depth  
 F1554 Grade 36 steel  
 Pier = 24 in. x 24 in.  
 Service shear load = 23 kip  
 Try  $L = 9''$ ,  $H = 4''$ ,  $t = t_{p1} = 2''$
- (2)  $V_u = 1.6 \times 23 = 36.8$  kip
- (3)  $A_b = L \times (H - G) = 9 \times (4 - 1.5) = 22.5$  in.<sup>2</sup>  
 Compressive strength =  $.85 \times .65 \times 4.000 = 49.725$  k, o.k.
- (4)  $A_v = 24 \times 24 / 2 - 22.5 = 265.5$  in.<sup>2</sup>
- (5) Resisting shear stress =  $4 \times .75 \times 4000^{(1/2)} / 1000$   
 = 0.1897 ksi  
 Resisting shear strength =  $265.5 \times .1897$   
 = 50.375 k, o.k.
- (6) Moment arm =  $H/2 + G/2 = 2.75''$   
 $M_u = 36.8 \times 2.75$
- (7)  $Z = 101.2 / (.9 \times 36) = 3.12345$  in.<sup>2</sup>
- (8)  $t$  solves to 1.178", o.k.
- (9) Weld 1 shear =  $36.8 / 2 = 18.4$  kip (each side)  
 Weld 2 shear =  $101.2 / 2 = 50.6$  kip (+-)  
 Total weld load, each side =  $(18.4^2 + 50.6^2)^{(1/2)}$   
 = 53.774 kip  
 $(L - 2 \times \text{throat}) \times \text{throat} \times .75 \times .6 \times 70 / 2^{(1/2)} = 53.774$   
 This quadratic is solved for throat = .1984  
 Size of symmetric fillet weld =  $\text{throat} \times 2^{(1/2)}$   
 = .2806"  
 Say 5/16" symmetric fillet weld, 8-3/8" long, each side.
- Note :  $t$  and  $t_{p1}$  may be reduced, but weld size increases

**EXAMPLE**

**2**  
**Single stud, combined tension and shear**

Design an embedment using a stud welded to an embedded plate.

Given:

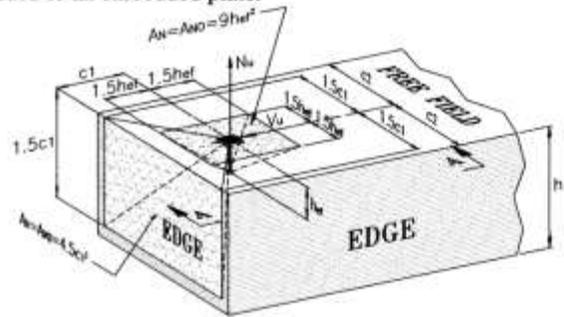
Edge  
 $c_1 = 12$  in.  
 $c_2 = 20$  in.  
 $h = 18$  in.

Concrete  
 $f'_c = 4000$  psi

Stud material (A108)  
 $f_y = 51$  ksi  
 $f_{ut} = 65$  ksi

Plate  
 $3 \times 3 \times 3/8$  in. thick  
 $F_y = 36$  ksi

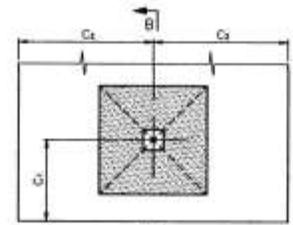
Loads  
 $N_u = 8$  kips  
 $V_u = 6$  kips



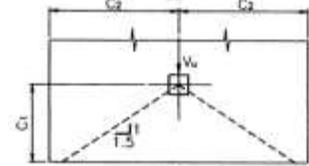
Where  $N_u$  and  $V_u$  are the applied factored external loads

Assumptions:

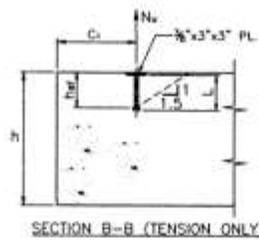
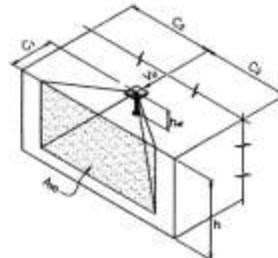
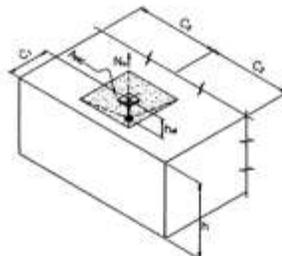
- Concrete is cracked
- $\phi$ -factors are based on Condition B in D.4.5 of the code (no supplementary reinforcement)



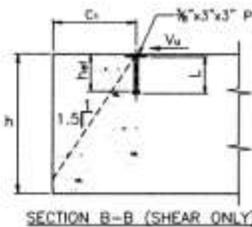
PLAN (TENSION ONLY)



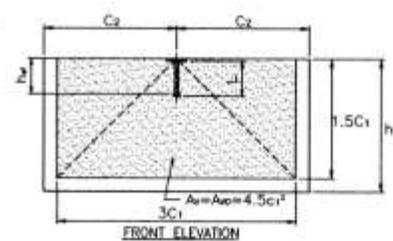
PLAN (SHEAR ONLY)



SECTION B-B (TENSION ONLY)



SECTION B-B (SHEAR ONLY)



FRONT ELEVATION

Try 3/4" dia., hef = 8", fc' = 4000psi  
 Normal weight concrete  
 See text "ACI FAILURE MODES"

### 1. Steel Strength

$$\begin{aligned} A_{se,n} &= .7854 * (.75 - .9743/10)^2 \\ &= .33460 \text{ in.}^2 \\ N_{sa} &= .33460 * 65000 \\ &= 21740 \text{ lbf} \\ \phi &= .75 \\ \phi N_{sa} &= 16305 \text{ lbf} \end{aligned}$$

### 2. Concrete Breakout in Tension

- (1) headed anchor welded to an embedded plate  
 see AWS D1.1, Chapter 7, Stud Welding  
 head diameter = 1.25 in.  
 concrete is cracked at service loading
  - (2)  $\phi = 0.7$ , Condition B
  - (3)  $k_c = 24$   
 $\lambda_a = 1.0$  (applies to lightweight concrete)
  - (4) not applicable
  - (5)  $N_b = 24 * 1 * 4000^{(1/2)} * 8^{(1.5)} = 34346 \text{ lbf}$   
 or  
 $N_b = 16 * 1 * 4000^{(1/2)} * 8^{(5/3)} = 32382 \text{ lbf}$
  - (6)  $\psi_{ec,n}$  doesn't apply
  - (7)  $\psi_{ed,n} = 1$
  - (8)  $\psi_{c,b} = 1$
  - (9) not applicable
  - (10)  $A_{nc} = 9 * 8^2 = 576 \text{ in.}^2$
  - (11)  $A_{nc} = (3 * 8) * (3 * 8) = 576 \text{ in.}^2$
  - (12)  $N_{cb} = (1/1) * 1 * 1 * 34346 = 34346 \text{ lbf}$
  - (13) not applicable
- $$\phi N_{cb} = 24842 \text{ lbf}$$

### 3. Pullout Strength

$$\begin{aligned} \psi_{cb} &= 1 \\ A_{brg} &= (\pi/4) * (1.25^2 - .75^2) = .78540 \text{ in.}^2 \\ N_{pn} &= 8 * 1 * .78540 * 4000 = 25133 \text{ lbf} \\ \phi N_{pn} &= .7 * N_{pn} = 17593 \text{ lbf} \end{aligned}$$

### 4. Side-Face Blowout - not applicable

## 5. Bond Strength - not applicable

## 6. Steel Strength in Shear

$$A_{se,v} = .33460 \text{ in.}^2$$

$$V_{sa} = 1.0 * .33460 * 65000 = 21749 \text{ lbf}$$

$$\phi V_{sa} = .65 * 21749 = 14137 \text{ lbf}$$

## 7. Concrete Breakout in Shear

$$x_1, x_2, h_a \geq 1.5 * c_{a1}$$

$$\text{thus } A_{vc} = A_{vco} = 1.5 * 8 * (3 * 8) = 288 \text{ in.}^2$$

$$l_e \leq 8 * d_a = 6 \text{ in.}$$

$$\psi_{ed,v} = \psi_{c,v} = \psi_{h,v} = 1$$

$$V_b = \text{smaller of}$$

$$7 * (6 / .75)^{(.2)} * .75^{(1/2)} * 12^{(1.5)} = 24157 \text{ lbf}$$

and

$$* 1 * 4000^{(91/2)} * 12^{(1.5)} = 23662 \text{ lbf}$$

$$V_{cb} = 1 * V_b = 23662 \text{ lbf}$$

$$\phi = .7$$

$$\phi V_{cb} = 16563 \text{ lbf}$$

## 8. Concrete Pryout in Shear

$$k_{cp} = 2, h_{ef} > 2.5 \text{ in.}$$

$$V_{cp} = 2 * 34346 = 68692 \text{ lbf, from step 2}$$

## 9. Summary

Step	$\phi N_n$ (lbf)	$\phi V_n$ (lbf)
----	-----	-----
1	16305	-
2	24842	-
3	17593	-
6	-	14137
7	-	16563
8	-	48024

$$\begin{aligned} N_{ua} / \phi N_n + V_{ua} / \phi V_n &= 8000 / 16305 + 6000 / 14137 \\ &= .491 + .424 = .915 < 1.2, \text{o.k.} \end{aligned}$$

## EXAMPLE 3 – TWO ADHESIVE ANCHORS

Given : Service load = 10000 lbf shear (not sustained)  
 Anchor steel = Grade 36  
 $f_c' = 4000$  psi  
 Installed with hammer drill  
 Dry concrete  
 Max. short term temperature = 130°F  
 Max. sustained temperature = 110°F  
 Concrete cracked under service load  
 Installation Condition B, Category 2  
 $ca_1 = 12$  in.  
 $ca_2 = ca_4 = 15$  in.  
 $ca_3 = 84$  in., use  $1.5 \cdot hef$   
 $s_1 = 0$ ,  $s_2 = 18$  in.  
 $h = 48$  in.  
 Try : 1 in. dia.,  $hef = 12$  in.

### 5. BOND STRENGTH OF ADHESIVE ANCHORS IN TENSION

$$\begin{aligned} \lambda_a &= 0.6 \\ A_{nco} &= (2 \cdot ca_1)^2 = 576 \text{ in.}^2 \\ A_{nc} &= (ca_1 + 1.5 \cdot hef) \cdot (ca_2 + s_2 + ca_4) = 1440 \text{ in.}^2 \\ &= \text{greater than } 2 \cdot A_{nco}, \text{ use } 2 \cdot 576 = 1152 \text{ in.}^2 \end{aligned}$$

From ESR-2322 (reference \_\_) for the above conditions :

$$\begin{aligned} \tau_{uncr} &= 1365 \text{ psi} \\ \tau_{cr} &= 600 \text{ psi} \\ \phi &= .55 \\ c_{ac} &= hef \cdot (\tau_{uncr} / 1160)^{0.4} \cdot (3.1 - .7 \cdot h / hef) \\ &\quad \text{use } 2.5 \cdot hef < h, \text{ for } h \\ c_{ac} &= 12 \cdot (1365 / 1160)^{0.4} \cdot (3.1 - .7 \cdot 2.5) \\ c_{ac} &= 17.2896 \text{ in.} \\ c_{na} &= 10 \cdot 1 \cdot (165 / 1100)^{1/2} \\ c_{nn} &= 11.1396 \text{ in.} \\ \psi_{cp,na} &= 12 / 17.2896 \text{ but not less than } 18 / 17.2896, \text{ use } \psi_{cp,na} = 1 \\ \Psi_{ec,na} &= 1 \\ \Psi_{ed,na} &= 1 \end{aligned}$$

$$\begin{aligned}
 N_{ba} &= .6 * 600 * \pi * 1 * 12 \\
 &= 13572 \text{ lbf} \\
 N_a &= 2 * 1 * 13572 = 27144 \text{ lbf} \\
 N_{ag} &= N_a = 27144 \text{ lbf} \\
 \phi N_{ag} &= 14929 \text{ lbf (need this number for step 8)}
 \end{aligned}$$

## 2. CONCRETE BREAKOUT STRENGTH IN TENSION

This failure mode included because it is used in Step 8, CONCRETE PRYOUT IN SHEAR

- (1) See given statement above
- (2) From above,  $c_{ac} = 17.2896 \text{ in.}$
- (3)  $K_c = 17, \lambda_a = .8$
- (4) Not applicable
- (5)  $N_b = 17 * .8 * 4000^{(1/2)} * 12^{(1.5)}$   
 $N_b = 35755 \text{ lbf}$
- (6)  $\psi_{ec,n} = 1$
- (7)  $\psi_{ed,n} = .7 + .3 * (12/18)$   
 $\psi_{ed,n} = .9$
- (8)  $\psi_{c,n} = 1$
- (9)  $\psi_{cp,n}$  not less than  $18/17.2896$ , use  $\psi_{cp,n} = 1$
- (10)  $A_{nc} = 9 * 12^2 = 1296 \text{ in.}^2$
- (11)  $A_{nc} = (12+0+18) * (15+18+15) = 1440 \text{ in.}^2$
- (12)  $N_{cb} = (1440/1296) * .9 * 35755 = 35795 \text{ lbf}$   
 $N_{cbg} = N_{ch}$
- (13)  $\phi N_{cbg} = .55 * 35795 = 19687 \text{ lbf}$

## 6. STEEL STRENGTH OF ANCHOR IN SHEAR

$$\begin{aligned}
 A_{se,v} &= .7854 * (1 - .9743/8)^2 = .6057 \text{ in.}^2 \\
 V_{sa} &= 2 * (.6057) * 58000 = 70261 \text{ lbf} \\
 \phi V_{sa} &= .65 * V_{sa} = 45670 \text{ lbf}
 \end{aligned}$$

## 7. CONCRETE BREAKOUT IN SHEAR

$$\begin{aligned}
 A_{vco} &= 4.5 * (c_{a1})^2 = 4.5 * 12^2 = 648 \text{ in.}^2 \\
 A_{vc} &= 1.5 * c_{a1} * (c_{a2\_s2} + c_{a4}) = 864 \text{ in.}^2 \\
 l_e &= 8 * 1 = 8 \text{ in.} \\
 \psi_{ec,v} &= 1 \\
 \psi_{ed,v} &= .7 + .3 * (15/18) = .95 \\
 \psi_{c,v} &= 1 \\
 (1.5 * c_{a1} / h_a)^{(1/2)} &= (18,48)^{(1/2)} = .6124 \\
 &\text{But not less than 1, use 1} \\
 &V_b \text{ is the smaller of:}
 \end{aligned}$$

$$\begin{aligned}
7*8^{(0.2)}*1^{(1/2)}*4000^{(1/2)}*12^{(1.5)} &= 26335 \text{ lbf} \\
9*.8*4000^{(1/2)}*12^{(1.5)} &= 18929 \text{ lbf} \\
V_{cb} = V_{cbg} = (864/648)*.95*18929 &= 23977 \text{ lbf} \\
\phi V_{cb} = \phi V_{cbg} &= 16783 \text{ lbf}
\end{aligned}$$

**8. CONCRETE PRYOUT IN SHEAR**

$$\begin{aligned}
K_{cp} &= 2.0 \text{ since } h_{ef} > 2.5 \text{ in.} \\
V_{cp} &= K_{cp}*N_{cp} \text{ where} \\
N_{cp} &= \text{lesser of } N_{cb} \text{ (step 2, 19687 lbf) and} \\
&\quad N_a \text{ (step 5, 14929 lbf).} \\
V_{cp} &= 2*14929 \\
V_{cp} &= 29858 \text{ lbf} \\
\phi V_{cp} &= .7*29858 = 20901 \text{ lbf}
\end{aligned}$$

**9. SUMMARY**

Step	$\phi V$ (lbf)
----	-----
6	45670
7	16763
8	20901

$$v_u/\phi v = 1.6*10000/16763 = .953 < 1, \text{ o.k.}$$

## EXAMPLE 4 - TWO BOLTS IN TENSION

**GIVEN :** The following drawing from the classic text "Design of Welded Structures", by Omer Blodgett (Ref. 17).  
 Base plate = PL 3-1/4"x19"x2'-5" (scaled)  
 Foundation = 8'-2" x 7'-2" x 60"  
 Base plate embedded in foundation  
 $f_c' = 4000$  psi  
 Concrete cracked under service loads  
 Service load = 84600 tension on 2 bolts  
 $ca_1 = ca_2 = ca_3 = ca_4 = 39"$   
 $s_1 = 0 \quad s_2 = 8"$   
 Anchors cast-in with heavy hex nut  
 Anchor steel = Grade 36 ( $f_u = 58000$  lbf)  
 Installation Condition B

**FIND :** A method for calculating anchor(s) diameter and hef given a tension load and ductility requirement. In this method, tensile anchor capacity is given by  $\phi A_{se,n} \cdot \text{no. bolts} \cdot f_u$ . This is set equal to the factored tensile load to find anchor diameter. The tensile load  $N_u$  is load factor  $\cdot$  service load =  $\phi \cdot \text{no. bolts} \cdot A_{se,n} \cdot f_y$ , for ductility.

**METHOD:** Failures 4 and 8 do not govern.  
 Failure 5 is not applicable.  
 A<sub>brg</sub> in Failure 3 can be adjusted by an oversized washer so it does not govern.  
 Assume  $(A_{nc}/A_{nco}) \cdot \psi \cdot \psi \cdot \psi \cdot \psi = 1$   
 $\lambda_a = 1$   
 $l_e/d_a = 8$   
 $1.6 \cdot 84600 = .7 \cdot 2 \cdot A_{se,n} \cdot 58000$   
 This solves to  $A_{se,n} = 1.667 \text{ in.}^2$   
 Use  $d_a = 1.75 \text{ in.}^2$   
 Anchor steel capacity governs in tension

## USE OF WING PLATES

When large wing plates are used to increase the leverage of an anchor bolt, the detail should always be checked for weakness in bearing against the side of the column flange.

### Problem

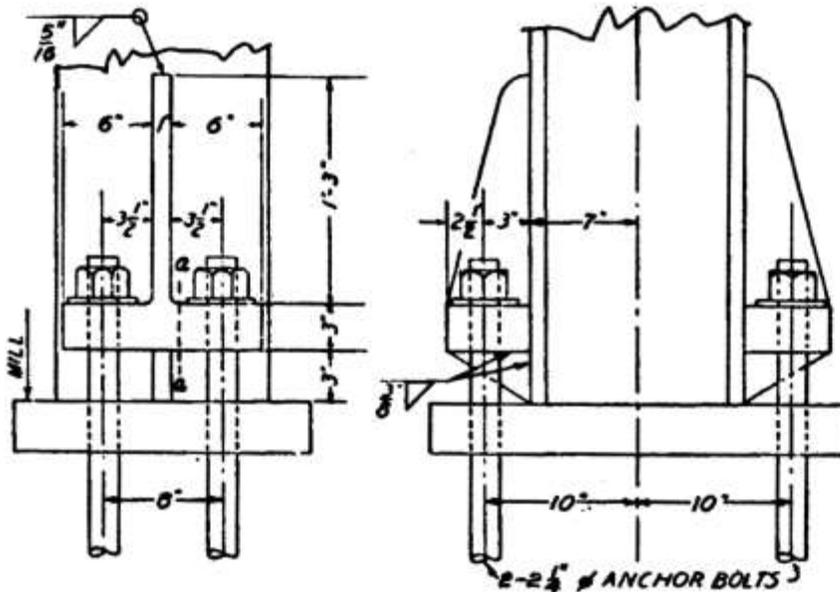


FIGURE 29

Figure 29 illustrates a wing-plate type of column base detail that is not limited with respect to size of bolts or strength of column flange. A similar detail, with bolts as large as 4½" diameter, has been used on a large terminal project.

The detail shown is good for four 2¼"-dia. anchor bolts. Two of these bolts have a gross area of 6.046 in.<sup>2</sup> and are good for 84,600 lbs tension at a stress of 14,000 psi.

$$\begin{aligned} \text{Tension to cause yielding} &= 1.90 \times 36000 \\ &= 68400 \text{ lbf} \end{aligned}$$

$$N_u = 1.6 \times 68400 = 109440 \text{ lbf}$$

Now the anchor steel tensile capacity must govern in tension so the hef value used must give a higher capacity than the anchor steel, i.e.,  $N_{sa}$ .

$$N_{sa} = n \cdot \text{bolts} \cdot \text{tensile stress area of } 1\text{-}3/4\text{" bolt} \cdot f_{uta}$$

$$N_{sa} = 2 \cdot 1.90 \cdot 58000 = 228400 \text{ lbf}$$

$$\phi N_{sa} = 0.75 \cdot N_{sa} = 165300 \text{ lbf}$$

Note that because of Condition B, all capacity reduction factors, except for anchor steel in tension (0.75) and in shear (0.65), in both tension and shear are 0.70.

Two values for hef are given by

$$\phi \cdot 24 \cdot 1 \cdot (f_c')^{1/2} \cdot \text{hef}^{1.5} \text{ and}$$

$$\phi \cdot 16 \cdot 1 \cdot (f_c')^{1/2} \cdot \text{hef}^{5/3}, \quad \phi = 0.70$$

Setting these equal to  $\phi N_{sa}$  obtains:

$$\text{hef} = 28.926\text{" and } 26.352\text{"}, \text{ respectively.}$$

$$\text{Use hef} = 26\text{"}$$

There are three (3) possibilities for capabilities in shear,, one corresponding to steel strength and two possibilities for concrete breakout. Pryout in shear does not govern.

$$\phi V_{sa} = \text{Steel strength} = 0.65 \cdot 0.6 \cdot N_{sa}$$

$$\phi V_{sa} = 85956 \text{ lbf}$$

The breakout strength is the lesser of  $7 \cdot (1e/d_o)^{.2} \cdot d_o^{1/2} \cdot (f_c')^{1/2} \cdot c_{a1}^{1.5}$  and  $9 \cdot 1 \cdot (f_c')^{1/2} \cdot c_{a1}^{1.5}$

The former obtains 200165 lbf and the latter 138634 lbf.

Thus steel shear strength governs, i.e., is

The lowest shear capacity of the shear failure loads.

The available shear fraction,  $1.2 - \nu / \phi N_s a$  is  $1.2 - .66207 = .53793$

Then max.  $V_u = .53793 * 85956 = 45163$  lbf.

This does not, however, include the additional shear due to bolt bending, or speak to what part of the shear is resisted by the bolts and that resisted by other constructions as reinforcement, bearing of edge of base plate, shear lugs, or others.

Additional shear may also be induced by displacement of the top of the stretch bolts by flexure of the column, rotation of the base plate, or construction error.

Consider a cantilever with a point load at the end.

$$\Delta = P * L^3 / 3 * E * I$$

This may be solved for P giving

$P = 50608$  lbf per inch of displacement of tip

Now the base plate scaled as

PL 3-1/4"x19"x2'-5" so that the bolt length is 9.25 in.

Using the bolt bending calculations in the Text,

$$L = 9.25 + .5 * 1.75 = 10.125 \text{ in.}$$

$$Z = 1.75^3 / 6 = .89323 \text{ in.}^3$$

$$M_{ao} = 1.2 * 58000 * .89323 = 62169 \text{ lbf-in.}$$

$$M_s = 62169 * (1 - .709360) = 18869 \text{ lbf-in.}$$

$$\alpha = 2$$

$$V_{add} = 2 * 18869 / 10.125 = 1864 \text{ lbf}$$

$M_v = V_u * L$ , where  $V_u$  here is the force caused by displacement.

The critical point is where  $M_v = M_s$ , or  
 $\Delta * 50608 * 10.125 = 18869$ , which solves for  
 $\Delta = .037$  in., which must be addressed  
in design.

$$\phi N_{sa} = 165300 \text{ lbf}$$

Concrete breakout strength is  $\phi N_{cbg}$

$$\phi N_{cbg} = 178275 \text{ lbf} > \phi N_{sa}, \text{ o.k.}$$

Abrg 1-3/4" heavy hex nut is 4.144 in.<sup>2</sup>

$$\phi N_{pn} = \text{pullout strength} = 92836 \text{ lbf/anchor}$$

$$\phi N_{png} = 183672 \text{ lbf}, > \phi N_{sa}, \text{ o.k.}$$

Thus steel strength governs in tension, and  
The connection is ductile , Q.E.D.

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- 
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## APPENDIX 1

ACI 318-11, Section 2.1, Code Notation

Units of length in inches, areas in square inches, forces in pounds force (lbf), pressures in pounds force/square inch (psi) unless noted otherwise.

**Abrg** = net bearing area of headed stud, headed anchor bolt, or headed deformed bar

**Ana** = projected influence area of  $\geq 1$  adhesive anchors, For calculation of bond strength in tension

**Anao** = projected influence area of a single adhesive anchor for calculation of bond strength in tension, not limited by edge distance or spacing

**Anc** = projected concrete failure area for tension,  $\geq 1$  anchor

**Anco** = projected failure area, 1 bolt, not limited by edge distance or spacing for calculation of strength in tension

**Ase,n** = effective cross-sectional area of anchor bolt in tension

**Ase,v** = effective cross-sectional area of anchor bolt in shear

**Avc** = projected concrete failure area for shear.

**Avco** = projected concrete failure area for shear, single anchor bolt, not limited by corners, spacing, or thickness

**ca1** = distance from the anchor center to the concrete edge in one direction, in. If shear is applied to anchor, ca1 is taken in direction of applied shear. If tension is applied to the anchor, ca1 is the minimum edge distance, in.

**ca2** = distance from the anchor center to the concrete edge perpendicular to ca1, in.

**cac** = critical edge distance required to develop the basic strength as controlled by concrete breakout or bond of a post-tensioned anchor in tension in uncracked concrete without supplementary reinforcement to control splitting

**ca,max** = max. distance from a.b.center, to concrete edge

$c_{a,min}$  = min. distance from a.b. center to concrete edge  
 $c_{na}$  = projected distance from center of an anchor shaft one side of the anchor required to develop the full full bond strength of an adhesive anchor  
 $d_a$  = outside diameter anchor or shaft diameter of headed stud, headed bolt, or hooked bolt  
 $d_a'$  = value substituted for  $d_a$  when an oversized anchor is used  
 $e_h$  = distance from inner surface of shaft of J- or L-bolt to outer tip  
 $e_n'$  = distance from resultant tension load to tension centroid of group of anchors  
 $e_v'$  = distance from resultant shear load on a group of anchors loaded in shear in the same direction, and the centroid of the group loaded in the same direction  
 $f_c'$  = specified compressive strength of concrete  
 $f_{uta}$  = specified tensile strength of anchor steel  
 $f_{ya}$  = specified yield strength of anchor steel  
 $h_a$  = thickness of member in which anchor is located, measured parallel to the anchor axis  
 $h_{ef}$  = effective embedment depth  
 $k_c$  = coefficient for concrete breakout strength in tension  
 $k_{cp}$  = coefficient for pryout strength  
 $l_e$  = load bearing length of anchor for shear  
 $n$  = number of items in group  
 $N_a$  = nominal bond strength in tension of single adhesive anchor  
 $N_{ag}$  = nominal bond strength in tension of a group of adhesive anchors  
 $N_b$  = basic concrete breakout strength in tension, cracked concrete, 1 anchor  
 $N_{ba}$  = basic bond strength of a single adhesive anchor in cracked concrete  
 $N_{bag}$  = nominal bond strength for a group of adhesive Anchors in tension  
 $N_{cb}$  = nominal concrete breakout strength in tension, 1 a.b.  
 $N_{cbg}$  = nominal concrete breakout strength in tension, >1a.b.  
 $N_n$  = nominal strength in tension  
 $N_p$  = nominal pullout strength in tension, 1 anchor,

cracked concrete

- $N_{pn}$  = nominal pullout strength in tension, 1 anchor
- $N_{sa}$  = nominal strength of  $\geq 1$  anchor, in tension, governed by steel
- $N_{sb}$  = side face blowout strength, 1 anchor
- $N_{sbg}$  = side face blowout strength,  $> 1$  anchor
- $N_{ua}$  = factored tensile factor applied to single anchor or group
- $s$  = center-to-center spacing of anchors
- $V_b$  = basic concrete breakout strength in shear, 1 anchor, cracked concrete
- $V_{cb}$  = nominal concrete breakout strength in shear, 1 anchor
- $V_{cbg}$  = nominal concrete breakout strength in shear  $> 1$  anchor
- $V_{cp}$  = nominal concrete pryout strength, 1 anchor
- $V_{cpg}$  = nominal concrete pryout strength,  $> 1$  anchor
- $V_n$  = nominal concrete shear strength
- $V_{sa}$  = nominal strength in shear 1 anchor governed by steel
- $V_{ua}$  = factored shear force,  $\geq 1$  anchor
- $\Psi_{c,n}$  = modifier for tensile strength for cracked versus uncracked concrete
- $\Psi_{c,v}$  = modifier for shear strength in anchors based on presence or absence of concrete cracking and presence or absence of supplementary reinforcement
- $\Psi_{cp,n}$  = modifier for tensile strength of post-installed anchors intended for use in uncracked concrete without supplementary reinforcement
- $\psi_{cp,na}$  = modify tensile strength of adhesive anchors, uncracked concrete, no supplementary reinforcement
- $\Psi_{ec,n}$  = modifier for tensile strength of anchors based on eccentricity of loads
- $\psi_{ec,na}$  = modify tensile strength of adhesive anchors, eccentric loads
- $\Psi_{ec,v}$  = modifier for shear strength of anchors based on eccentricity of loads
- $\Psi_{ed,n}$  = modifier for tensile strength of anchors based on edge distances
- $\Psi_{ed,v}$  = modifier for shear strength of anchors based on edge distances

$\psi_{ed,na}$  = modify tensile strength of adhesive anchors, based on edge distances

$\psi_{h,v}$  = modify shear strength for concrete thickness

$\tau_{cr}$  = characteristic bond stress of adhesive anchor in cracked concrete

$\tau_{uncr}$  = characteristic bond stress of adhesive anchor in uncracked concrete

$\lambda$  = modification factor reflecting the reduced properties of lightweight concrete relative to normalweight concrete.

$\lambda_a$  = modification reflecting reduced mechanical properties of lightweight concrete in certain applications

$\phi$  = strength reduction factor

## APPENDIX 2

### ANCHOR BOLTS DESIGN CITATION LIST

Reference : ACI 318-11, appendix D.

Section Title	Step	Section
General Requirements		see text
Steel strength in tension	1	D.5.1
Concrete breakout strength	2	D.5.2
Pull-out strength cast-in and post-installed expansion anchor	3	D.5.3
Concrete side-face blowout strength of headed anchor	4	D.5.4
Bond strength of adhesive anchor in tension	5	D.5.5
Steel strength of anchor in shear	6	D.6.1
Concrete breakout strength of anchor in shear	7	D.6.2
Concrete pryout strength in shear	8	D.6.3
Intersection of tensile and shear	9	D.7

■ = include    x = do not include    (Table D.4.1.1 + above)

Step	Group Effect	Cast-in Anchor	Expansion Anchor	Adhesive Anchor
1	x	■	■	■
2	■	■	■	■
3	■	■	■	x
4	■	■	■	x
5	x	x	x	■
6	x	■	■	■
7	■	■	■	■
8	■	■	■	■
9	■	■	■	■

**STEP 1: STEEL STRENGTH OF ANCHOR IN TENSION (D.5.1)**

$A_{se,n} = .7854 * (D - .9743/nt)^2$  where

$nt = \text{no. turns/inch}$  (AISC)

$f_{uta} \leq 1.9 * f_{ya}$  and  $\leq 125000$  psi

$N_{sa} = n * A_{se,n} * f_{uta}$  (D-2)

**STEP 2: CONCRETE BREAKOUT STRENGTH OF ANCHOR IN TENSION (D.5.2)**

$k_c = 24$  for cast-in and 17 for post-installed, the post-installed may be increased above 17 by product-specific tests but in no case greater than 24

$\lambda = .75, .85, 1.0$  for all-lightweight concrete, sand-lightweight, concrete, normalweight concrete, respectively (D.6.1)

$\lambda_a = 1.0\lambda$  cast-in.  $0.8\lambda$  for concrete failure, adhesive anchors, (D.3.6)

$0.6\lambda$  for concrete bond failure, adhesive anchors

If an additional plate or washer is added at the head of the anchor the projected area may be (D.5.2.8) calculated from the perimeter of the plate or washer. The effective perimeter should not exceed the thickness of the washer or plate. Where anchors are located less than  $1.5 * h_{ef}$  from three or more edges, the value of  $h_{ef}$  used for the calculation of  $A_{nc}$  in accordance with D.5.2.1, as well as in Equations (D-3) through (D-10) shall be the larger of  $c_{a,max}/1.5$  and  $s/3$ , where  $s$  is the maximum spacing of anchors in a group (D.5.2.3)

For a single anchor in tension in cracked concrete

$N_b = k_c * \lambda_a * (f_c')^{1/2} * h_{ef}^{1.5}$  (D-6)

Alternatively, for cast-in headed studs and headed bolts where  $11 \text{ in.} \leq h_{ef} \leq 25 \text{ in.}$ ,

$N_b = 16 * \lambda_a * (f_c')^{1/2} * h_{ef}^{5/3}$  (D-7)

$\psi_{ec,n} = 1 / (1 + 2 * e_n' / 3 * h_{ef}) \leq 1.0$  (D-8)

If  $c_{a,min} \geq 1.5 * h_{ef}$  then  $\psi_{ed,n} = 1.0$  (D-9)

If  $c_{a,min} < 1.5 * h_{ef}$ , then

$$\psi_{ed,n} = 0.7 + 0.3 \cdot c_{a,min} / 1.5 \cdot h_{ef} \quad (D-10)$$

$$\Psi_{c,n} = 1.0, \text{ cracking at service loads} \quad (D.5.2.6)$$

$$\Psi_{c,n} = 1.25, \text{ cast-in-anchors, no cracking} \quad (D.5.2.6)$$

$$\Psi_{c,n} = 1.40, \text{ post-installed anchors, no cracking} \quad (D.5.2.6)$$

$$c_{ac} \text{ for adhesive anchors} = 2 \cdot h_{ef} \quad (D.8.6)$$

$$c_{ac} \text{ for undercut anchors} = 2.5 \cdot h_{ef} \quad (D.8.6)$$

unless determined by tests (eg., ESR-2322)

If cracking:

$$\text{If } c_{a,min} \geq c_{ac} \text{ then } \Psi_{cp,n} = 1.0 \quad (D-11)$$

$$\text{If } c_{a,min} < c_{ac} \text{ and } \geq 1.5 \cdot h_{ef} / c_{ac} \quad (D-12)$$

then  $\psi_{cp,n} = c_{a,min} / c_{ac}$  but not less than  
1.5  $\cdot h_{ef} / c_{ac}$  for post-installed anchors

$$\psi_{cp,n} \text{ for cast-in} = 1.0$$

See Fig. RD.5.2.1 (a) for calculation of  $A_{nc}$  and (b)

$$\text{for calculation of } A_{nc} . \quad A_{nc} \leq n \cdot A_{nco} \quad (D-5)$$

$$A_{nco} = 9 \cdot h_{ef}^2 \text{ if edge distance } \geq 1.5 \cdot h_{ef}$$

$$1 \text{ anchor } N_{cb} = (A_{nc} / A_{nco}) \cdot \psi_{ed,n} \cdot \psi_{c,n} \cdot \psi_{cp,n} \cdot N_b \quad (D-3)$$

$$\text{else } N_{cbg} = (A_{nc} / A_{nco}) \cdot \psi_{ec,n} \cdot \psi_{ed,n} \cdot \psi_{c,n} \cdot \psi_{cp,n} \cdot N_b \quad (D-4)$$

### STEP 3: PULLOUT STRENGTH OF CAST-IN AND EXPANSION ANCHORS (D.5.3)

for single headed stud or headed bolt,

$$N_p = 8 \cdot A_{brg} \cdot f_c' \quad (D-14)$$

for single hooked bolt,

$$N_p = 0.9 \cdot (f_c') \cdot e_h \cdot d_o \text{ where } 3 \cdot d_o \leq e_h \leq 4.5 \cdot d_o \quad (D-15)$$

Expansion anchors pullout strength obtained by tests

$$t_0 \text{ ACI 355.2} \quad (D.5.3.2)$$

$$\psi_{c,p} = 1.4 \text{ no cracking, } 1.0 \text{ if cracking} \quad (D-13)$$

$$N_{pn} = \psi_{c,p} \cdot N_p$$

### STEP 4: CONCRETE SIDE-FACE BLOWOUT STRENGTH OF ANCHOR IN TENSION (D.5.4)

$\lambda_a$  - see Step 2

$A_{brg}$  - see Step 3

for a single headed anchor with deep embedment close to an edge, ( $h_{ef} > 2.5 \cdot c_{a1}$ ):

$$N_{sb} = 160 \cdot c_{a1} \cdot A_{brg}^{(0.5)} \cdot \lambda_a \cdot (f_c')^{(1/2)} \quad (D-16)$$

$$N_{sb} = (1+s/6*c_{a1}) * N_{sb} \quad (D-17)$$

for multiple headed anchors with deep embedment close to an edge ( $hef > 2.5c_{a1}$ ) and anchor spacing less than  $6*c_{a1}$ .

#### STEP 5: BOND STRENGTH OF ADHESIVE ANCHOR (D.5.5)

$\lambda_a$  - see Step 2

for outdoor work, dry to fully saturated moisture content, at installation, and maximum temperature of 175°F,

$$\tau_{cr} = 200 \text{ psi and } \tau_{uncr} = 650 \text{ psi} \quad (\text{Table D.5.5.2})$$

$$N_{ba} = \lambda_a * \tau_{cr} * \Pi * d_a * hef \quad (D-22)$$

If adhesive anchor designed to resist sustained loads,  $0.55 * \phi * N_{ba} \geq N_{u, \text{sustained}}$  (D-1)

$$c_{na} = 10 * d_a * (\tau_{uncr} / 1100)^{1/2} \quad (D-21)$$

$$A_{nao} = (2 * c_{na})^2 \quad (D-20)$$

$A_{na} \leq n * A_{nao}$ , n = number of anchors in group

See Fig. RD.5.5.1 for calculation of  $A_{nao}$ ,  $A_{na}$

if  $c_{a, \text{min}} \leq c_{na}$ ,  $A_n = (2 * c_{a, \text{min}})^2$  (author's opinion)

else  $A_n = (2 * c_{na})^2$

$$\psi_{ec, na} = 1 / (1 + 2 * e' / n / 3 * hef) \quad (D-23)$$

$$\psi_{ed, na} \text{ if } c_{a, \text{min}} \geq c_{na} \text{ then } = 1 \quad (D-24)$$

$$\text{else } = 0.7 + 0.3 * (c_{a, \text{min}} / c_{na}) \quad (D-25)$$

$$c_{ac} = 2 * hef, \text{ or tests to ACI 355.2 or ACI 355.4} \quad (D.8.6)$$

$$\psi_{cp, na} \text{ if } c_{a, \text{min}} \geq c_{ac} \text{ then } = 1 \quad (D-26)$$

$$\text{else } = c_{a, \text{min}} / c_{ac} \text{ but not less than } c_{na} / c_{ac} \quad (D-27)$$

$$N_a = (A_{na} / A_{nbo}) * \psi_{ed, na} * \psi_{cp, na} * N_{ba} \quad (D-18)$$

$$N_{ag} = (A_{na} / A_{nbo}) * \psi_{ec, na} * \psi_{ed, na} * \psi_{cp, na} * N_{ba} \quad (D-19)$$

#### STEP 6: STEEL STRENGTH OF ANCHOR IN SHEAR (D.6.1)

Where concrete breakout is a potential failure mode, the required steel shear strength shall be consistent with the assumed breakout surface (D.6.2.1)

$$A_{se, v} = .7854 * (D - .9743 / nt)^2$$

Type of Anchor

V<sub>sa</sub>

-----

-----

cast-in headed stud anchor

$$1.0 * A_{se, v} * f_{uta}$$

(D-28)

cast-in headed bolt and hooked anchors  $0.6 \cdot A_{se,v} \cdot f_{uta}$   
 for post-installed anchors where sleeves do  
 not extend through the shear plane (D-29)  
 post-installed anchors where sleeves extend through  
 the shear plane ACI 355.2 tests  
 where anchors are used with built-up grout pads,  
 multiply values above by 0.80 (D.6.1.3)

## STEP 7: CONCRETE BREAKOUT STRENGTH IN SHEAR (D.6.2)

$l_e = h_{ef}$  for anchors with constant stiffness over the  
 full length of the anchor

$$l_e \leq 8 \cdot d_a$$

$\lambda = 0.75, 0.85,$  and  $1.0$  for all-lightweight, sand-  
 lightweight, and normalweight concrete, respectively  
 (8.6.1)

$\lambda_a = .6 \cdot \lambda$  adhesive concrete bond failure,  $.8 \cdot \lambda$  (D.3.6)  
 expansion and adhesive anchor concrete failure, and  
 $1.0 \cdot \lambda$  for cast-in and undercut anchor concrete failure.

Figure	Used to find
RD.6.2.1 (a) and (b)	$ca_1, A_{vc}$
RD.6.2.1(c)	shear force parallel to edge
RD.6.2.1(d)	shear force at corner, $ca_1, ca_2$
RD.6.2.9(a)	hairpin anchor reinforcement
RD.6.2.9(b)	edge and anchor reinforcement for shear

$V_b =$  is the smaller of (a) and (b):

$$(a) V_b = 7 \cdot \left( \left( \frac{l_e}{d_a} \right)^{0.2} \cdot \lambda_a \right) \cdot f_{c'}^{1/2} \cdot ca_1^{1.5} \quad (D-33)$$

$$(b) V_b = 9 \cdot \lambda_a \cdot f_{c'}^{1/2} \cdot ca_1^{1.5} \quad (D-34)$$

$$A_{vco} = 4.5 \cdot (ca_1)^2 \quad (D-32)$$

$$A_{vc} \leq \text{number of anchors in group} \cdot A_{vco}$$

$$\Psi_{ec,v} = 1 / (1 + 2 \cdot e_{v'} / 3 \cdot ca_1) \leq 1 \quad (D-36)$$

$$\Psi_{ed,v} = 1.0 \text{ if } ca_2 \geq 1.5 \cdot ca_1 \quad (D-37)$$

$$\text{else } 0.7 + 0.3 \cdot ca_2 / (1.5 \cdot ca_1) \quad (D-38)$$

$$\psi_{c,v} = 1.4 \text{ if no cracking} \quad (D.6.2.7)$$

$$= 1.2 \text{ cracked and } \#4 \text{ bar between between anchor} \\ \text{and edge}$$

= 1.0 if bar smaller than #4 or no bar between anchor and edge

$$\psi_{h,v} = (1.5 \cdot c_{a1} / h_a)^{(1/2)} \text{ where } h_a < 1.5 \cdot c_{a1} \quad (D-39)$$

else  $\psi_{h,v} = 1.0$

for shear force perpendicular to edge of a single anchor:

$$V_{cb} = (A_{vc} / A_{vco}) \cdot \psi_{ed,v} \cdot \psi_{c,v} \cdot \psi_{h,v} \cdot V_b \quad (D-30)$$

And for a group of anchors

$$V_{cbg} = (A_{vc} / A_{vco}) \cdot \psi_{ec,v} \cdot \psi_{ed,v} \cdot \psi_{c,v} \cdot \psi_{h,v} \cdot V_b \quad (D-31)$$

For shear force parallel to edge, use twice the values for  $V_{cb}$  and

$V_{cbg}$  above.

For corner locations, calculate  $V_{cb}$  above and  $V_{cbg}$  above for both directions, using the smaller value.

#### STEP 8: CONCRETE PRYOUT STRENGTH IN SHEAR (D.6.3)

$$K_{cp} = 1.0 \text{ for } h_{ef} < 2.5 \text{ inches and} \quad (D.6.3.1)$$

$$= 2.0 \text{ for } h_{ef} \geq 2.5 \text{ inch}$$

$$V_{cp} = k_{cp} \cdot N_{cp} \text{ where:} \quad (D-40)$$

= use  $N_{cb}$  (step 2) for cast-in, expansion, or undercut anchors and the lesser of  $N_{cb}$  (step 2) and  $N_a$  (step 5) for adhesive anchors

$$V_{cpg} = k_{cpg} \cdot N_{cpg} \text{ where:} \quad (D-41)$$

= use  $N_{cbg}$  (step 2) for cast-in, expansion, or undercut anchors and the lesser of  $N_{cbg}$  (step 2) and  $N_{ag}$  (step 5) for adhesive anchors

#### STEP 9: INTERACTION OF TENSILE AND SHEAR FORCES (D.7)

Determine load factors from applicable Code.

$$N_{ua} = \text{factor} \cdot \text{service load}$$

$$V_{ua} = \text{factor} \cdot \text{service load}$$

Using the lowest values of  $\phi N_n$  and  $\phi V_n$  for all combinations of Steps 0 through 9,

From the Code body (D.7) ,  
if  $Vua/\phi Vn \leq 0.2$ , use  $\phi Nn \geq Nua$  (D.7.1)

and if  
if  $Nua/\phi Nn \leq 0.2$ , use  $\phi Vn \geq Vua$  (D.7.2)

else  
 $Nua/\phi Nn + Vua/\phi Vn \leq 1.2$  (D-42)

From the commentary (RD.7)  
 $(Nua/\phi Nn)^{(5/3)} + (Vua/\phi Vn)^{(5/3)} \leq 1.0$

## APPENDIX 3

```

/*****
*
*          Anc.c : 12-02-15 : ml
*
*          ca1   s1   ca3   ca4
*          -----
*          ca4 |   |   |   |
*              |   |   |   |
*          -----
*          s2  |   |   |   |
*              |   |   |   |
*          -----
*          ca2 |   |   |   |
*              |   |   |   |
*          -----
*
*****/

#include<math.h>
#include<stdio.h>
#include<stdlib.h>

int main(void)
{
    int i;
    double gca1,gca2,gca3,gca4,hef,gs1,gs2; /* inputs */
    double x0,x1,y0,y1,xbar,ybar;
    double gbar[2],Area[4];
    double Areatotal,Darea,ratio;          /* outputs */
    FILE *inn;
    FILE *out;
    void centroid
        (double,double,double,double,double,double,double[2]);
    inn = fopen("Anc.in","r");
    out = fopen("Anc.out","w+");
    fscanf(inn,"%lf %lf %lf %lf %lf %lf
        %lf",&gca1,&gca2,&gca3,&gca4,&hef,&gs1,&gs2);
    fclose(inn);

    gbar[0] = 0.0;

```

```
gbar[1]      =    0.0;
for(i=0;i<=3;i++)
{
    Area[i]   =    0.0;
}

centroid(gca1,gca2,gca3,gca4,gs1,gs2,gbar);

xbar        =    gbar[0];
ybar        =    gbar[1];
fprintf(out,"xbar      = ");
fprintf(out,"%19.6e\n",xbar);
fprintf(out,"ybar      = ");
fprintf(out,"%19.6e\n",ybar);

x0          =    xbar;
x1          =    gca1+gs1+gca3-xbar;
y0          =    ybar;
y1          =    gca2+gs2+gca4-ybar;

Area[0]     =    sqrt(4*y0*y0*(hef*hef+x0*x0))/2;
Area[1]     =
sqrt((x1+x0)*(x1+x0)*(hef*hef+y1*y1))/2;
Area[2]     =
sqrt((y1+y0)*(y1+y0)*(hef*hef+x1*x1))/2;
Area[3]     =
sqrt((x1+x0)*(x1+x0)*(hef*hef+y0*y0))/2;

Areatotal  =    0.0;

for(i=0;i<=3;i++)
{
    fprintf(out,"Area");fprintf(out,"%1d",i);
    fprintf(out,"      =      ");
    fprintf(out,"%16.6e\n",Area[i]);
    Areatotal += Area[i];
}

fprintf(out,"Areatotal = ");
fprintf(out,"%19.6e\n",Areatotal);
Darea      =    (gca1+gs1+gca3)*(gca2+gs2+gca4);
```

```
fprintf(out,"DArea      = ");
fprintf(out,"%19.6e\n",Darea);
ratio          =   Areatotal/Darea;
fprintf(out,"ratio      = ");
fprintf(out,"%19.6e\n",ratio);

fclose(out);
return 0;
}

void centroid(double ca1,double ca2,double ca3,double
              ca4,double s1,double s2,double bar[2])
{
    int i,j;
    double xx[3],yy[3],A[3][3];
    double zzarea,xxbar,yybar;

    xx[0]      =   ca1/2.0;
    xx[1]      =   ca1+s1/2.0;
    xx[2]      =   ca1+s1+ca3/2.0;
    yy[0]      =   ca2/2.0;
    yy[1]      =   ca2+s2/2.0;
    yy[2]      =   ca2+s2+ca4/2.0;

    A[0][0]    =   ca2*ca1;
    A[0][1]    =   ca2*s1;
    A[0][2]    =   ca2*ca3;
    A[1][0]    =   s2*ca1;
    A[1][1]    =   s2*s1;
    A[1][2]    =   s2*ca3;
    A[2][0]    =   ca4*ca1;
    A[2][1]    =   ca4*s1;
    A[2][2]    =   ca4*ca3;

    xxbar      =   0.0;
    yybar      =   0.0;
    zzarea     =   (ca1+s1+ca3) * (ca2+s2+ca4);

    for(i=0;i<=2;i++)
    {
        for(j=0;j<=2;j++)
        {
```

```
        xxbar += xx[i]*A[j][i];
        yybar += yy[i]*A[i][j];
    }
}
bar[0] = xxbar/zzarea;
bar[1] = yybar/zzarea;
}
```